

Assignment : Trigonometric Identity

Verify each identity.

1) $\frac{\sec^2 x}{\tan^2 x} = \frac{1}{\sin^2 x}$

2) $-\tan^2 x \cos^2 x = -\sin^2 x$

3) $-\sin^2 x \sec^2 x = 1 - \sec^2 x$

4) $\cot x \csc^2 x \sec x = \frac{1 + \cot^2 x}{\sin x}$

5) $\csc^2 x - \cos x \sec x = \frac{1}{\tan^2 x}$

6) $\frac{\tan x}{1 - \sec^2 x} = -\csc x \cos x$

7) $\frac{\csc^2 x - 1}{\cos^2 x} = \csc^2 x$

8) $\cot x + \tan x = \frac{\sec x}{\sin x}$

9) $\csc^2 x (1 - \sec^2 x) = -\sec^2 x$

10) $-\csc^2 x \cos^2 x = -\csc^2 x + 1$

Answers to Assignment : Trigonometric Identity-3

1) $\frac{\sec^2 x}{\tan^2 x}$ Decompose into sine and cosine

$$\frac{\left(\frac{1}{\cos x}\right)^2}{\left(\frac{\sin x}{\cos x}\right)^2}$$

Simplify

$$\frac{1}{\sin^2 x}$$

■

2) $-\tan^2 x \cos^2 x$ Decompose into sine and cosine

$$-\left(\frac{\sin x}{\cos x}\right)^2 \cdot \cos^2 x$$

Simplify

$$-\sin^2 x$$

■

3) $-\sin^2 x \sec^2 x$ Use $\sec x = \frac{1}{\cos x}$ 4) $\cot x \csc^2 x \sec x$ Use $\cot x = \frac{\cos x}{\sin x}$

$$-\frac{\sin^2 x}{\cos^2 x}$$

Use $\tan x = \frac{\sin x}{\cos x}$ $\frac{\csc^2 x \sec x \cos x}{\sin x}$ Use $\sec x = \frac{1}{\cos x}$

$$-\tan^2 x$$

Use $\tan^2 x + 1 = \sec^2 x$ $\frac{\csc^2 x \sec x}{\sin x \sec x}$ Cancel common factors

$$1 - \sec^2 x$$

■ $\frac{\csc^2 x}{\sin x}$ Use $\cot^2 x + 1 = \csc^2 x$

$$\frac{1 + \cot^2 x}{\sin x}$$

■

5) $\csc^2 x - \cos x \sec x$ Use $\cot^2 x + 1 = \csc^2 x$

$$\cot^2 x + 1 - \cos x \sec x$$

Decompose into sine and cosine

$$\left(\frac{\cos x}{\sin x}\right)^2 + 1 - \cos x \cdot \frac{1}{\cos x}$$

Simplify

$$\frac{\cos^2 x}{\sin^2 x}$$

Use $\tan x = \frac{\sin x}{\cos x}$

$$\frac{1}{\tan^2 x}$$

■

$$6) \frac{\tan x}{1 - \sec^2 x}$$

Use $\tan^2 x + 1 = \sec^2 x$

$$7) \frac{\csc^2 x - 1}{\cos^2 x}$$

Use $\cot^2 x + 1 = \csc^2 x$

$$\frac{\tan x}{-\tan^2 x}$$

Decompose into sine and cosine

$$\frac{\cot^2 x}{\cos^2 x}$$

Decompose into sine and cosine

$$\frac{\frac{\sin x}{\cos x}}{-\left(\frac{\sin x}{\cos x}\right)^2}$$

Simplify

$$\frac{\left(\frac{\cos x}{\sin x}\right)^2}{\cos^2 x}$$

Simplify

$$-\frac{\cos x}{\sin x}$$

Use $\csc x = \frac{1}{\sin x}$

$$\frac{1}{\sin^2 x}$$

Use $\csc x = \frac{1}{\sin x}$

$$\csc^2 x$$

$$8) \cot x + \tan x$$

Decompose into sine and cosine

$$\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$$

Simplify

$$\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$$

Use $\sin^2 x + \cos^2 x = 1$

$$\frac{1}{\sin x \cos x}$$

Use $\sec x = \frac{1}{\cos x}$

$$\frac{\sec x}{\sin x}$$

$$9) \csc^2 x(1 - \sec^2 x)$$

Use $\tan^2 x + 1 = \sec^2 x$

$$-\csc^2 x \tan^2 x$$

Decompose into sine and cosine

$$-\left(\frac{1}{\sin x}\right)^2 \cdot \left(\frac{\sin x}{\cos x}\right)^2$$

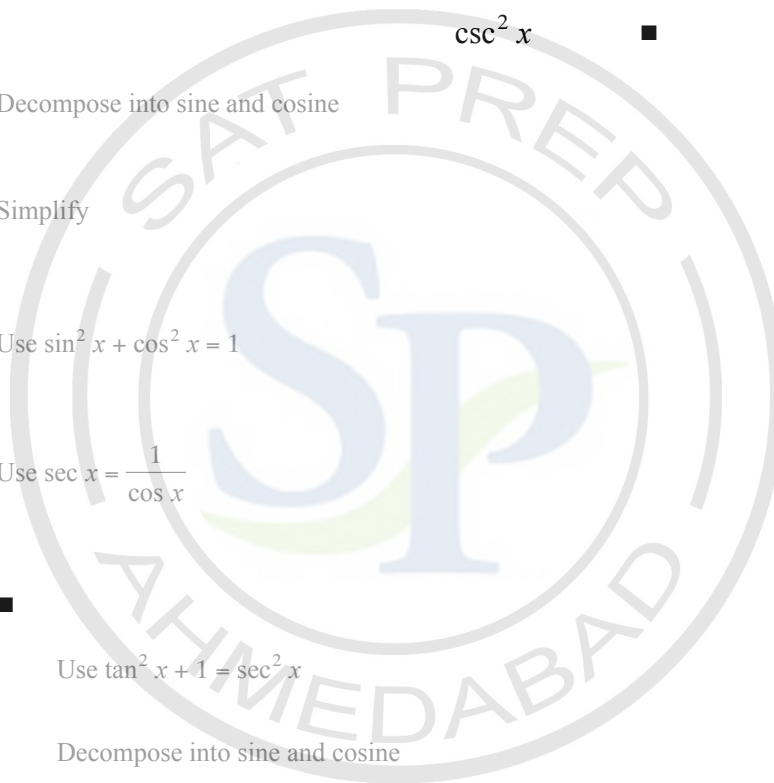
Simplify

$$-\frac{1}{\cos^2 x}$$

Use $\sec x = \frac{1}{\cos x}$

$$-\sec^2 x$$

■



$$10) -\csc^2 x \cos^2 x \quad \text{Use } \csc x = \frac{1}{\sin x}$$

$$-\frac{\cos^2 x}{\sin^2 x} \quad \text{Use } \cot x = \frac{\cos x}{\sin x}$$

$$-\cot^2 x \quad \text{Use } \cot^2 x + 1 = \csc^2 x$$

$$-\csc^2 x + 1 \quad \blacksquare$$

