SAT PREP
Assignment : Rolle's and MVT Theorem

Determine whether Rolle's Theorem can be applied to f on the interval $[a,b]$. If Rolle's Theorem can be applied, find all values of c in the interval (a,b) such that $f'(c)=0$.

1. $f(x)=x^2-2x,\ [0,2]$
2. $f(x)=(x-1)(x-2)(x-3),\ [1,3]$
3. $f(x)=x^{2/3}-1,\ [-8,8]$
4. $f(x)=\dfrac{x^2-2x-3}{x+2},\ [-1,3]$
5. $f(x)=\sin x,\ [0,2\pi]$
6. $f(x)=\tan x,\ [0,\pi]$
7. $f(x)=\sin x,\ \left[0,\dfrac{\pi}{2}\right]$

Determine whether the Mean Value Theorem can be applied to f on the interval $[a,b]$. If MVT can be applied, find all values of c in (a,b) such that $f'(c)=\dfrac{f(b)-f(a)}{b-a}$.

8. $f(x)=x^2,\ [-2,1]$
9. $f(x)=x^{2/3},\ [0,1]$
10. $f(x)=\sqrt{2-x},\ [-7,2]$
11. $f(x)=\sin x,\ [0,\pi]$
12. $f(x)=\dfrac{1}{x},\ [-1,1]$

Getting at the Concept:

13. Let f be continuous on $[a,b]$ and differentiable on (a,b). If there exists c in (a,b) such that $f'(c)=0$, does it follow that $f(a)=f(b)$? Explain.

14. When an object is removed from a furnace and placed in an environment with a constant temperature of 90ºF, its core temperature is 1500ºF. Five hours later the core temperature is 390ºF. Explain why there must exist a time in the interval when the temperature is decreasing at a rate of 222ºF per hour.

Answers:

1. $c = 1$ 2. $c = \dfrac{6 \pm \sqrt{3}}{3}$

3. Rolle's Theorem cannot be applied to f since f is not differentiable at $x = 0$ which is on $(-8, 8)$

4. $c = -2 + \sqrt{5}$ 5. $c = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

6. Rolle's Theorem cannot be applied since f is not continuous at $x = \dfrac{\pi}{2}$ which is on $[0, \pi]$.

7. Rolle's Thrm cannot be applied since $f(0) \neq f\left(\dfrac{\pi}{2}\right)$

8. $c = \dfrac{-1}{2}$ 9. $c = \dfrac{8}{27}$ 10. $c = \dfrac{-1}{4}$ 11. $c = \dfrac{\pi}{2}$

12. MVT cannot be applied since f is not continuous @ $x = 0$ which is on $[-1, 1]$

13. No. Ex: $f(x) = x^2, [-2, 3]$ $f'(0) = 0$ *but* $f(-2) \neq f(3)$

14. MVT, $f'(c) = \dfrac{390 - 1500}{5 - 0} = -222$