SATPREP Name_____

Assignment : Related rates

Date_____

Solve each related rate problem.

1) A hypothetical square grows so that the length of its diagonals are increasing at a rate of 2 m/min. How fast is the area of the square increasing when the diagonals are 9 m each?

2) A hypothetical square grows so that the length of its sides are increasing at a rate of 5 m/min. How fast is the area of the square increasing when the sides are 8 m each?

3) A conical paper cup is 10 cm tall with a radius of 10 cm. The bottom of the cup is punctured so that the water leaks out at a rate of $\frac{9\pi}{4}$ cm³/sec. At what rate is the water level changing when the water level is 2 cm?

4) A hypothetical cube grows at a rate of 8 m^3/min. How fast are the sides of the cube increasing when the sides are 8 m each?

5) An observer stands 300 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of $\frac{90000}{a}$ ft/sec, where a is the altitude of the rocket. Assume the scenario can be modeled as a right triangle. How fast is the angle of elevation (in radians/sec) from the observer to rocket changing when the rocket is 400 ft from the ground?

6) A 5 ft tall person is walking away from a 19 ft tall lamppost at a rate of 5 ft/sec. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 17 ft from the lamppost?

7) An observer stands 400 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of $\frac{90000}{a}$ ft/sec, where a is the altitude of the rocket. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 300 ft from the ground?

8) A 5 ft tall person is walking towards a 20 ft tall lamppost at a rate of 4 ft/sec. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 17 ft from the lamppost?

9) A hypothetical cube grows at a rate of 64 m^3/min. How fast are the sides of the cube increasing when the sides are 7 m each?

10) A spherical balloon is inflated so that its radius increases at a rate of 4 cm/sec. How fast is the volume of the balloon increasing when the radius is 6 cm?

Answers to Assignment : Related rates

1) A = area of square x = length of diagonals t = time

Equation: $A = \dfrac{x^2}{2}$ Given rate: $\dfrac{dx}{dt} = 2$ Find: $\left.\dfrac{dA}{dt}\right|_{x=9}$

$$\left.\frac{dA}{dt}\right|_{x=9} = x \cdot \frac{dx}{dt} = 18 \text{ m}^2/\text{min}$$

2) A = area of square s = length of sides t = time

Equation: $A = s^2$ Given rate: $\dfrac{ds}{dt} = 5$ Find: $\left.\dfrac{dA}{dt}\right|_{s=8}$

$$\left.\frac{dA}{dt}\right|_{s=8} = 2s \cdot \frac{ds}{dt} = 80 \text{ m}^2/\text{min}$$

3) V = volume of material in cone h = height t = time

Equation: $V = \dfrac{\pi h^3}{3}$ Given rate: $\dfrac{dV}{dt} = -\dfrac{9\pi}{4}$ Find: $\left.\dfrac{dh}{dt}\right|_{h=2}$

$$\left.\frac{dh}{dt}\right|_{h=2} = \frac{1}{\pi h^2} \cdot \frac{dV}{dt} = -\frac{9}{16} \text{ cm/sec}$$

4) V = volume of cube s = length of sides t = time

Equation: $V = s^3$ Given rate: $\dfrac{dV}{dt} = 8$ Find: $\left.\dfrac{ds}{dt}\right|_{s=8}$

$$\left.\frac{ds}{dt}\right|_{s=8} = \frac{1}{3s^2} \cdot \frac{dV}{dt} = \frac{1}{24} \text{ m/min}$$

5) a = altitude of rocket θ = angle of elevation from observer to rocket t = time

Equation: $\tan\theta = \dfrac{a}{300}$ Given rate: $\dfrac{da}{dt} = \dfrac{90000}{a}$ Find: $\left.\dfrac{d\theta}{dt}\right|_{a=400}$

$$\left.\frac{d\theta}{dt}\right|_{a=400} = \frac{1}{300\sec^2\theta} \cdot \frac{da}{dt} = \frac{27}{100} \text{ radians/sec}$$

6) x = distance from person to lamppost y = length of shadow t = time

Equation: $\dfrac{x+y}{19} = \dfrac{y}{5}$ Given rate: $\dfrac{dx}{dt} = 5$ Find: $\left.\dfrac{dy}{dt}\right|_{x=17}$

$$\left.\frac{dy}{dt}\right|_{x=17} = \frac{5}{14} \cdot \frac{dx}{dt} = \frac{25}{14} \text{ ft/sec}$$

7) a = altitute of rocket z = distance from observer to rocket t = time

Equation: $a^2 + 160000 = z^2$ Given rate: $\dfrac{da}{dt} = \dfrac{90000}{a}$ Find: $\left.\dfrac{dz}{dt}\right|_{a=300}$

$$\left.\frac{dz}{dt}\right|_{a=300} = \frac{a}{z} \cdot \frac{da}{dt} = 180 \text{ ft/sec}$$

8) x = distance from person to lamppost y = length of shadow t = time

Equation: $\frac{x+y}{20} = \frac{y}{5}$ Given rate: $\frac{dx}{dt} = -4$ Find: $\left.\frac{dy}{dt}\right|_{x=17}$

$\left.\frac{dy}{dt}\right|_{x=17} = \frac{1}{3} \cdot \frac{dx}{dt} = -\frac{4}{3}$ ft/sec

9) V = volume of cube s = length of sides t = time

Equation: $V = s^3$ Given rate: $\frac{dV}{dt} = 64$ Find: $\left.\frac{ds}{dt}\right|_{s=7}$

$\left.\frac{ds}{dt}\right|_{s=7} = \frac{1}{3s^2} \cdot \frac{dV}{dt} = \frac{64}{147}$ m/min

10) V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dr}{dt} = 4$ Find: $\left.\frac{dV}{dt}\right|_{r=6}$

$\left.\frac{dV}{dt}\right|_{r=6} = 4\pi r^2 \cdot \frac{dr}{dt} = 576\pi$ cm³/sec