SATPREP Name___________________________________ ID: 1

Assignment:Divisibility

Date________________

Use mathematical induction to prove that each statement is true for all positive integers.

1) $7n^3 + 2n$ is divisible by 3

2) $5n^2 + 5n$ is divisible by 2

3) $14n^2 - 6n$ is divisible by 4

4) 4 is a factor of $5^n + 3$

5) $3n^3 + 6n$ is divisible by 9

6) $21n^2 - 15n$ is divisible by -6

7) $10n^3 + 8n$ is divisible by 6

8) 2 is a factor of $3^n - 1$

9) $15n^2 - 3n$ is divisible by 2

10) $2n^3 + 4n$ is divisible by 6

Answers to Assignment:Divisibility (ID: 1)

1) Let P_n be the statement $7n^3 + 2n$ is divisible by 3

Anchor Step

P_1 is true: $7 \cdot 1^3 + 2$ is divisible by 3

Inductive Hypothesis

Assume that $7k^3 + 2k$ is divisible by 3. Therefore, $7k^3 + 2k = 3r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $7(k + 1)^3 + 2(k + 1)$ is divisible by 3

$7(k^3 + 3k^2 + 3k + 1) + 2k + 2$

$7k^3 + 21k^2 + 21k + 7 + 2k + 2$

$7k^3 + 2k + 21k^2 + 21k + 9$

$3r + 21k^2 + 21k + 9$

$3(r + 7k^2 + 7k + 3)$

Conclusion

By induction P_n is true for all $n \geq 1$.

2) Let P_n be the statement $5n^2 + 5n$ is divisible by 2

Anchor Step

P_1 is true: $5 \cdot 1^2 + 5$ is divisible by 2

Inductive Hypothesis

Assume that $5k^2 + 5k$ is divisible by 2. Therefore, $5k^2 + 5k = 2r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $5(k + 1)^2 + 5(k + 1)$ is divisible by 2

$5k^2 + 10k + 5 + 5k + 5$

$5k^2 + 5k + 10k + 10$

$2r + 10k + 10$

$2(r + 5k + 5)$

Conclusion

By induction P_n is true for all $n \geq 1$.

3) Let P_n be the statement $14n^2 - 6n$ is divisible by 4

Anchor Step

P_1 is true: $14 \cdot 1^2 - 6$ is divisible by 4

Inductive Hypothesis

Assume that $14k^2 - 6k$ is divisible by 4. Therefore, $14k^2 - 6k = 4r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $14(k + 1)^2 - 6(k + 1)$ is divisible by 4

$14k^2 + 28k + 14 - 6k - 6$

$14k^2 - 6k + 28k + 8$

$4r + 28k + 8$

$4(r + 7k + 2)$

Conclusion

By induction P_n is true for all $n \geq 1$.

4) Let P_n be the statement 4 is a factor of $5^n + 3$

Anchor Step

P_1 is true: 4 is a factor of $5^1 + 3$

Inductive Hypothesis

Assume that 4 is a factor of $5^k + 3$. Therefore, $5^k + 3 = 4r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: 4 is a factor of $5^{(k+1)} + 3$

$5 \cdot 5^k + 3$
$(4 + 1) \cdot 5^k + 3$
$4 \cdot 5^k + 5^k + 3$
$4 \cdot 5^k + 4r$
$4(5^k + r)$

Conclusion

By induction P_n is true for all $n \geq 1$.

5) Let P_n be the statement $3n^3 + 6n$ is divisible by 9

Anchor Step

P_1 is true: $3 \cdot 1^3 + 6$ is divisible by 9

Inductive Hypothesis

Assume that $3k^3 + 6k$ is divisible by 9. Therefore, $3k^3 + 6k = 9r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $3(k + 1)^3 + 6(k + 1)$ is divisible by 9

$3(k^3 + 3k^2 + 3k + 1) + 6k + 6$
$3k^3 + 9k^2 + 9k + 3 + 6k + 6$
$3k^3 + 6k + 9k^2 + 9k + 9$
$9r + 9k^2 + 9k + 9$
$9(r + k^2 + k + 1)$

Conclusion

By induction P_n is true for all $n \geq 1$.

6) Let P_n be the statement $21n^2 - 15n$ is divisible by -6

Anchor Step

P_1 is true: $21 \cdot 1^2 - 15$ is divisible by -6

Inductive Hypothesis

Assume that $21k^2 - 15k$ is divisible by -6. Therefore, $21k^2 - 15k = -6r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $21(k + 1)^2 - 15(k + 1)$ is divisible by -6

$21k^2 + 42k + 21 - 15k - 15$
$21k^2 - 15k + 42k + 6$
$-6r + 42k + 6$
$-6(r - 7k - 1)$

Conclusion

By induction P_n is true for all $n \geq 1$.

7) Let P_n be the statement $10n^3 + 8n$ is divisible by 6

Anchor Step

P_1 is true: $10 \cdot 1^3 + 8$ is divisible by 6

Inductive Hypothesis

Assume that $10k^3 + 8k$ is divisible by 6. Therefore, $10k^3 + 8k = 6r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $10(k+1)^3 + 8(k+1)$ is divisible by 6

$$10(k^3 + 3k^2 + 3k + 1) + 8k + 8$$
$$10k^3 + 30k^2 + 30k + 10 + 8k + 8$$
$$10k^3 + 8k + 30k^2 + 30k + 18$$
$$6r + 30k^2 + 30k + 18$$
$$6(r + 5k^2 + 5k + 3)$$

Conclusion

By induction P_n is true for all $n \geq 1$.

8) Let P_n be the statement 2 is a factor of $3^n - 1$

Anchor Step

P_1 is true: 2 is a factor of $3^1 - 1$

Inductive Hypothesis

Assume that 2 is a factor of $3^k - 1$. Therefore, $3^k - 1 = 2r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: 2 is a factor of $3^{(k+1)} - 1$

$$3 \cdot 3^k - 1$$
$$(2 + 1) \cdot 3^k - 1$$
$$2 \cdot 3^k + 3^k - 1$$
$$2 \cdot 3^k + 2r$$
$$2(3^k + r)$$

Conclusion

By induction P_n is true for all $n \geq 1$.

9) Let P_n be the statement $15n^2 - 3n$ is divisible by 2

Anchor Step

P_1 is true: $15 \cdot 1^2 - 3$ is divisible by 2

Inductive Hypothesis

Assume that $15k^2 - 3k$ is divisible by 2. Therefore, $15k^2 - 3k = 2r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $15(k+1)^2 - 3(k+1)$ is divisible by 2

$$15k^2 + 30k + 15 - 3k - 3$$
$$15k^2 - 3k + 30k + 12$$
$$2r + 30k + 12$$
$$2(r + 15k + 6)$$

Conclusion

By induction P_n is true for all $n \geq 1$.

10) Let P_n be the statement $2n^3 + 4n$ is divisible by 6

Anchor Step

P_1 is true: $2 \cdot 1^3 + 4$ is divisible by 6

Inductive Hypothesis

Assume that $2k^3 + 4k$ is divisible by 6. Therefore, $2k^3 + 4k = 6r$ for some integer r.

Inductive Step

We now show that P_{k+1} is true: $2(k+1)^3 + 4(k+1)$ is divisible by 6

$2(k^3 + 3k^2 + 3k + 1) + 4k + 4$

$2k^3 + 6k^2 + 6k + 2 + 4k + 4$

$2k^3 + 4k + 6k^2 + 6k + 6$

$6r + 6k^2 + 6k + 6$

$6(r + k^2 + k + 1)$

Conclusion

By induction P_n is true for all $n \geq 1$.