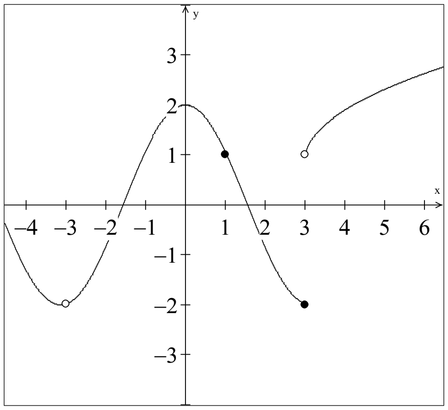
SATPREP

Assignment – LIMITS 2 (Explore One-Sided Limits Graphically & Analytically)

1. Given the graph of $f(x)$, determine the following.



a) $\lim_{x\to -3^-} f(x)$ b) $\lim_{x\to -3^+} f(x)$ c) $\lim_{x\to -3} f(x)$

d) $\lim_{x\to 1^+} f(x)$ e) $\lim_{x\to 1^-} f(x)$ f) $\lim_{x\to 1} f(x)$

g) $\lim_{x\to 3^-} f(x)$ h) $\lim_{x\to 3^+} f(x)$ i) $\lim_{x\to 3} f(x)$

j) $f(-3)$ k) $f(1)$ l) $f(3)$

2. Sketch each piecewise function below and determine, if it exists, the given limit. If the limit does not exist, provide an explanation.

a) $f(x) = \begin{cases} 2 & , \quad x < 1 \\ 3 & , \quad x = 1 \\ x+1, & \quad x > 1 \end{cases}$

Find $\lim_{x\to 1} f(x)$

b) $f(x) = \begin{cases} 4 - x^2 \;, & -2 < x \le 2 \\ x - 2 \;, & x > 2 \end{cases}$

Find $\lim_{x\to 2} f(x)$

c) $f(x) = \begin{cases} |x+2| + 1 \;, & x < -1 \\ -x + 1 & , -1 \le x \le 1 \\ x^2 - 2x + 2 \;, & x > 1 \end{cases}$

Find $\lim_{x\to 1} f(x)$

3. For each function below, determine, if it exists, the given limit. If the limit does not exist, provide an explanation.

a) $f(x) = \begin{cases} 2x-1, & x \le -2 \\ -x+2, & x > -2 \end{cases}$

Find $\lim_{x\to -2^+} f(x)$

b) $f(x) = \begin{cases} -x^2+4x-3, & x < 1 \\ x-7, & x \ge 1 \end{cases}$

Find $\lim_{x\to 1^-} f(x)$

c) $f(x) = \begin{cases} x^2-2x+1, & x < -1 \\ -\dfrac{x}{2}+\dfrac{7}{2}, & x \ge -1 \end{cases}$

Find $\lim_{x\to -1} f(x)$

d) $f(x) = \begin{cases} x+3, & x \in (-\infty, 0] \\ -x+2, & x \in (0,2) \\ (x-2)^2, & x \in [2, \infty) \end{cases}$

Find $\lim_{x\to 0} f(x)$ *and* $\lim_{x\to 2} f(x)$

e) $f(x) = \begin{cases} (x+1)^2 - 1, & -2 \le x < 0 \\ \dfrac{5}{4}\sin\left(\dfrac{\pi x}{2}\right), & 0 \le x < 2 \\ (x-3)^2 - 1, & 2 \le x \le 4 \end{cases}$

Find $\lim_{x\to 2} f(x)$

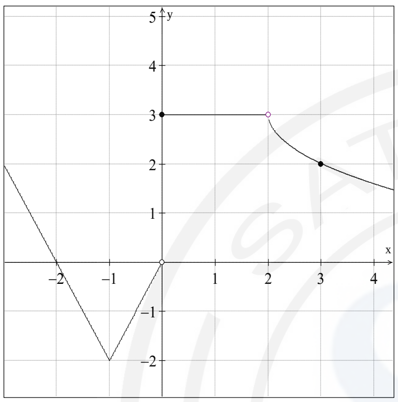
4. Rewrite $f(x)=\frac{3|-x+2|}{-x+2}$ as a piecewise function and then determine the following limits:

a) $\lim_{x\to 2^-} f(x)$ b) $\lim_{x\to 2^+} f(x)$ c) $\lim_{x\to 2} f(x)$

5. a) What is the possible defining function for the piecewise graph below?

b) i) Does the limit exist as x approaches 0?

ii) Does the limit exist as x approaches 2?



6. The function $f(t)$ is defined by $f(t)=\begin{cases}3t+b & t<1\\ 2-bt^2 & t\geq 1\end{cases}$ where b is a constant.

Compute $\lim_{t\to 1^+} f(t)$ *and* $\lim_{t\to 1^-} f(t)$ in terms of b.

