SATPREP

Assignment – Tangent and Normal

1. Find the standard form of the equation of the normal to the graph of $y = x^3$ at the point $x = \frac{1}{3}$.

2. At what point(s) does the normal line to the curve $y = x^2 - 3x + 5$ at the point (3, 5) intersect the curve?

3. Find the equations of the tangent *and* the normal to the curve $f(x) = x^3 - 3x^2$ at (1, -2).

4. Find the equation of the normal to the curve $x^2 + y^2 = 25$ at (3, -4).

5. Find the equations of the lines that are tangent to *and* normal to the curve $x^2 y^2 = 9$ at (-1, 3).

6. Find the equation of the line normal to the curve $(y - x)^2 = 2x + 4$ at (6, 2).

7. Find the two points where the curve $x^2 + xy + y^2 = 7$ crosses the x-axis, and show that the tangents to the curve at these points are parallel. What is the common slope of these tangents?

8. The line that is normal to the curve $y = x^2 + 2x - 3$ at (1, 0) intersects the curve at what other point?

9. Find the tangents to the curve $y = x^3 - 6x + 2$ that are parallel to the line $y = 6x - 2$.

10. Find the normals to the curve $xy + 2x - y = 0$ that are parallel to the line $2x + y = 0$.

Answers:

1. $81x + 27y = 28$
2. $(3,5); (-\frac{1}{3}, \frac{55}{9})$
3. tangent: $3x + y = 1$; normal: $x - 3y = 7$
4. $4x + 3y = 0$
5. tangent: $3x - y = -6$; normal: $x + 3y = 8$
6. $4x + 3y = 30$
7. $(\sqrt{7}, 0); (-\sqrt{7}, 0)$; $slopes = -2$
8. normal: $x + 4y = 1$; $(-\frac{13}{4}, \frac{17}{16})$
9. $6x - y = -18$; $6x - y = 14$
10. $2x + y = -3$; $2x + y = 3$

Normal Line Sample Problems and Solutions

Basic Principle: The slope of the normal line is the opposite reciprocal of the tangent line's slope, because the normal line is perpendicular to the tangent line. This can also be described as the normal line is *orthogonal to the curve*.

1. Find the standard form of the equation of the tangent <u>and</u> the normal to the graph of $y = x^2$ at the point $x = \frac{1}{4}$.

   By plugging the value $x = \frac{1}{4}$ into $y = x^2$, you get $y = \frac{1}{16}$… so the point $(\frac{1}{4}, \frac{1}{16})$ is on the curve. This is where both the tangent and the normal will intersect the graph. Next find the derivative, since both the tangent and the normal require you to have the slope. Therefore, $y' = 2x$. Now you can find both the slope of the tangent and the normal. $y' = 2(\frac{1}{4}) = \frac{1}{2}$ … So $m_{tan} = \frac{1}{2}$ … Tangent line: $y - \frac{1}{16} = \frac{1}{2}(x - \frac{1}{4})$… Distribute 16 on to both sides … $16y - 1 = 8(x - \frac{1}{4})$ … simplify and reorganize into standard form. $8x - 16y = 1$ … Now for the normal line, all you have to do is use the opposite reciprocal of the tangent line's slope … which is m_{normal} = -2. Therefore, the equation is $y - \frac{1}{16} = -2(x - \frac{1}{4})$ … $16y - 1 = -32(x - \frac{1}{4})$ … $32x + 16y = 9$.

2. Find the equation of the normal to the graph of the curve $9x^2 + 4y^2 = 45$ at (-1, 3).

   To find the normal line, we only need to find the slope at (-1, 3). First find the tangent line's slope, and then use its opposite reciprocal. To find the derivative, we must use implicit differentiation. $18x + 8y\dfrac{dy}{dx} = 0$ … $\dfrac{dy}{dx} = \dfrac{-9x}{4y}$ … Plug in (-1, 3) for slope of the tangent is … $\dfrac{dy}{dx} = \dfrac{-9(-1)}{4(3)} = \dfrac{3}{4}$ … so the normal line slope is $-\dfrac{4}{3}$ … so the equation is $y - 3 = -\dfrac{4}{3}(x+1)$ … $3y - 9 = -4(x+1)$ … $4x + 3y = 5$.

3. A line is normal to the curve $y = 4x - x^2$ at (3, 3). Find the other point of intersect between the normal and the parabola.

   First, take the derivative … $y' = 4 - 2x$… so this gives slope of tangent = 4 – 2(3) = -2. This makes the normal line slope = $\frac{1}{2}$. Hence the equation of the normal line is $y - 3 = \frac{1}{2}(x - 3)$ … also known as … $y = \frac{1}{2}x + \frac{3}{2}$. Now to find the intersection between the normal and the parabola, set the their equation equal. $\frac{1}{2}x + \frac{3}{2} = 4x - x^2$… Solve … $x + 3 = 8x - 2x^2$ … $2x^2 - 7x + 3 = 0$ … $(2x-1)(x-3) = 0$ … So $x = \frac{1}{2}$ *or* 3 … We already know about the x = 3 … Plug $\frac{1}{2}$ in to either the parabola or the line and you will get $y = \frac{7}{4}$. Graph the two on the TI-83 and you will see this verified. Graph the tangent line too for the full effect.