Assignment : Trigonometric Identity

Verify each identity.

1) $\dfrac{\cot x + 1}{\csc^2 x} = \sin x \cdot \left(\cos x + \sin x\right)$

2) $\dfrac{\tan x}{\cot x + \sec x} = \dfrac{\sin^2 x}{\sin x + \cos^2 x}$

3) $\dfrac{\sec^3 x}{\csc^2 x} = \dfrac{\sec^2 x - 1}{\cos x}$

4) $\sin^2 x\left(1 - \csc^2 x\right) = -\cos^2 x$

5) $\dfrac{\csc^2 x \tan^2 x}{\sec x} = \dfrac{\sec x}{\sin^2 x + \cos^2 x}$

6) $\dfrac{\csc^2 x}{\cos x \cot x} = \dfrac{1 + \tan^2 x}{\sin x}$

7) $\dfrac{\cot^2 x}{\cos^2 x} = 1 + \cot^2 x$

8) $\csc^2 x\left(1 - \sec^2 x\right) = -\sec^2 x$

9) $\dfrac{\csc^2 x - 1}{\cos^2 x} = \dfrac{\csc x}{\sin x}$

10) $\csc^2 x + \sec^2 x = \dfrac{\sec^2 x}{\sin^2 x}$

Answers to Assignment : Trigonometric Identity

1) $\dfrac{\cot x + 1}{\csc^2 x}$ Decompose into sine and cosine

$\dfrac{\dfrac{\cos x}{\sin x} + 1}{\left(\dfrac{1}{\sin x}\right)^2}$ Simplify

$\sin x \cdot (\cos x + \sin x)$ ■

2) $\dfrac{\tan x}{\cot x + \sec x}$ Decompose into sine and cosine

$\dfrac{\dfrac{\sin x}{\cos x}}{\dfrac{\cos x}{\sin x} + \dfrac{1}{\cos x}}$ Simplify

$\dfrac{\sin^2 x}{\sin x + \cos^2 x}$ ■

3) $\dfrac{\sec^3 x}{\csc^2 x}$ Use $\csc x = \dfrac{1}{\sin x}$

$\sec^3 x \sin^2 x$ Use $\sec x = \dfrac{1}{\cos x}$

$\dfrac{\sin^2 x}{\cos^3 x}$ Use $\tan x = \dfrac{\sin x}{\cos x}$

$\dfrac{\tan^2 x}{\cos x}$ Use $\tan^2 x + 1 = \sec^2 x$

$\dfrac{\sec^2 x - 1}{\cos x}$ ■

4) $\sin^2 x\left(1 - \csc^2 x\right)$ Use $\cot^2 x + 1 = \csc^2 x$

$-\sin^2 x \cot^2 x$ Decompose into sine and cosine

$-\sin^2 x \cdot \left(\dfrac{\cos x}{\sin x}\right)^2$ Simplify

$-\cos^2 x$ ■

5) $\dfrac{\csc^2 x\tan^2 x}{\sec x}$ Decompose into sine and cosine

$\dfrac{\left(\dfrac{1}{\sin x}\right)^2 \cdot \left(\dfrac{\sin x}{\cos x}\right)^2}{\dfrac{1}{\cos x}}$ Simplify

$\dfrac{1}{\cos x}$ Use $\sin^2 x + \cos^2 x = 1$

$\dfrac{1}{\cos x \cdot \left(\sin^2 x + \cos^2 x\right)}$ Use $\sec x = \dfrac{1}{\cos x}$

$\dfrac{\sec x}{\sin^2 x + \cos^2 x}$ ■

6) $\dfrac{\csc^2 x}{\cos x\cot x}$ Decompose into sine and cosine

$\dfrac{\left(\dfrac{1}{\sin x}\right)^2}{\cos x \cdot \dfrac{\cos x}{\sin x}}$ Simplify

$\dfrac{1}{\sin x\cos^2 x}$ Use $\sec x = \dfrac{1}{\cos x}$

$\dfrac{\sec^2 x}{\sin x}$ Use $\tan^2 x + 1 = \sec^2 x$

$\dfrac{1 + \tan^2 x}{\sin x}$ ■

7) $\dfrac{\cot^2 x}{\cos^2 x}$ Use $\cot x = \dfrac{\cos x}{\sin x}$

$\dfrac{\cos^2 x}{\cos^2 x\sin^2 x}$ Cancel common factors

$\dfrac{1}{\sin^2 x}$ Use $\csc x = \dfrac{1}{\sin x}$

$\csc^2 x$ Use $\cot^2 x + 1 = \csc^2 x$

$1 + \cot^2 x$ ■

8) $\csc^2 x\left(1 - \sec^2 x\right)$ Use $\tan^2 x + 1 = \sec^2 x$

$-\csc^2 x\tan^2 x$ Decompose into sine and cosine

$-\left(\dfrac{1}{\sin x}\right)^2 \cdot \left(\dfrac{\sin x}{\cos x}\right)^2$ Simplify

$-\dfrac{1}{\cos^2 x}$ Use $\sec x = \dfrac{1}{\cos x}$

$-\sec^2 x$ ■

9) $\dfrac{\csc^2 x - 1}{\cos^2 x}$ Use $\cot^2 x + 1 = \csc^2 x$

$\dfrac{\cot^2 x}{\cos^2 x}$ Decompose into sine and cosine

$\dfrac{\left(\dfrac{\cos x}{\sin x}\right)^2}{\cos^2 x}$ Simplify

$\dfrac{1}{\sin^2 x}$ Use $\csc x = \dfrac{1}{\sin x}$

$\dfrac{\csc x}{\sin x}$ ■

10) $\csc^2 x + \sec^2 x$ Decompose into sine and cosine

$\left(\dfrac{1}{\sin x}\right)^2 + \left(\dfrac{1}{\cos x}\right)^2$ Simplify

$\dfrac{\cos^2 x + \sin^2 x}{\sin^2 x \cos^2 x}$ Use $\sin^2 x + \cos^2 x = 1$

$\dfrac{1}{\cos^2 x \sin^2 x}$ Use $\sec x = \dfrac{1}{\cos x}$

$\dfrac{\sec^2 x}{\sin^2 x}$ ■