

Assignment: Area between the curve

Date _____

For each problem, find the area of the region enclosed by the curves.

1) $y = x^2 - 4x + 3, \quad y = -x^2 + 2x + 3,$
 $x = 1, \quad x = 4$

2) $y = \frac{x^2}{2} + x - \frac{3}{2}, \quad y = -2x + 2,$
 $x = -2, \quad x = 3$

3) $y = -x^2 + 5, \quad y = -\frac{x^2}{2} + 4x - 5,$
 $x = 0, \quad x = 3$

4) $y = \frac{x^2}{2} - 2x - 4, \quad y = 2,$
 $x = -3, \quad x = 2$

5) $y = -x^3 + 6x, \quad y = -x^2$

6) $y = x^3 + x^2 - 4x, \quad y = 2x$

$$7) \ y = x^3 - 7x^2 + 11x, \ y = x^2 - 4x$$

$$8) \ y = \frac{x^3}{2} + \frac{x^2}{2} - 2x, \ y = \frac{x^2}{2}$$

$$9) \ y = 2\sin x, \ y = 2\cos x,$$

$$x = -\frac{\pi}{4}, \ x = \frac{\pi}{4}$$

$$10) \ y = -\csc x \cot x, \ y = -2\csc^2 x,$$

$$x = \frac{\pi}{3}, \ x = \frac{3\pi}{4}$$

$$11) \ y = -\sec^2 x, \ y = 2\sec^2 x,$$

$$x = -\frac{\pi}{4}, \ x = 0$$

$$12) \ y = 2\csc^2 x, \ y = 2\sin x,$$

$$x = -\frac{\pi}{2}, \ x = -\frac{\pi}{4}$$

$$13) \ y = -2\sqrt{x}, \ y = \sqrt{x},$$

$$x = 0, \ x = 4$$

$$14) \ y = \frac{2}{x^2}, \ y = -3,$$

$$x = 2, \ x = 3$$

Answers to Assignment: Area between the curve

1)
$$\int_1^3 (-x^2 + 2x + 3 - (x^2 - 4x + 3)) dx + \int_3^4 (x^2 - 4x + 3 - (-x^2 + 2x + 3)) dx$$

$$= \frac{31}{3} \approx 10.333$$

3)
$$\int_0^2 \left(-x^2 + 5 - \left(-\frac{x^2}{2} + 4x - 5 \right) \right) dx + \int_2^3 \left(-\frac{x^2}{2} + 4x - 5 - (-x^2 + 5) \right) dx$$

$$= \frac{83}{6} \approx 13.833$$

5)
$$\int_{-2}^0 (-x^2 - (-x^3 + 6x)) dx + \int_0^3 (-x^3 + 6x + x^2) dx$$

$$= \frac{253}{12} \approx 21.083$$

7)
$$\int_0^3 (x^3 - 7x^2 + 11x - (x^2 - 4x)) dx + \int_3^5 (x^2 - 4x - (x^3 - 7x^2 + 11x)) dx$$

$$= \frac{253}{12} \approx 21.083$$

9)
$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (2\cos x - 2\sin x) dx$$

$$= 2\sqrt{2} \approx 2.828$$

11)
$$\int_{-\frac{\pi}{4}}^0 (2\sec^2 x + \sec^2 x) dx$$

$$= 3$$

14)
$$\int_2^3 \left(\frac{2}{x^2} + 3 \right) dx$$

$$= \frac{10}{3} \approx 3.333$$

2)
$$\int_{-2}^1 \left(-2x + 2 - \left(\frac{x^2}{2} + x - \frac{3}{2} \right) \right) dx + \int_1^3 \left(\frac{x^2}{2} + x - \frac{3}{2} - (-2x + 2) \right) dx$$

$$= \frac{137}{6} \approx 22.833$$

4)
$$\int_{-3}^{-2} \left(\frac{x^2}{2} - 2x - 6 \right) dx + \int_{-2}^2 \left(2 - \left(\frac{x^2}{2} - 2x - 4 \right) \right) dx$$

$$= \frac{47}{2} = 23.5$$

6)
$$\int_{-3}^0 (x^3 + x^2 - 6x) dx + \int_0^2 (2x - (x^3 + x^2 - 4x)) dx$$

$$= \frac{253}{12} \approx 21.083$$

8)
$$\int_{-2}^0 \left(\frac{x^3}{2} + \frac{x^2}{2} - 2x - \frac{x^2}{2} \right) dx + \int_0^2 \left(\frac{x^2}{2} - \left(\frac{x^3}{2} + \frac{x^2}{2} - 2x \right) \right) dx$$

$$= 4$$

10)
$$\int_{\frac{\pi}{3}}^{\frac{3\pi}{4}} (-\csc x \cot x + 2\csc^2 x) dx$$

$$= \sqrt{2} + 2 \approx 3.414$$

12)
$$\int_{-\frac{\pi}{2}}^{-\frac{\pi}{4}} (2\csc^2 x - 2\sin x) dx$$

$$= 2 + \sqrt{2} \approx 3.414$$

13)
$$\int_0^4 (\sqrt{x} + 2\sqrt{x}) dx$$

$$= 16$$