SATPREP Name_____

Assignment : Optimisation

Date_____

Solve each optimization problem.

1) A cryptography expert is deciphering a computer code. To do this, the expert needs to minimize the product of a positive rational number and a negative rational number, given that the positive number is exactly 6 greater than the negative number. What final product is the expert looking for?

2) A company has started selling a new type of smartphone at the price of $\$140 - 0.1x$ where x is the number of smartphones manufactured per day. The parts for each smartphone cost \$60 and the labor and overhead for running the plant cost \$5000 per day. How many smartphones should the company manufacture and sell per day to maximize profit?

3) Which points on the graph of $y = 8 - x^2$ are closest to the point $(0, 4)$?

4) Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 256 ft^3 of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?

5) An architect is designing a composite window by attaching a semicircular window on top of a rectangular window, so the diameter of the top window is equal to and aligned with the width of the bottom window. If the architect wants the perimeter of the composite window to be 12 ft, what dimensions should the bottom window be in order to create the composite window with the largest area?

6) Two vertical poles, one 6 ft high and the other 18 ft high, stand 32 feet apart on a flat field. A worker wants to support both poles by running rope from the ground to the top of each post. If the worker wants to stake both ropes in the ground at the same point, where should the stake be placed to use the least amount of rope?

7) Which points on the graph of $y = 7 - x^2$ are closest to the point $(0, 3)$?

8) A cryptography expert is deciphering a computer code. To do this, the expert needs to minimize the product of a positive rational number and a negative rational number, given that the positive number is exactly 8 greater than the negative number. What final product is the expert looking for?

9) Two vertical poles, one 8 ft high and the other 16 ft high, stand 18 feet apart on a flat field. A worker wants to support both poles by running rope from the ground to the top of each post. If the worker wants to stake both ropes in the ground at the same point, where should the stake be placed to use the least amount of rope?

10) A company has started selling a new type of smartphone at the price of $\$120 - 0.1x$ where x is the number of smartphones manufactured per day. The parts for each smartphone cost \$50 and the labor and overhead for running the plant cost \$4000 per day. How many smartphones should the company manufacture and sell per day to maximize profit?

Answers to Assignment : Optimisation

1) P = the product of the two numbers x = the positive number
   Function to minimize: $P = x(x - 6)$ where $-\infty < x < \infty$
   Smallest product of the two numbers: -9
2) p = the profit per day x = the number of items manufactured per day
   Function to maximize: $p = x(140 - 0.1x) - (60x + 5000)$ where $0 \le x < \infty$
   Optimal number of smartphones to manufacture per day: 400
3) d = the distance from point $(0, 4)$ to a point on the parabola x = the x-coord. of a point on the parabola
   Function to minimize: $d = \sqrt{x^2 + (8 - x^2 - 4)^2}$ where $-\infty < x < \infty$
   Points on the parabola that are closest to the point $(0, 4)$: $\left(-\frac{\sqrt{14}}{2}, \frac{9}{2}\right), \left(\frac{\sqrt{14}}{2}, \frac{9}{2}\right)$
4) A = the area of the glass x = the length of the sides of the square bottom
   Function to minimize: $A = x^2 + 4x \cdot \frac{256}{x^2}$ where $0 < x < \infty$
   Dimensions of the aquarium: 8 ft by 8 ft by 4 ft tall
5) A = the area of the composite window x = the width of the bottom window = the diameter of the top window
   Function to maximize: $A = x\left(\frac{12}{2} - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{1}{2}\pi \cdot \left(\frac{x}{2}\right)^2$ where $0 < x < \frac{48}{4 + \pi}$
   Dimensions of the bottom window: $\frac{24}{4 + \pi}$ ft (width) by $\frac{12}{4 + \pi}$ ft (height)
6) L = the total length of rope x = the horizontal distance from the short pole to the stake
   Function to minimize: $L = \sqrt{x^2 + 6^2} + \sqrt{(32 - x)^2 + 18^2}$ where $0 \le x \le 32$
   Stake should be placed: 8 ft from the short pole (or 24 ft from the long pole)
7) $\left(-\frac{\sqrt{14}}{2}, \frac{7}{2}\right), \left(\frac{\sqrt{14}}{2}, \frac{7}{2}\right)$
8) -16
9) 6 ft from the short pole (or 12 ft from the long pole)
10) 350