

## SATPREP

### Assignment : Integration by trigonometric substitution

$$a^2 + u^2 \quad (u = a \tan \theta) \quad ; \quad a^2 - u^2 \quad (u = a \sin \theta) \quad ; \quad u^2 - a^2 \quad (u = a \sec \theta)$$

1. Evaluate the following indefinite integrals.

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| (1) $\int \frac{1}{\sqrt{1-4x^2}} dx$          | (2) $\int \frac{1}{x^2+25} dx$                             | (3) $\int \frac{x}{x^4+16} dx$                           |
| (4) $\int \frac{1}{\sqrt{2-5x^2}} dx$          | (5) $\int \frac{3}{x\sqrt{x^2-9}} dx$                      | (6) $\int \frac{x}{\sqrt{16-9x^4}} dx$                   |
| (7) $\int \frac{1}{x\sqrt{16x^2-9}} dx$        | (8) $\int \frac{e^x}{7+e^{2x}} dx$                         | (9) $\int \frac{\sin x}{\sqrt{2-\cos^2 x}} dx$           |
| (10) $\int \frac{1}{\sqrt{x}(1+x)} dx$         | (11) $\int \frac{1}{x^2\sqrt{4-x^2}} dx$                   | (12) $\int \frac{1}{x\sqrt{x^2+4}} dx$                   |
| (13) $\int \frac{\sqrt{9-x^2}}{x^2} dx$        | (14) $\int \frac{1}{x\sqrt{25-x^2}} dx$                    | (15) $\int \frac{1}{\sqrt{x^2-a^2}} dx$                  |
| (16) $\int \sqrt{1-x^2} dx$                    | (17) $\int \frac{1}{x^2\sqrt{x^2-7}} dx$                   | (18) $\int x^2\sqrt{16-x^2} dx$                          |
| (19) $\int \frac{1}{(x^2+4)^{\frac{3}{2}}} dx$ | (20) $\int \frac{1}{(4x^2-9)^{\frac{3}{2}}} dx$            | (21) $\int \frac{1}{x^4\sqrt{16+x^2}} dx$                |
| (22) $\int \frac{1}{x\sqrt{x^4-4}} dx$         | (23) $\int \frac{\sec^2 x}{(4-\tan^2 x)^{\frac{3}{2}}} dx$ | (24) $\int \frac{e^{-x}}{(9e^{-2x}+1)^{\frac{3}{2}}} dx$ |
| (25) $\int \frac{1}{\sqrt{x^2+2x}} dx$         | (26) $\int \frac{1}{x^4\sqrt{x^2+3}} dx$                   | (27) $\int \frac{\sqrt{x^2-25}}{x} dx$                   |

## ANSWERS

1.  $\frac{1}{2} \arcsin 2x + C$
2.  $\frac{1}{5} \arctan\left(\frac{x}{5}\right) + C$
3.  $\frac{1}{8} \arctan\left(\frac{x^2}{4}\right) + C$
4.  $\frac{1}{\sqrt{5}} \arcsin\left(\frac{x\sqrt{5}}{\sqrt{2}}\right) + C$
5.  $\sec^{-1}\left(\frac{x}{3}\right) + C$
6.  $\frac{1}{6} \arcsin\left(\frac{3x^2}{4}\right) + C$
7.  $\frac{1}{3} \sec^{-1}\left(\frac{4x}{3}\right) + C$
8.  $\frac{1}{\sqrt{7}} \arctan\left(\frac{e^x}{\sqrt{7}}\right) + C$
9.  $-\arcsin\left(\frac{\cos x}{\sqrt{2}}\right) + C$
10.  $2 \arctan \sqrt{x} + C$
11.  $-\frac{\sqrt{4-x^2}}{4x} + C$
12.  $\frac{1}{2} \ln \left| \frac{\sqrt{x^2+4}-2}{x} \right| + C$
13.  $-\frac{\sqrt{9-x^2}}{x} - \arcsin\left(\frac{x}{3}\right) + C$
14.  $\frac{1}{5} \ln \left| \frac{5-\sqrt{25-x^2}}{x} \right| + C$
15.  $\ln \left| \frac{x+\sqrt{x^2-a^2}}{a} \right| + C$
16.  $\frac{1}{2} \arcsin x + \frac{x\sqrt{1-x^2}}{2} + C$
17.  $\frac{\sqrt{x^2-7}}{7x} + C$
18.  $32 \arcsin\left(\frac{x}{4}\right) - \frac{x\sqrt{16-x^2}}{4}(8-x^2) + C$
19.  $\frac{x}{4\sqrt{x^2+4}} + C$
20.  $-\frac{x}{9\sqrt{4x^2-9}} + C$
21.  $\frac{(x^2-8)\sqrt{16+x^2}}{384x^3} + C$
22.  $\frac{1}{4} \sec^{-1}\left(\frac{x^2}{2}\right) + C$
23.  $\frac{\tan x}{4\sqrt{4-\tan^2 x}} + C$
24.  $-\frac{e^{-x}}{\sqrt{9e^{-2x}+1}} + C$
25.  $\ln |x+1+\sqrt{x^2+2x}| + C$
26.  $\frac{(2x^2-3)\sqrt{x^2+3}}{27x^3} + C$
27.  $\sqrt{x^2-25} - 5 \sec^{-1}\left(\frac{x}{5}\right) + C$