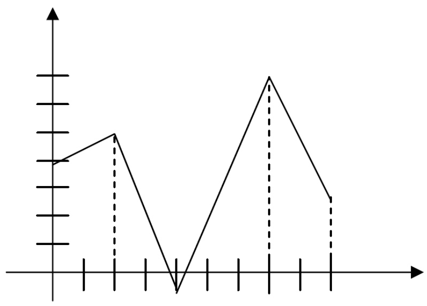
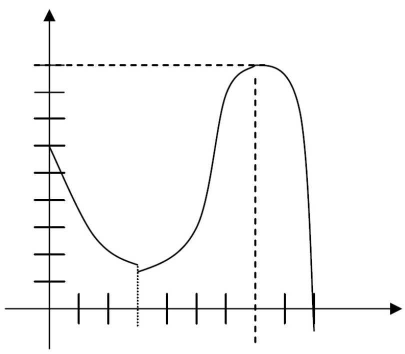
SATPREP

Assignment : Extrema, Rolle's and MVT

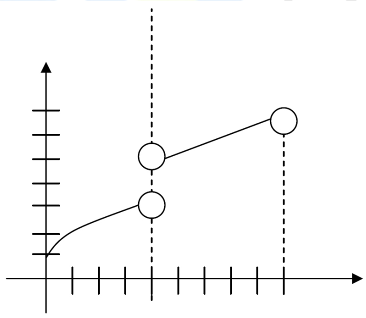
In problems #1–3, find the coordinates of all absolute and relative extrema.



1. Continuous on [0,9]



2. Continuous on [0,9]



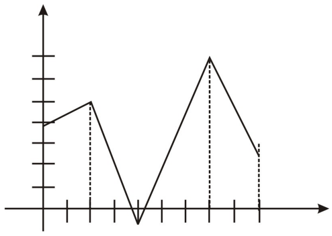
3. Continuous on [0, 4) U (4, 9)

In problems #4–6, find the absolute extrema on the closed interval indicated, and sketch the graph.

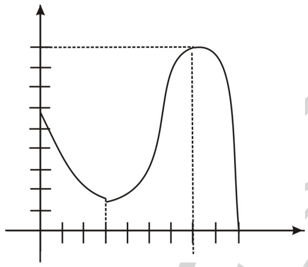
4. $f(x)=-x^2-6x+4$, [-4,1]
5. $f(x) = x^3-x^4$, [0,2]
6. $f(x) = x^2 + \dfrac{4}{x^2}$, [-2,0]
7. Verify Rolle's Theorem for $f(x) = 3x^3 - 12x$ by finding values of x for which $f(x) = 0$ and $f'(x) = 0$.
8. Verify Rolle's Theorem for $f(x) = x^2 - \dfrac{2}{x-1}$. [-1,0]
9. Verify that the Mean Value Theorem works for $f(x) = \dfrac{x+2}{x}$ on the interval [1, 2].
10. Prove that the equation $x^3+a_1x^2+a_2x=0$ has a positive root at x = r, and that the equation $3x^2+2a_1x+a_2=0$ has a positive root less than r.

Review Answers

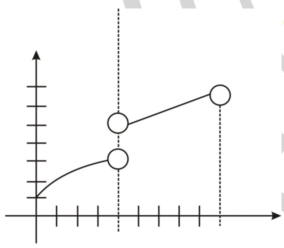
1. Absolute maximum (7, 7); absolute minimum (4, -1); relative maximum (2, 5) and (7, 7); relative minimum (0, 4), (4, -1), and (9, 3).



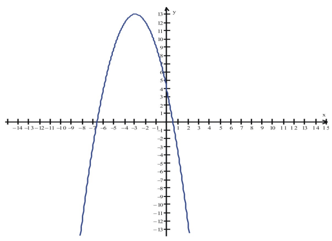
2. Absolute maximum (7, 9); absolute minimum (9, 0); relative maximum (0, 6) and (7, 9); relative minimum (3, 1.5) and (9, 0).



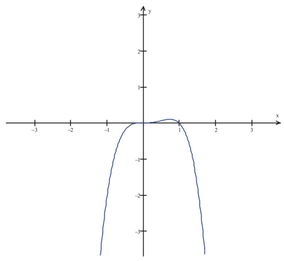
3. Absolute minimum (0, 1); relative minimum (0, 1); there is no max since the function is not continuous on a closed interval.



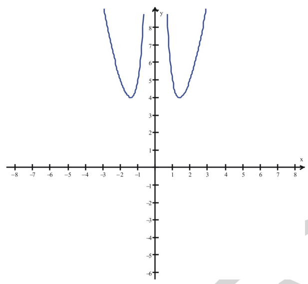
4. Absolute maximum (-3, 13); absolute minimum (1, -3)



5. Absolute maximum $\left(\frac{3}{4}, f\left(\frac{3}{4}\right) \approx .1055\right)$, absolute minimum (2, -8)



6. Absolute minimum at $(-\sqrt{2}, 4)$



7. $f(x) = 0$ at $x = 0, \pm 2$. $f'(x) = 0, x = \pm\frac{2\sqrt{3}}{3}$; by Rolle's Theorem, there is a critical value in each of the intervals (-2, 0) and (0, 2), and we found those to be $x = \pm\sqrt{2}$.
8. $f(x) = 2$ at $x = -1, 0$. $f'(x) = 0$ at $x \approx -0.84$; by Rolle's Theorem, there is a critical value in the interval (-1, 0) and we found it to be $x \approx -0.48$.
9. Need to find $c \in (1, 2)$ such that $f(2) - f(1) = (2 - 1)f'(c)$; $c = \sqrt{2}$.
10. Let $f(x) = x^3 + a_1x^2 + a_2x$. Observe that $f(x) = f(r) = 0$. By Rolle's Theorem, there must exist $c \in (0, r)$ such that $f'(0) = 0$.