Solution LIMITS 2 (Explore One-Sided Limits Graphically & Analytically)

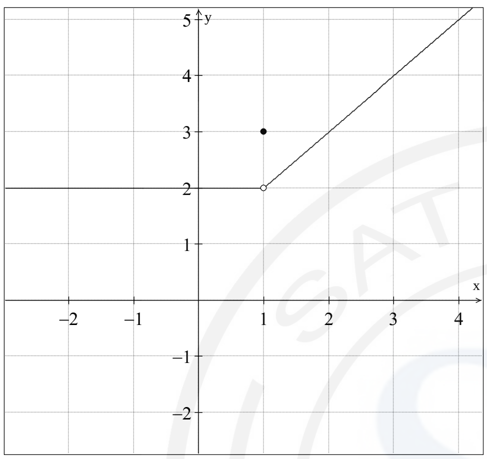
1. a) $\lim\limits_{x\to -3^-} f(x) = -2$ b) $\lim\limits_{x\to -3^+} f(x) = -2$ c) $\lim\limits_{x\to -3} f(x) = -2$

   d) $\lim\limits_{x\to 1^+} f(x) = 1$ e) $\lim\limits_{x\to 1^-} f(x) = 1$ f) $\lim\limits_{x\to 1} f(x) = 1$

   g) $\lim\limits_{x\to 3^-} f(x) = -2$ h) $\lim\limits_{x\to 3^+} f(x) = 1$ i) $\lim\limits_{x\to 3} f(x)$ *does not exist*

   j) $f(-3)$ = undefined k) $f(1)$ = 1 l) $f(3)$ = -2

2. a)

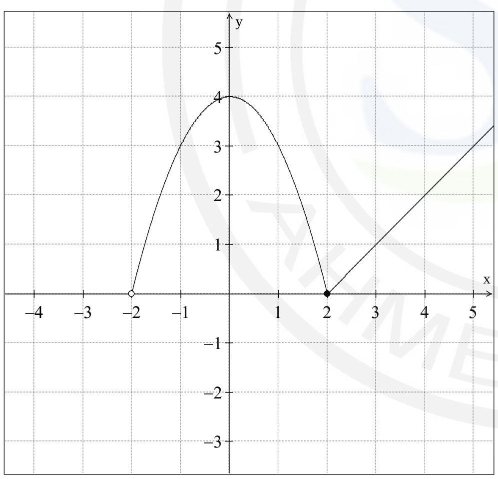


$\lim\limits_{x\to 1^-} f(x) = 2$ and $\lim\limits_{x\to 1^+} f(x) = 2$

Since the limits from the left and right of 1 are equal,

$$\lim_{x\to 1} f(x) = 2$$

b)

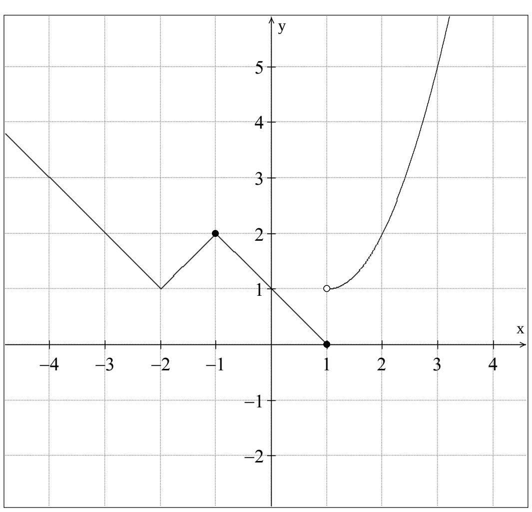


$\lim\limits_{x\to 2^-} f(x) = 0$ *and* $\lim\limits_{x\to 2^+} f(x) = 0$

Since the limits from the left and right of 2 are equal,

$$\lim_{x\to 2} f(x) = 0$$

c)



$\lim\limits_{x\to 1^-} f(x) = 0$ and $\lim\limits_{x\to 1^+} f(x) = 1$

Since the limits from the left and right of 1 are not equal,

$\lim\limits_{x\to 1} f(x)$ does not exist.

3\. a) $\lim_{x \to -2^+} f(x) = 4$ b) $\lim_{x \to 1^-} f(x) = 0$ c) $Since \lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = 4,$ $then \lim_{x \to -1} f(x) = 4$

d)

$\lim_{x \to 0^-} f(x) = 3$

$\lim_{x \to 0^+} f(x) = 2$

$Since \lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x),$

$then \lim_{x \to 0} f(x)\ DNE$

$\lim_{x \to 2^-} f(x) = 0$

$\lim_{x \to 2^+} f(x) = 0$

$Since \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = 0,$

$then \lim_{x \to 0} f(x) = 0$

e)

$\lim_{x \to 2^-} f(x) = 0$

$\lim_{x \to 2^+} f(x) = 0$

$Since \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = 0,$

$then \lim_{x \to 0} f(x) = 0$

4\.

$$f(x) = \begin{cases} \dfrac{3(-x+2)}{-x+2}, & x \le 2 \\ \dfrac{3(x-2)}{-x+2}, & x > 2 \end{cases} = \begin{cases} 3, & x \le 2 \\ -3, & x > 2 \end{cases}$$

$a) \lim_{x \to 2^-} f(x) = 3$ $b) \lim_{x \to 2^+} f(x) = -3$ $c)\ Since \lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x),\ then \lim_{x \to 2} f(x)\ DNE$

5\. a) A possible defining equation is $f(x) = \begin{cases} 2|x+1| - 2 & x < 0 \\ 3 & 0 \le x < 2 \\ -\sqrt{x-2} + 3 & x > 2 \end{cases}$

b) i) The limit as x approaches 0 does not exist since the limit from the left does not equal the limit from the right.

$\lim_{x \to 0^-} f(x) = 0$ $\lim_{x \to 0^+} f(x) = 3$ Therefore $\lim_{x \to 0} f(x)$ does not exist

ii) The limit as x approaches 2 equals 3 since the limit from the left equals the limit from the right.

$\lim_{x \to 2^-} f(x) = 3$ $\lim_{x \to 2^+} f(x) = 3$ Therefore $\lim_{x \to 2} f(x) = 3$

6\. $\lim_{t \to 1^+} f(t) = 2 - b$ and $\lim_{t \to 1^-} f(t) = 3 + b$

If the limit as x approaches 1 does exist, the value of b could be calculated.