

Assignment : Trigonometric identity

Date _____

Verify each identity.

1) $\cot x - \csc x = \frac{\cos x - 1}{\sin x}$

2) $\sec^2 x \sin x \csc x = \frac{1}{\cos^2 x}$

3) $\frac{\cot x}{\cos x} = \frac{1}{\sin x}$

4) $\frac{1 - \sec^2 x}{\sec^2 x} = \cos^2 x - 1$

5) $\csc x \cdot (1 + \csc x) = \frac{1 + \sin x}{\sin^2 x}$

6) $\frac{\sec x}{\sec^2 x - \tan^2 x} = \frac{1}{\cos x}$

7) $1 - \sec^2 x \cot^2 x = -\cot^2 x$

8) $\tan x - \cot x \sec^2 x = -\cot x$

9) $\frac{\cot x}{\csc^2 x - 1} = \sin x \sec x$

10) $\sec x + \csc^2 x \cos x = \frac{\csc^2 x}{\cos x}$

$$11) 1 - \tan^2 x \csc^2 x = -\sin^2 x \sec^2 x$$

$$12) \frac{1}{\csc^2 x \cos^2 x} = \sec^2 x - 1$$

Use identities to find the value of each expression.

13) Find $\cos \theta$ and $\cot \theta$

$$\text{if } \sin \theta = \frac{2}{3} \text{ and } \sec \theta > 0.$$

14) Find $\sec \theta$ and $\tan \theta$

$$\text{if } \cos \theta = \frac{1}{2} \text{ and } \csc \theta < 0.$$

15) Find $\sec \theta$ and $\cos \theta$

$$\text{if } \csc \theta = -\frac{5}{3} \text{ and } \cot \theta > 0.$$

16) Find $\csc \theta$ and $\sin \theta$

$$\text{if } \cos \theta = -\frac{5}{8} \text{ and } \cot \theta < 0.$$

17) Find $\cot \theta$ and $\sec \theta$

$$\text{if } \csc \theta = -3 \text{ and } \cot \theta < 0.$$

18) Find $\tan \theta$ and $\csc \theta$

$$\text{if } \cot \theta = -\frac{3}{5} \text{ and } \sin \theta > 0.$$

Answers to Assignment : Trigonometric identity

1) $\cot x - \csc x$ Decompose into sine and cosine

$$\frac{\cos x}{\sin x} - \frac{1}{\sin x} \quad \text{Simplify}$$

$$\frac{\cos x - 1}{\sin x} \quad \blacksquare$$

2) $\sec^2 x \sin x \csc x$ Decompose into sine and cosine

$$\left(\frac{1}{\cos x}\right)^2 \sin x \cdot \frac{1}{\sin x} \quad \text{Simplify}$$

$$\frac{1}{\cos^2 x} \quad \blacksquare$$

3) $\frac{\cot x}{\cos x}$ Use $\cot x = \frac{\cos x}{\sin x}$ 4) $\frac{1 - \sec^2 x}{\sec^2 x}$ Decompose into sine and cosine

$$\frac{\cos x}{\cos x \sin x} \quad \text{Cancel common factors}$$

$$\frac{1}{\sin x} \quad \blacksquare$$

$$\frac{1 - \left(\frac{1}{\cos x}\right)^2}{\left(\frac{1}{\cos x}\right)^2} \quad \text{Simplify}$$

$$\cos^2 x - 1 \quad \blacksquare$$

5) $\csc x \cdot (1 + \csc x)$ Decompose into sine and cosine

$$\frac{1}{\sin x} \left(1 + \frac{1}{\sin x}\right) \quad \text{Simplify}$$

$$\frac{1 + \sin x}{\sin^2 x} \quad \blacksquare$$

6) $\frac{\sec x}{\sec^2 x - \tan^2 x}$ Use $\tan^2 x + 1 = \sec^2 x$

$$\sec x \quad \text{Use } \sec x = \frac{1}{\cos x}$$

$$\frac{1}{\cos x} \quad \blacksquare$$

$$7) 1 - \sec^2 x \cot^2 x$$

Use $\tan^2 x + 1 = \sec^2 x$

$$1 - \cot^2 x \tan^2 x - \cot^2 x$$

Decompose into sine and cosine

$$1 - \left(\frac{\cos x}{\sin x}\right)^2 \cdot \left(\frac{\sin x}{\cos x}\right)^2 - \left(\frac{\cos x}{\sin x}\right)^2$$

Simplify

$$-\frac{\cos^2 x}{\sin^2 x}$$

Use $\cot x = \frac{\cos x}{\sin x}$

$$-\cot^2 x$$

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$$8) \tan x - \cot x \sec^2 x$$

Use $\tan^2 x + 1 = \sec^2 x$

$$\tan x - \cot x \tan^2 x - \cot x$$

Decompose into sine and cosine

$$\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} \cdot \left(\frac{\sin x}{\cos x}\right)^2 - \frac{\cos x}{\sin x}$$

Simplify

$$-\frac{\cos x}{\sin x}$$

Use $\cot x = \frac{\cos x}{\sin x}$

$$-\cot x$$

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$$9) \frac{\cot x}{\csc^2 x - 1}$$

Use $\cot^2 x + 1 = \csc^2 x$

$$\frac{\cot x}{\cot^2 x}$$

Cancel common factors

$$\frac{1}{\cot x}$$

Use $\cot x = \frac{\cos x}{\sin x}$

$$\frac{\sin x}{\cos x}$$

Use $\sec x = \frac{1}{\cos x}$

$$\sin x \sec x$$

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10) $\sec x + \csc^2 x \cos x$ Decompose into sine and cosine

$$\frac{1}{\cos x} + \left(\frac{1}{\sin x}\right)^2 \cos x$$

Simplify

$$\frac{\sin^2 x + \cos^2 x}{\cos x \sin^2 x}$$

Use $\sin^2 x + \cos^2 x = 1$

$$\frac{1}{\sin^2 x \cos x}$$

Use $\csc x = \frac{1}{\sin x}$

$$\frac{\csc^2 x}{\cos x}$$

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11) $1 - \tan^2 x \csc^2 x$ Use $\cot^2 x + 1 = \csc^2 x$

$$1 - \tan^2 x \cot^2 x - \tan^2 x$$

Decompose into sine and cosine

$$1 - \left(\frac{\sin x}{\cos x}\right)^2 \cdot \left(\frac{\cos x}{\sin x}\right)^2 - \left(\frac{\sin x}{\cos x}\right)^2$$

Simplify

$$-\frac{\sin^2 x}{\cos^2 x}$$

Use $\sec x = \frac{1}{\cos x}$

$$-\sin^2 x \sec^2 x$$

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12) $\frac{1}{\csc^2 x \cos^2 x}$ Use $\csc x = \frac{1}{\sin x}$ 13) $\frac{\sqrt{5}}{3}$ and $\frac{\sqrt{5}}{2}$ 14) 2 and $-\sqrt{3}$

$$\frac{\sin^2 x}{\cos^2 x}$$

Use $\tan x = \frac{\sin x}{\cos x}$

$$\tan^2 x$$

Use $\tan^2 x + 1 = \sec^2 x$

$$\sec^2 x - 1$$

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15) $-\frac{5}{4}$ and $-\frac{4}{5}$ 16) $\frac{8\sqrt{39}}{39}$ and $\frac{\sqrt{39}}{8}$ 17) $-2\sqrt{2}$ and $\frac{3\sqrt{2}}{4}$ 18) $-\frac{5}{3}$ and $\frac{\sqrt{34}}{5}$