Assignment: Volume -2

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the x-axis.

1)
$$y = 2\sqrt{\cos x}$$
, $y = 0$, $x = -\frac{\pi}{4}$, $x = \frac{\pi}{6}$ 2) $y = \sec x$, $y = 0$, $x = -\frac{\pi}{3}$, $x = 0$

2)
$$y = \sec x$$
, $y = 0$, $x = -\frac{\pi}{3}$, $x = 0$

3)
$$y = -x^2 + 5$$
, $y = -x + 3$

4)
$$y = \sqrt{x + 1}$$
, $y = x^2 + 1$

5)
$$y = 2$$
, $y = \sqrt{\cos x}$, $x = -\frac{\pi}{2}$, $x = 0$

6)
$$y = 2\sec x$$
, $y = \sec x$, $x = 0$, $x = \frac{\pi}{4}$

For each problem, find the volume of the specified solid.

7) The base of a solid is the region enclosed by the semicircle $y = \sqrt{9 - x^2}$ and the x-axis. Cross-sections perpendicular to the *x*-axis are squares.

8) The base of a solid is the region enclosed by the semicircle $y = \sqrt{9 - x^2}$ and the *x*-axis. Cross-sections perpendicular to the *x*-axis are rectangles with heights half that of the side in the *x y*-plane.

9) The base of a solid is the region enclosed by y = 4 and $y = \frac{x^2}{9}$. Cross-sections perpendicular to the *x*-axis are squares.

10) The base of a solid is the region enclosed by y = 4 and $y = \frac{x^2}{4}$. Cross-sections perpendicular to the *x*-axis are squares.

Answers to Assignment: Volume -2

1)
$$\pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{6}} (2\sqrt{\cos x})^2 dx$$
 2) $\pi \int_{-\frac{\pi}{3}}^{0} \sec^2 x dx$
$$= (2 + 2\sqrt{2})\pi \approx 15.169$$
 2) $\pi \int_{-\frac{\pi}{3}}^{0} \sec^2 x dx$
$$= \sqrt{3} \cdot \pi \approx 5.441$$

2)
$$\pi \int_{-\frac{\pi}{3}}^{0} \sec^2 x \, dx$$
$$= \sqrt{3} \cdot \pi \approx 5.441$$

3)
$$\pi \int_{-1}^{2} ((-x^2 + 5)^2 - (-x + 3)^2) dx$$

= $\frac{153}{5} \pi \approx 96.133$

4)
$$\pi \int_0^1 ((\sqrt{x} + 1)^2 - (x^2 + 1)^2) dx$$

= $\frac{29}{30} \pi \approx 3.037$

5)
$$\pi \int_{-\frac{\pi}{2}}^{0} \left(2^{2} - \left(\sqrt{\cos x}\right)^{2}\right) dx$$

= $(2\pi - 1)\pi \approx 16.598$

5)
$$\pi \int_{-\frac{\pi}{2}}^{0} \left(2^{2} - (\sqrt{\cos x})^{2}\right) dx$$
 6) $\pi \int_{0}^{\frac{\pi}{4}} \left((2\sec x)^{2} - \sec^{2} x\right) dx$ 7) $\int_{-3}^{3} (\sqrt{9 - x^{2}})^{2} dx$ 8) $\frac{1}{2} \int_{-3}^{3} (\sqrt{9 - x^{2}})^{2} dx$ 9) $\int_{-6}^{6} \left(4 - \frac{x^{2}}{9}\right)^{2} dx$ 10) $\int_{-4}^{4} \left(4 - \frac{x^{2}}{4}\right)^{2} dx$

8)
$$\frac{1}{2} \int_{-3}^{3} (\sqrt{9 - x^2})^2 dx$$
$$= 18$$

9)
$$\int_{-6}^{6} \left(4 - \frac{x^2}{9}\right)^2 dx$$
$$= \frac{512}{5} = 102.4$$

10)
$$\int_{-4} \left(4 - \frac{x^2}{4}\right)^2 dx$$
$$= \frac{1024}{15} \approx 68.267$$