SATPREP Name_____

Assignment:Related Rates -2

Date_____

Solve each related rate problem.

1) A 6 ft tall person is walking away from a 16 ft tall lamppost at a rate of 6 ft/sec. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 10 ft from the lamppost?

2) A 25 ft ladder is leaning against a wall and sliding towards the floor. The top of the ladder is sliding down the wall at a rate of $\frac{5}{y}$ ft/sec, where y is the distance from the top of the ladder to the floor. How fast is the base of the ladder sliding away from the wall when the base of the ladder is 24 ft from the wall?

3) An observer stands 800 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of $\frac{90000}{a}$ ft/sec, where a is the altitude of the rocket. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 600 ft from the ground?

4) A 25 ft ladder is leaning against a wall and sliding towards the floor. The foot of the ladder is sliding away from the base of the wall at a rate of $\frac{2}{x}$ ft/sec, where x is the distance from the base of the ladder to the wall. How fast is the top of the ladder sliding down the wall when the top of the ladder is 7 ft from the ground?

5) A 7 ft tall person is walking towards a 17 ft tall lamppost at a rate of $\frac{6}{x}$ ft/sec, where x is the distance from the person to the lamppost. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 16 ft from the lamppost?

6) A conical paper cup is 10 cm tall with a radius of 10 cm. The cup is being filled with water at a rate of $\frac{16\pi}{3}$ cm³/sec. How fast is the water level rising when the water level is 3 cm?

7) A spherical balloon is inflated at a rate of 36π cm³/sec. How fast is the radius of the balloon increasing when the radius is 5 cm?

8) A spherical balloon is inflated at a rate of $\frac{256\pi}{3V}$ cm³/sec, where V is the volume of the balloon. How fast is the radius of the balloon increasing when the radius is 3 cm?

9) A spherical balloon is deflated at a rate of $\frac{256\pi}{3}$ cm³/sec. At what rate is the radius of the balloon changing when the radius is 3 cm?

10) A 7 ft tall person is walking away from a 19 ft tall lamppost at a rate of $\frac{4}{x}$ ft/sec, where x is the distance from the person to the lamppost. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 15 ft from the lamppost?

11) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The radius of the spill increases at a rate of 5 m/min. How fast is the area of the spill increasing when the radius is 9 m?

12) A hypothetical square grows so that the length of its sides are increasing at a rate of 2 m/min. How fast is the area of the square increasing when the sides are 4 m each?

13) Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 9 cm/min. How fast is the area of the pool increasing when the radius is 2 cm?

14) A crowd gathers around a movie star, forming a circle. The radius of the crowd increases at a rate of 9 ft/sec. How fast is the area taken up by the crowd increasing when the radius is 9 ft?

15) A hypothetical square grows so that the length of its diagonals are increasing at a rate of 6 m/min. How fast is the area of the square increasing when the diagonals are 4 m each?

Answers to Assignment:Related Rates -2

1) x = distance from person to lamppost y = length of shadow t = time

Equation: $\frac{x+y}{16} = \frac{y}{6}$ Given rate: $\frac{dx}{dt} = 6$ Find: $\left.\frac{dy}{dt}\right|_{x=10}$

$\left.\frac{dy}{dt}\right|_{x=10} = \frac{3}{5} \cdot \frac{dx}{dt} = \frac{18}{5}$ ft/sec

2) x = horizontal distance from base of ladder to wall y = vertical distance from top of ladder to floor t = time

Equation: $x^2 + y^2 = 25^2$ Given rate: $\frac{dy}{dt} = -\frac{5}{y}$ Find: $\left.\frac{dx}{dt}\right|_{x=24}$

$\left.\frac{dx}{dt}\right|_{x=24} = -\frac{y}{x} \cdot \frac{dy}{dt} = \frac{5}{24}$ ft/sec

3) a = altitute of rocket z = distance from observer to rocket t = time

Equation: $a^2 + 640000 = z^2$ Given rate: $\frac{da}{dt} = \frac{90000}{a}$ Find: $\left.\frac{dz}{dt}\right|_{a=600}$

$\left.\frac{dz}{dt}\right|_{a=600} = \frac{a}{z} \cdot \frac{da}{dt} = 90$ ft/sec

4) x = horizontal distance from base of ladder to wall y = vertical distance from top of ladder to floor t = time

