# SATPREP

## Assignment : L'Hopital's Rule

| L'Hopital's Rule: | Suppose that f(a)=g(a)=0, that f & g are differentiable on an open interval containing a, and that g'(x) ≠ 0 on ; then $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \lim_{x \to a} \dfrac{f'(x)}{g'(x)}$ |
|---|---|
| Examples: | $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2} = \dfrac{0}{0} \Rightarrow$ Use L'Hopital's Rule - Take derivative of numerator and denominator $\Rightarrow$ $\lim_{x \to 2} \dfrac{2x}{1} = \boxed{4}$ |
|  | $\lim_{x \to 1} \dfrac{x - 1}{\sqrt{x} - 1} = \dfrac{0}{0} \Rightarrow$ Use L'Hopitals: $\lim_{x \to 1} \dfrac{1}{\frac{1}{2\sqrt{x}}} = \lim_{x \to 1} \dfrac{2\sqrt{x}}{1} = \boxed{2}$ |
|  | $\lim_{x \to 2} \dfrac{x^2 - 4}{x + 2} = \dfrac{0}{4} = \boxed{0} \Rightarrow$ You cannot use L'Hopital's Rule on this problem!  Why?? |

Evaluate each Limit.  Use L'Hopital's Rule where appropriate.

| 1 | $\lim_{x \to 2} \dfrac{x^3 - 8}{x^2 - 4}$ | 2 | $\lim_{x \to 0} \dfrac{\sin 5x}{x}$ |
|---|---|---|---|
| 3 | $\lim_{x \to 2} \dfrac{\sqrt{2 + x} - 2}{x - 2}$ | 4 | $\lim_{x \to 1} \dfrac{\sqrt[3]{x} - 1}{x - 1}$ |
| 5 | $\lim_{x \to 2} \dfrac{x^2 - 4x + 4}{x^3 - 12x + 16}$ | 6 | $\lim_{x \to \frac{\pi}{2}} \dfrac{1 - \sin x}{1 + \cos 2x}$ |
| 7 | $\lim_{x \to 1} \dfrac{x^3 - 1}{4x^3 - x - 3}$ | 8 | $\lim_{x \to 3} \dfrac{x - 4}{x - 2}$ |

Find each derivative.

| 9 | $f(x) = e^{\sqrt{x}}$ | 10 | $y = e^{4 \ln x}$ |
|---|---|---|---|
| 11 | $y = 3^{x+1}$ | 12 | $y = \sqrt{4x^2 + 4x}$ |
| 13 | $g(x) = \cos^3 5x$ | 14 | $y = \ln(xe^{2x})$ |
| 15 | $y = \ln|\sin x|$ | 16 | $f(x) = x^{\cot 2x}$ |

Answers:

| 1) 3 | 2) 5 | 3) $\dfrac{1}{4}$ | 4) $\dfrac{1}{3}$ |
|---|---|---|---|
| 5) $\dfrac{1}{6}$ | 6) $\dfrac{1}{4}$ | 7) $\dfrac{3}{11}$ | 8) -1 |

| 9) $f'(x) = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$ | 10) $y' = 4x^3$ | 11) $\dfrac{dy}{dx} = 3^{x+1}(\ln 3)$ | 12) $y' = \dfrac{2x + 1}{\sqrt{x^2 + x}}$ |
|---|---|---|---|
| 13) | 14) | 15) $y' = \cot x$ | 16) |