SATPREP

Assignment : Integration by Substitution -4

Find the integrals of the functions in Exercises 1 to 22:

1. $\sin^2 (2x + 5)$
2. $\sin 3x \cos 4x$
3. $\cos 2x \cos 4x \cos 6x$
4. $\sin^3 (2x + 1)$
5. $\sin^3 x \cos^3 x$
6. $\sin x \sin 2x \sin 3x$
7. $\sin 4x \sin 8x$
8. $\dfrac{1 - \cos x}{1 + \cos x}$
9. $\dfrac{\cos x}{1 + \cos x}$
10. $\sin^4 x$
11. $\cos^4 2x$
12. $\dfrac{\sin^2 x}{1 + \cos x}$
13. $\dfrac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$
14. $\dfrac{\cos x - \sin x}{1 + \sin 2x}$
15. $\tan^3 2x \sec 2x$
16. $\tan^4 x$
17. $\dfrac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$
18. $\dfrac{\cos 2x + 2\sin^2 x}{\cos^2 x}$
19. $\dfrac{1}{\sin x \cos^3 x}$
20. $\dfrac{\cos 2x}{\left(\cos x + \sin x \right)^2}$
21. $\sin^{-1} (\cos x)$
22. $\dfrac{1}{\cos (x - a) \cos (x - b)}$

Answer

1. $\dfrac{x}{2}-\dfrac{1}{8}\sin(4x+10)+C$

2. $-\dfrac{1}{14}\cos 7x+\dfrac{1}{2}\cos x+C$

3. $\dfrac{1}{4}\left[\dfrac{1}{12}\sin 12x+x+\dfrac{1}{8}\sin 8x+\dfrac{1}{4}\sin 4x\right]+C$

4. $-\dfrac{1}{2}\cos(2x+1)+\dfrac{1}{6}\cos^3(2x+1)+C$

5. $\dfrac{1}{6}\cos^6 x-\dfrac{1}{4}\cos^4 x+C$

6. $\dfrac{1}{4}\left[\dfrac{1}{6}\cos 6x-\dfrac{1}{4}\cos 4x-\dfrac{1}{2}\cos 2x\right]+C$

7. $\dfrac{1}{2}\left[\dfrac{1}{4}\sin 4x-\dfrac{1}{12}\sin 12x\right]+C$

8. $2\tan\dfrac{x}{2}-x+C$

9. $x-\tan\dfrac{x}{2}+C$

10. $\dfrac{3x}{8}-\dfrac{1}{4}\sin 2x+\dfrac{1}{32}\sin 4x+C$

11. $\dfrac{3x}{8}+\dfrac{1}{8}\sin 4x+\dfrac{1}{64}\sin 8x+C$

12. $x-\sin x+C$

13. $2(\sin x+x\cos\alpha)+C$

14. $-\dfrac{1}{\cos x+\sin x}+C$

15. $\dfrac{1}{6}\sec^3 2x-\dfrac{1}{2}\sec 2x+C$

16. $\dfrac{1}{3}\tan^3 x-\tan x+x+C$

17. $\sec x-\operatorname{cosec} x+C$

18. $\tan x+C$

19. $\log|\tan x|+\dfrac{1}{2}\tan^2 x+C$

20. $\log|\cos x+\sin x|+C$

21. $\dfrac{\pi x}{2}-\dfrac{x^2}{2}+C$

22. $\dfrac{1}{\sin(a-b)}\log\left|\dfrac{\cos(x-a)}{\cos(x-b)}\right|+C$