SATPREP Name_____

Assignment: Optimization -2

Date_____

Solve each optimization problem.

1) A supermarket employee wants to construct an open-top box from a 10 by 16 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?

2) A cryptography expert is deciphering a computer code. To do this, the expert needs to minimize the product of a positive rational number and a negative rational number, given that the positive number is exactly 8 greater than the negative number. What final product is the expert looking for?

3) A rancher wants to construct two identical rectangular corrals using 400 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?

4) A geometry student wants to draw a rectangle inscribed in a semicircle of radius 8. If one side must be on the semicircle's diameter, what is the area of the largest rectangle that the student can draw?

5) Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 1372 ft³ of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?

6) A geometry student wants to draw a rectangle inscribed in the ellipse $x^2 + 4y^2 = 16$. What is the area of the largest rectangle that the student can draw?

7) An architect is designing a composite window by attaching a semicircular window on top of a rectangular window, so the diameter of the top window is equal to and aligned with the width of the bottom window. If the architect wants the perimeter of the composite window to be 10 ft, what dimensions should the bottom window be in order to create the composite window with the largest area?

8) Two vertical poles, one 8 ft high and the other 16 ft high, stand 45 feet apart on a flat field. A worker wants to support both poles by running rope from the ground to the top of each post. If the worker wants to stake both ropes in the ground at the same point, where should the stake be placed to use the least amount of rope?

Answers to Assignment: Optimization -2

1) V = the volume of the box x = the length of the sides of the squares
   Function to maximize: $V = (16 - 2x)(10 - 2x) \cdot x$ where $0 < x < 5$
   Sides of the squares: 2 in
2) P = the product of the two numbers x = the positive number
   Function to minimize: $P = x(x - 8)$ where $-\infty < x < \infty$
   Smallest product of the two numbers: -16
3) A = the total area of the two corrals x = the length of the non-adjacent sides of each corral
   Function to maximize: $A = 2x \cdot \dfrac{400 - 4x}{3}$ where $0 < x < 100$
   Dimensions of each corall: 50 ft (non-adjacent sides) by $\dfrac{200}{3}$ ft (adjacent sides)
4) A = the area of the rectangle x = half the base of the rectangle
   Function to maximize: $A = 2x\sqrt{8^2 - x^2}$ where $0 < x < 8$
   Area of largest rectangle: 64
5) A = the area of the glass x = the length of the sides of the square bottom
   Function to minimize: $A = x^2 + 4x \cdot \dfrac{1372}{x^2}$ where $0 < x < \infty$
   Dimensions of the aquarium: 14 ft by 14 ft by 7 ft tall
6) A = the area of the rectangle x = half the base of the rectangle
   Function to maximize: $A = 2x \cdot 2 \cdot \dfrac{\sqrt{16 - x^2}}{2}$ where $0 < x < 4$
   Area of largest rectangle: 16
7) $\dfrac{20}{4 + \pi}$ ft (width) by $\dfrac{10}{4 + \pi}$ ft (height)
8) 15 ft from the short pole (or 30 ft from the long pole)