Equation: $x^2 + y^2 = 25^2$ Given rate: $\frac{dx}{dt} = \frac{2}{x}$ Find: $\left.\frac{dy}{dt}\right|_{y=7}$

$\left.\frac{dy}{dt}\right|_{y=7} = -\frac{x}{y} \cdot \frac{dx}{dt} = -\frac{2}{7}$ ft/sec, therefore: $\frac{2}{7}$ ft/sec down the wall

5) x = distance from person to lamppost y = length of shadow t = time

Equation: $\frac{x+y}{17} = \frac{y}{7}$ Given rate: $\frac{dx}{dt} = -\frac{6}{x}$ Find: $\left.\frac{dy}{dt}\right|_{x=16}$

$\left.\frac{dy}{dt}\right|_{x=16} = \frac{7}{10} \cdot \frac{dx}{dt} = -\frac{21}{80}$ ft/sec

6) V = volume of material in cone h = height t = time

Equation: $V = \frac{\pi h^3}{3}$ Given rate: $\frac{dV}{dt} = \frac{16\pi}{3}$ Find: $\left.\frac{dh}{dt}\right|_{h=3}$

$\left.\frac{dh}{dt}\right|_{h=3} = \frac{1}{\pi h^2} \cdot \frac{dV}{dt} = \frac{16}{27}$ cm/sec

7) V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dV}{dt} = 36\pi$ Find: $\left.\frac{dr}{dt}\right|_{r=5}$

$\left.\frac{dr}{dt}\right|_{r=5} = \frac{1}{4\pi r^2} \cdot \frac{dV}{dt} = \frac{9}{25}$ cm/sec

8) V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dV}{dt} = \frac{256\pi}{3V}$ Find: $\left.\frac{dr}{dt}\right|_{r=3}$

$$\left.\frac{dr}{dt}\right|_{r=3} = \frac{1}{4\pi r^2} \cdot \frac{dV}{dt} = \frac{16}{243\pi} \text{ cm/sec}$$

9) V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dV}{dt} = -\frac{256\pi}{3}$ Find: $\left.\frac{dr}{dt}\right|_{r=3}$

$$\left.\frac{dr}{dt}\right|_{r=3} = \frac{1}{4\pi r^2} \cdot \frac{dV}{dt} = -\frac{64}{27} \text{ cm/sec}$$

10) x = distance from person to lamppost y = length of shadow t = time

Equation: $\frac{x+y}{19} = \frac{y}{7}$ Given rate: $\frac{dx}{dt} = \frac{4}{x}$ Find: $\left.\frac{dy}{dt}\right|_{x=15}$

$$\left.\frac{dy}{dt}\right|_{x=15} = \frac{7}{12} \cdot \frac{dx}{dt} = \frac{7}{45} \text{ ft/sec}$$

11) A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dr}{dt} = 5$ Find: $\left.\frac{dA}{dt}\right|_{r=9}$

$$\left.\frac{dA}{dt}\right|_{r=9} = 2\pi r \cdot \frac{dr}{dt} = 90\pi \text{ m}^2\text{/min}$$

12) A = area of square s = length of sides t = time

Equation: $A = s^2$ Given rate: $\frac{ds}{dt} = 2$ Find: $\left.\frac{dA}{dt}\right|_{s=4}$

$$\left.\frac{dA}{dt}\right|_{s=4} = 2s \cdot \frac{ds}{dt} = 16 \text{ m}^2\text{/min}$$

13) A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dr}{dt} = 9$ Find: $\left.\frac{dA}{dt}\right|_{r=2}$

$$\left.\frac{dA}{dt}\right|_{r=2} = 2\pi r \cdot \frac{dr}{dt} = 36\pi \text{ cm}^2\text{/min}$$

14) A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dr}{dt} = 9$ Find: $\left.\frac{dA}{dt}\right|_{r=9}$

$$\left.\frac{dA}{dt}\right|_{r=9} = 2\pi r \cdot \frac{dr}{dt} = 162\pi \text{ ft}^2\text{/sec}$$

15) A = area of square x = length of diagonals t = time

Equation: $A = \frac{x^2}{2}$ Given rate: $\frac{dx}{dt} = 6$ Find: $\left.\frac{dA}{dt}\right|_{x=4}$

$$\left.\frac{dA}{dt}\right|_{x=4} = x \cdot \frac{dx}{dt} = 24 \text{ m}^2/\text{min}$$