SATPREP
Assignment : Optimization

1. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area? Ans 1200ft

2. We need to enclose a field with a rectangular fence. We have 500 ft of fencing material and a building is on one side of the field and so won't need any fencing. Determine the dimensions of the field that will enclose the largest area. Ans 250 ft.

3. We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost \$10/ft^2 and the material used to build the sides cost \$6/ft^2. If the box must have a volume of 50 ft^3 determine the dimensions that will minimize the cost to build the box. Ans \$637.60

4. We want to construct a box with a square base and we only have 10 m^2 of material to use in construction of the box. Assuming that all the material is used in the construction process determine the maximum volume that the box can have. Ans 2.1517 m^3

5. A manufacturer needs to make a cylindrical can that will hold 1.5 liters of liquid. Determine the dimensions of the can that will minimize the amount of material used in its construction. Ans 12.4070 cm

6. We have a piece of cardboard that is 14 in by 10 in and we're going to cut out the corners as shown below and fold up the sides to form a box, also shown below. Determine the height of the box that will give a maximum volume. Ans 120.1644 in^3.

7. A window is being built and the bottom is a rectangle and the top is a semicircle. If there is 12 m of framing materials what must the dimensions of the window be to let in the most light? Ans 3.3606 m

8. Determine the area of the largest rectangle that can be inscribed in a circle of radius 4. Ans $2\sqrt{2}$

9. Determine the points on $y = x^2 + 1$ that are closest to (0, 2).

Ans $\left(\frac{-1}{\sqrt{2}}, \frac{3}{2}\right) and \left(\frac{1}{\sqrt{2}}, \frac{3}{2}\right)$

10. A man launches his boat from point A on a bank of a straight river, 3 km wide, and wants to reach point B, 8 km downstream on the opposite bank, as quickly as possible. He could proceed in any of three ways:

    1. Row his boat directly across the river to point C and then run to B
    2. Row directly to B
    3. Row to some point D between C and B and then run to B

    If he can row 6 km/h and run 8 km/h, where should he land to reach B as soon as possible?

    Ans 3.4 km