## **SATPREP**

## Assignment: Integration by Substitution

Integrate the functions in Exercises 1 to 37:

1. 
$$\frac{2x}{1+x^2}$$

$$2. \ \frac{(\log x)^2}{x}$$

3. 
$$\frac{1}{x + x \log x}$$

4. 
$$\sin x \sin (\cos x)$$

5. 
$$\sin(ax+b)\cos(ax+b)$$

6. 
$$\sqrt{ax+b}$$

7. 
$$x\sqrt{x+2}$$

8. 
$$x\sqrt{1+2x^2}$$

9. 
$$(4x+2)\sqrt{x^2+x+1}$$
 10.  $\frac{1}{x-\sqrt{x}}$ 

$$10. \quad \frac{1}{x - \sqrt{x}}$$

11. 
$$\frac{x}{\sqrt{x+4}}, x > 0$$

12. 
$$(x^3-1)^{\frac{1}{3}}x^5$$

13. 
$$\frac{x^2}{(2+3x^3)^3}$$

13. 
$$\frac{x^2}{(2+3x^3)^3}$$
 14.  $\frac{1}{x(\log x)^m}, x > 0, m \ne 1$ 

15. 
$$\frac{x}{9-4x^2}$$

16. 
$$e^{2x+3}$$

17. 
$$\frac{x}{e^{x^2}}$$

18. 
$$\frac{e^{\tan^{-1}x}}{1+x^2}$$
 19.  $\frac{e^{2x}-1}{e^{2x}+1}$ 

19. 
$$\frac{e^{2x}-1}{e^{2x}+1}$$

$$20. \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$$

**21.** 
$$tan^2 (2x - 3)$$

**22.** 
$$\sec^2(7-4x)$$

23. 
$$\frac{\sin^{-1} x}{\sqrt{1-x^2}}$$

24. 
$$\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$$

24. 
$$\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$$
 25.  $\frac{1}{\cos^2 x (1 - \tan x)^2}$ 

$$26. \ \frac{\cos\sqrt{x}}{\sqrt{x}}$$

$$27. \sqrt{\sin 2x} \cos 2x$$

$$28. \quad \frac{\cos x}{\sqrt{1+\sin x}}$$

**29.** 
$$\cot x \log \sin x$$

$$30. \ \frac{\sin x}{1 + \cos x}$$

31. 
$$\frac{\sin x}{(1+\cos x)^2}$$
 32.  $\frac{1}{1+\cot x}$ 

32. 
$$\frac{1}{1+\cot x}$$

33. 
$$\frac{1}{1-\tan x}$$

$$34. \quad \frac{\sqrt{\tan x}}{\sin x \cos x}$$

34. 
$$\frac{\sqrt{\tan x}}{\sin x \cos x}$$
 35. 
$$\frac{(1 + \log x)^2}{x}$$

36. 
$$\frac{(x+1)(x+\log x)^2}{x}$$
 37.  $\frac{x^3\sin(\tan^{-1}x^4)}{1+x^8}$ 

37. 
$$\frac{x^3 \sin(\tan^{-1}x^4)}{1+x^8}$$

## Answer:

1. 
$$\log(1+x^2) + C$$

1. 
$$\log (1 + x^2) + C$$
 2.  $\frac{1}{3} (\log |x|)^3 + C$  3.  $\log |1 + \log x| + C$ 

$$3. \quad \log |1 + \log x| + C$$

$$4. \quad \cos(\cos x) + C$$

**4.** 
$$\cos(\cos x) + C$$
 **5.**  $-\frac{1}{4a}\cos 2(ax+b) + C$ 

6. 
$$\frac{2}{3a}(ax+b)^{\frac{3}{2}}+C$$

6. 
$$\frac{2}{3a}(ax+b)^{\frac{3}{2}}+C$$
 7.  $\frac{2}{5}(x+2)^{\frac{5}{2}}-\frac{4}{3}(x+2)^{\frac{3}{2}}+C$ 

8. 
$$\frac{1}{6}(1+2x^2)^{\frac{3}{2}} + C$$

8. 
$$\frac{1}{6}(1+2x^2)^{\frac{3}{2}} + C$$
 9.  $\frac{4}{3}(x^2+x+1)^{\frac{3}{2}} + C$  10.  $2\log\left|\sqrt{x}-1\right| + C$ 

11. 
$$\frac{2}{3}\sqrt{x+4}(x-8)+C$$

11. 
$$\frac{1}{3}\sqrt{x^{3}+4(x^{3}-6)+C}$$
  
12.  $\frac{1}{7}(x^{3}-1)^{\frac{7}{3}} + \frac{1}{4}(x^{3}-1)^{\frac{4}{3}} + C$   
13.  $-\frac{1}{18(2+3x^{3})^{2}} + C$ 

13. 
$$-\frac{1}{18(2+3x^3)^2}$$
 + C

14. 
$$\frac{(\log x)^{1-m}}{1-m} + C$$

14. 
$$\frac{(\log x)^{1-m}}{1-m}$$
 + C 15.  $-\frac{1}{8}\log|9-4x^2|$  16.  $\frac{1}{2}e^{2x+3}$  + C

16. 
$$\frac{1}{2}e^{2x+3} + C$$

17. 
$$-\frac{1}{2e^{x^2}} + C$$

18. 
$$e^{\tan^{-1}x} + C$$

17. 
$$-\frac{1}{2e^{x^2}} + C$$
 18.  $e^{\tan^{-1}x} + C$  19.  $\log(e^x + e^{-x}) + C$ 

20. 
$$\frac{1}{2}\log(e^{2x}+e^{-2x})+C$$

20. 
$$\frac{1}{2}\log(e^{2x} + e^{-2x}) + C$$
  
21.  $\frac{1}{2}\tan(2x-3) - x + C$   
22.  $-\frac{1}{4}\tan(7-4x) + C$   
23.  $\frac{1}{2}(\sin^{-1}x)^2 + C$ 

22. 
$$-\frac{1}{4}\tan(7-4x) + 6$$

23. 
$$\frac{1}{2}(\sin^{-1}x)^2 + C$$

**24.** 
$$\frac{1}{2}\log|2\sin x + 3\cos x| + C$$

25. 
$$\frac{1}{(1-\tan x)}$$
 + C

**26.** 
$$2\sin\sqrt{x} + C$$

**26.** 
$$2\sin\sqrt{x} + C$$
 **27.**  $\frac{1}{3}(\sin 2x)^{\frac{3}{2}} + C$  **28.**  $2\sqrt{1+\sin x} + C$ 

28. 
$$2\sqrt{1+\sin x} + 0$$

**29.** 
$$\frac{1}{2}(\log \sin x)^2 + C$$
 **30.**  $-\log (1+\cos x)$  **31.**  $\frac{1}{1+\cos x} + C$ 

31. 
$$\frac{1}{1+\cos x} + C$$

32. 
$$\frac{x}{2} - \frac{1}{2} \log |\cos x + \sin x| + C$$

33. 
$$\frac{x}{2} - \frac{1}{2} \log |\cos x - \sin x| + C$$

34. 
$$2\sqrt{\tan x} + C$$

34. 
$$2\sqrt{\tan x} + C$$
 35.  $\frac{1}{3}(1 + \log x)^3 + C$  36.  $\frac{1}{3}(x + \log x)^3 + C$ 

36. 
$$\frac{1}{3}(x+\log x)^3+0$$

37. 
$$-\frac{1}{4}\cos(\tan^{-1}x^4) + C$$