SAT PREP

Complex No and Powers of *i*

The Number *i* - i is the unique number for which $i = \sqrt{-1}$ and $i^2 = -1$.

Imaginary Number – any number that can be written in the form $a + bi$, where a and b are real numbers and $b \neq 0$.

Complex Number – any number that can be written in the form $a + bi$, where a and b are real numbers. (Note: a and b both **can** be 0.) *The union of the set of all imaginary numbers and the set of all real numbers is the set of complex numbers.*

Addition / Subtraction - Combine like terms (i.e. the real parts with real parts and the imaginary parts with imaginary parts).

Example - $(2 - 3i) - (4 - 6i) = 2 - 3i - 4 + 6i$

$= -2 + 3i$

Multiplication - When multiplying square roots of negative real numbers, begin by expressing them in terms of i.

Example - $\sqrt{-4} \cdot \sqrt{-8} = \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{-1} \cdot \sqrt{8}$

$= i \cdot 2 \cdot i \cdot 2\sqrt{2}$

$= i^2 \cdot 4\sqrt{2}$

$= (-1) \cdot 4\sqrt{2}$

$= -4\sqrt{2}$

Note: The answer is not $+4\sqrt{2}$, which could be calculated erroneously if the radicands were simply multiplied as
$\sqrt{-4} \cdot \sqrt{-8} \neq \sqrt{(-4)(-8)} \neq \sqrt{32}$

Multiplication (Cont'd) – When multiplying two complex numbers, begin by F O I L ing them together and then simplify.

Example - $(2 + 3i) \cdot (8 - 7i)$ = $16 - 14i + 24i - 21i^2$

= $16 + 10i - 21i^2$

= $16 + 10i - 21\,(-1)$

= $16 + 10i + 21$

= $37 + 10i$

Division – When dividing by a complex number, multiply the top and bottom by the complex conjugate of the denominator. Then F O I L the top and the bottom and simplify. The answer should be written in standard form ($a + bi$.)

Example - $\frac{2+3i}{1-5i}$ = $\frac{(2+3i)}{(1-5i)} \cdot \frac{(1+5i)}{(1+5i)}$ (Multiply by complex conjugate)

= $\frac{2+10i+3i+15\,i^2}{1+5i-5i-25\,i^2}$ = $\frac{2+13i+15(-1)}{1-25(-1)}$

= $\frac{2+13i-15}{1+25}$ = $\frac{-13+13i}{26}$

= $\frac{-1+i}{2}$ = $\frac{-1}{2} + \frac{1}{2}\,i$

Example - $\frac{14}{i}$ = $\frac{14}{i} \cdot \frac{-i}{-i}$ (Multiply by complex conjugate)

= $\frac{-14\,i}{-i^2}$ = $\frac{-14i}{-(-1)}$

= $\frac{-14i}{1}$ = $-14\,i$

Powers of i – Given a number, i^n, the number can be simplified by using the following chart.

i^n	Is Equivalent to…	Because…
i^0	1	$a\ number\ raised\ to\ the\ 0\ power\ is\ 1$
i^1	i	$a\ number\ raised\ to\ the\ 1\ power\ is\ that\ same\ number$
i^2	(-1)	$i^2 = -1$ *(definition of i)*
i^3	$-i$	$i^3 = i^2 \cdot i = (-1) \cdot i = -i$
i^4	1	$i^4 = i^2 \cdot i^2 = (-1) \cdot (-1) = 1$
i^5	i	$i^5 = i^4 \cdot i = (1) \cdot i = i$

Because the powers of i will cycle through $1, i, -1, and - i$, this repeating pattern of four terms can be used to simplify i^n.

Example - Simplify i^{25}

Step 1 - Divide 25 *(the power)* by 4.

$\frac{25}{4}$ = quotient of 6 with a remainder of 1

Step 2 - Note the quotient (i.e. 6) and the remainder (i.e. 1).

Step 3 - Rewrite the problem.

$$i^{25} = (i^4)^{quotient} \cdot i^{remainder} = (i^4)^6 \cdot i^1$$

Step 4 - Simplify by recalling that $i^4 = 1$

$$(i^4)^6 \cdot i^1 = (1)^6 \cdot i^1 = 1 \cdot i = i$$

Note: Because the powers of i cycle through $1, i, -1, and\ i$, these types of problems can always be simplified by noting what the remainder is in step 2 above. In fact, the problem can be re-written as…

$i^n = i^{remainder}$ (Divide n by 4 and determine the remainder).

The remainder will always be either 0, 1, 2, or 3.

Example - Simplify i^{59}

$i^{59} = i^3$ *(because $\frac{59}{4}$ has a remainder of 3.)*

So, $i^{59} = i^3 = -i$

Imaginary and Complex Numbers Practice

Simplify:

1) (4 + 2i) + (-3 – 5i)
2) (-3 + 4i) – (5 + 2i)
3) (-8 – 7i) – (5 – 4i)
4) (3 – 2i)(5 + 4i)
5) $(3 - 4i)^2$
6) $(3 - 2i)(5 + 4i) - (3 - 4i)^2$
7) Write $\frac{3+7i}{5-3i}$ in standard form
8) Simplify i^{925}
9) Simplify i^{460}
10) Write $\frac{1-4i}{5+2i}$ in standard form
11) $\sqrt{-16}$
12) $\sqrt{-8}$
13) $\sqrt{-6}\,\sqrt{-6}$
14) 4 + $\sqrt{-25}$
15) $\frac{6-\sqrt{-8}}{-2}$

Answers:

(1) 1 – 3i	(2) -8 + 2i	(3) -13 – 3i	(4) 23 + 2i
(5) -7 – 24i	(6) 30 + 26i	(7) $\frac{-3}{17}+\frac{22}{17}i$	(8) i
(9) 1	(10) $\frac{-2}{29}-\frac{22}{29}i$	(11) 4i	(12) $2\sqrt{2}$ i
(13) -6	(14) 4 +5i	(15) -3 + $\sqrt{2}$ i	