SATPREP

The fundamental Counting Principles.

There are two principles of counting that will enable you to find the number of outcomes without listing and counting each one.

The Multiplication Principle

If there are n_1 ways to choose a first item, n_2 ways to choose a second item, n_3 ways to choose a third item, and so on, the the total number of ways to choose all the items is given by the product
$\mathbf{n_1 \times n_2 \times n_3 \times \ldots}$

Example
The Shoe store sells 9 different styles of running shoes, each available in 2 colors. How many combinations of color and style are there?

Choose :
style and color = choices
$\underline{9} \times \underline{2} = 18$

The Addition Principle

If one outcome can occur in *r* ways, and a second mutually exclusive outcome, can occur in *s* ways, then there are a total of (r + s) possible outcomes.

You can order one item from a list of 5 hamburgers and 3 pizzas. How many choices do you have?
Choose:
burger or pizza = choices
5 + 3 = 8

Permutations Permutation - an arrangement of objects in which order is important

The letters a, b, and c can be arranged in six different orders: abc /acb/ bac /bca cab/cba
Each of these arrangements is called a permutation of the letters a, b, c

Find the number of permutations of the letters p, q, r, and s.

$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 24$ ways

Factorial (n!) - $\mathbf{n! = n (n - 1)(n - 2)...(2)(1)}$

The number of permutations of n objects is n! (! is the mathematical symbol for factorial)

Find 5!

5! =

Simplifying of factorials in fractions can make the problem easier:

$$\frac{7!}{4!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$$

$$= 7 \times 6 \times 5$$
$$= 210$$

The number of permutations of n objects taken n at a time is ${}_nP_n$.

$\mathbf{{}_nP_n = n\,!}$

$0! = 1$

${}_6P_6$ ${}_6P_6 = 6! = 720$

The symbol nPr is used to show the total number of n items taken r at a time.

An easier way to think of this is to look at boxes, n item being placed in r boxes.

$\boxed{1}$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	…..	$\boxed{r}$	- Number of Boxes
n	n-1	n-2	n-3		$n - r + 1$	- Number Of Arrangements

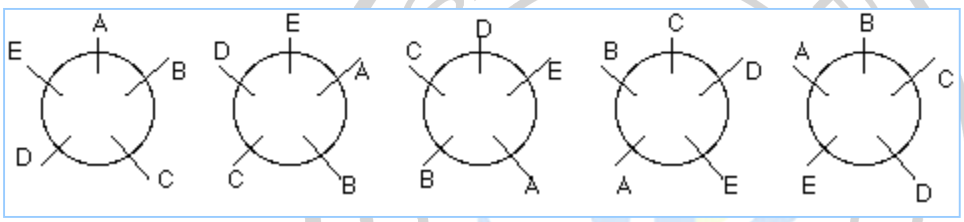
The following formula is the general equation to find the permutation:

$${}_{n}\mathbf{P}_{r} = \frac{\mathbf{n!}}{\mathbf{(n\text{-}r)!}}$$

* **Important note**: Calculators that have the $\boxed{_n\mathrm{P}_r}$ key are enabled to calculate permutations. To calculate ${}_8P_4$, press 8 then the key, then 4. Your answer will be 1680.

Circular Permutation

A circular permutation is the number of different ways you can arrange items in a circle.
In how many ways can 5 different people be seated around a circular table?
If it were 5 people in a straight line: 5! = 120 arrangements
However, around a circular table the 5 arrangements may look different, but the relative position of the people has not changed.



The total number of arrangements is:
5! / 5 = 4! = 24

${}_nP_n / n = (n - 1)!$

A jeweler has 15 different sized pearls to string on a circular band.
a) In how many ways can this be done?

${}_{15}P_{15} / 15 = 14! = 8.71 \times 10^{10}$

b) If three of the pearls are identical in size, then what is the total number of arrangements?
${}_{15}P_{15} / 15\text{x}3!\ 14! / 6 = 1.45 \times 10^{10}$

Distinguishable Permutations

If, in a collection of n objects, n_1 are alike, n_2 are alike of another kind, … ,n_k are alike of a further kind, and

$$\boldsymbol{n = n_1 + n_2 + \ldots\ldots + n_k,}$$

then the number of distinguishable permutations of the n object is

$$\frac{n!}{n_1!n_2!..n_k!}$$

An example we could use could be the word MISSISSIPPI. We want to find the number of distinguishable permutations of the letters.

There are a collection of eleven objects in which four are of one kind (S), four are of another kind(I), two are a third kind (P), and one is of a fourth kind (M). Using the equation we can establish the answer to be 34 650.

$$\frac{11!}{4!4!2!1!} = 34\,650$$

Combinations

Combination –Is a selection in which the order is not important (groupings)

How many groups of three letters are there if choosing from the letters;
A, B, C, D, E?

There are ${}_5P_3 = 5! / (5-3)! = 60$ ways to arrange 3 letters. But for each arrangement of three letters, 6 arrangements are the same group.

For example, arrangements using the letters A B C are ABC, ACB, BAC, BCA, CAB, and CBA. The number of arrangements is 3! = 6. Note however that all 6 arrangements are the same group. We must, then, divide the 60 ways to arrange the letters by 6 or 3! to find the number of groups.

The theorem on combinations states that the number of combinations or r elements can be obtained from a set of n elements:

$${}_nC_r = \frac{n!}{(n-r)!\,r!} \qquad 1 \le r \le n.$$

Combinations can be used with probability as well as permutations.

If 3 coins were tossed simultaneously and you want to find the number of possible outcomes. You can look at the sample space: HHH, HHT, HTT, THH, HTH, THT, TTH, and TTT there are 8 possible outcomes.
To find the probability of tossing three heads you can use the combination formula and the probability formula.

$$P\text{ (3 heads)} = \frac{\text{number of favorable outcomes}}{\text{number of all possible outcomes}}$$

$$P\text{ (3heads)} = \frac{{}_3C_3}{8} \quad \leftarrow \text{(from the three heads, you choose three)} \quad = \frac{1}{8}$$

Keep in note that if $r = n$, then the formula for ${}_nC_r$ becomes

$$\boldsymbol{nCn = \frac{n!}{(n-n)!n!} = \frac{n!}{0!n!} = 1}$$