SATPREP **Name** ____________________

Assignment: Complex Number **Date** ____________________

1. Given that $z \in \mathbb{C}$, solve the equation $z^3 - 8\text{i} = 0$, giving your answers in the form $z = r(\cos\theta + \text{i}\sin\theta)$.

2. Let $z_1 = r\left(\cos\frac{\pi}{4} + \text{i}\sin\frac{\pi}{4}\right)$ and $z_2 = 1 + \sqrt{3}\,\text{i}$.

(a) Write z_2 in modulus-argument form.

(b) Find the value of r if $\left|z_1 z_2^3\right| = 2$.

3. The complex number z satisfies $\text{i}(z + 2) = 1 - 2z$, where $\text{i} = \sqrt{-1}$. Write z in the form $z = a + b\text{i}$, where a and b are real numbers.

4. Find, in its simplest form, the argument of $(\sin\theta + \text{i}(1 - \cos\theta))^2$ where θ is an acute angle.

5. Let $z = x + y\text{i}$. Find the values of x and y if $(1 - \text{i})z = 1 - 3\text{i}$.

6. Let $z_1 = a\left(\cos\frac{\pi}{4} + \text{i}\sin\frac{\pi}{4}\right)$ and $z_2 = b\left(\cos\frac{\pi}{3} + \text{i}\sin\frac{\pi}{3}\right)$.

Express $\left(\frac{z_1}{z_2}\right)^3$ in the form $z = x + y\text{i}$.

7. The complex number z satisfies the equation

$$\sqrt{z} = \frac{2}{1 - \text{i}} + 1 - 4\text{i}.$$

Express z in the form $x + \text{i}y$ where $x, y \in \mathbb{Z}$.

8. The complex number z is defined by

$$z = 4\left(\cos\frac{2\pi}{3} + \text{i}\sin\frac{2\pi}{3}\right) + 4\sqrt{3}\left(\cos\frac{\pi}{6} + \text{i}\sin\frac{\pi}{6}\right).$$

(a) Express z in the form $re^{\text{i}\theta}$, where r and θ have exact values.

(b) Find the cube roots of z, expressing in the form $re^{\text{i}\theta}$, where r and θ have exact values.

9. Let $z_1 = \frac{\sqrt{6} - \text{i}\sqrt{2}}{2}$, and $z_2 = 1 - \text{i}$.

(a) Write z_1 and z_2 in the form $r(\cos\theta + \text{i}\sin\theta)$, where $r > 0$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

(b) Show that $\frac{z_1}{z_2} = \cos\frac{\pi}{12} + \text{i}\sin\frac{\pi}{12}$.

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Answer of assignment Complex Number

1.

$$\Rightarrow z_1 = 2\left(\cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}\right)$$

$$\Rightarrow z_2 = 2\left(\cos\frac{5\pi}{6} + \mathrm{i}\sin\frac{5\pi}{6}\right)$$

$$\Rightarrow z_3 = 2\left(\cos\frac{3\pi}{2} + \mathrm{i}\sin\frac{3\pi}{2}\right) \text{or } z_3 = 2\left(\cos -\frac{\pi}{2} + \mathrm{i}\sin -\frac{\pi}{2}\right)$$

2. (a)

$$\Rightarrow z_2 = 2\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right) \quad \left(\text{accept } 2\mathrm{e}^{\mathrm{i}\frac{\pi}{3}},\ 2\mathrm{e}^{1.05\mathrm{i}}\right)$$

(b) $r = \dfrac{1}{4}$

3. $-\mathrm{i}.(a = 0, b = -1)$

4. $\alpha = \theta$

5. $z = 2 - \mathrm{i}$

6. $x = \dfrac{\sqrt{2}a^3}{2b^3}, y = \dfrac{-\sqrt{2}a^3}{2b^3}$, or $x = \dfrac{a^3}{\sqrt{2}b^3}, y = \dfrac{-a^3}{\sqrt{2}b^3}$

7. $-5 - 12\mathrm{i}$ (or $x = -5, y = -12$)

8. (a) $\left(z = 8\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{3}\right)}\right)$

(b) $z^{\frac{1}{3}} = 2\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{9}\right)} \left(= 2\mathrm{e}^{\mathrm{i}20°}\right)$

$z^{\frac{1}{3}} = 2\mathrm{e}^{\mathrm{i}\left(\frac{7\pi}{9}\right)} \left(= 2\mathrm{e}^{\mathrm{i}140°}\right), z^{\frac{1}{3}} = 2\mathrm{e}^{\mathrm{i}\left(\frac{13\pi}{9}\right)} \left(= 2\mathrm{e}^{\mathrm{i}260°}\right)$

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9. (a) $z_1 = \sqrt{2}\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$ $z_2 = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

(b) $\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}$

(c) $\cos\frac{\pi}{12} = \frac{\sqrt{2+\sqrt{3}}}{2}$, $\sin\frac{\pi}{12} = \frac{\sqrt{2-\sqrt{3}}}{2}$

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