SATPREP Name____________________

Assignment : Double Angle Identity

Date__________

Verify each identity.

1) $2\cos^2 x\tan^2 x = \sin^2 x - \cos 2x + \cos^2 x$

2) $\dfrac{2\cot^2 x}{\sin^2 x} = \csc^4 x(1 + \cos 2x)$

3) $\dfrac{\sec^2 x}{\cot x\tan x} = \dfrac{1}{1 + \cos 2x - \cos^2 x}$

4) $\dfrac{\cos^2 x + \cot^2 x}{\cot^2 x} = \cos^2 x + 1 - \cos 2x$

5) $\tan^2 x\cos 2x + 2\sin^2 x = \dfrac{2\cos 2x + 1}{\cot^2 x}$

6) $\tan x + 2\sin^2 x = \dfrac{1 + \sin 2x}{\cot x}$

7) $\dfrac{\tan 2x}{2\tan^2 x\sin x\cos x} = \dfrac{\cot^2 x}{\cos 2x}$

8) $\dfrac{2(1 + \cos 2x)}{\cot^2 x} = 2\sin^2 x + 1 - \cos 2x$

Solve each equation for $0 \le \theta < 2\pi$.

9) $2\sin 2\theta = \sin\theta + 3\sin 2\theta$

10) $-3\cos^2\theta + \sin^2 2\theta = 0$

11) $\cos 2\theta = -3\sin^2 \theta + 2\sin \theta$

12) $-\cos 2\theta = 6\sin^2 \theta + 4\sin \theta$

Answers to Assignment : Double Angle Identity

1) $2\cos^2 x\tan^2 x$ — Decompose into sine and cosine

$2\cos^2 x \cdot \left(\frac{\sin x}{\cos x}\right)^2$ — Simplify

$2\sin^2 x$ — Use $\cos 2x = \cos^2 x - \sin^2 x$

$\sin^2 x - \cos 2x + \cos^2 x$ ■

2) $\frac{2\cot^2 x}{\sin^2 x}$ — Use $\cot x = \frac{\cos x}{\sin x}$

$\frac{2\cos^2 x}{\sin^4 x}$ — Use $\cos 2x = 2\cos^2 x - 1$

$\frac{1 + \cos 2x}{\sin^4 x}$ — Use $\csc x = \frac{1}{\sin x}$

$\csc^4 x(1 + \cos 2x)$ ■

3) $\frac{\sec^2 x}{\cot x\tan x}$ — Decompose into sine and cosine

$\frac{\left(\frac{1}{\cos x}\right)^2}{\frac{\cos x}{\sin x} \cdot \frac{\sin x}{\cos x}}$ — Simplify

$\frac{1}{\cos^2 x}$ — Use $\cos 2x = 2\cos^2 x - 1$

$\frac{1}{1 + \cos 2x - \cos^2 x}$ ■

4) $\frac{\cos^2 x + \cot^2 x}{\cot^2 x}$ — Decompose into sine and cosine

$\frac{\cos^2 x + \left(\frac{\cos x}{\sin x}\right)^2}{\left(\frac{\cos x}{\sin x}\right)^2}$ — Simplify

$\sin^2 x + 1$ — Use $\cos 2x = \cos^2 x - \sin^2 x$

$\cos^2 x + 1 - \cos 2x$ ■

5) $\tan^2 x\cos 2x + 2\sin^2 x$ — Use $\cos 2x = 2\cos^2 x - 1$

$2\tan^2 x\cos^2 x - \tan^2 x + 2\sin^2 x$ — Decompose into sine and cosine

$2 \cdot \left(\frac{\sin x}{\cos x}\right)^2 \cdot \cos^2 x - \left(\frac{\sin x}{\cos x}\right)^2 + 2\sin^2 x$ — Simplify

$\frac{\sin^2 x(4\cos^2 x - 1)}{\cos^2 x}$ — Use $\cos 2x = 2\cos^2 x - 1$

$\frac{\sin^2 x(2\cos 2x + 1)}{\cos^2 x}$ — Use $\cot x = \frac{\cos x}{\sin x}$

$\frac{2\cos 2x + 1}{\cot^2 x}$ ■

6) $\tan x + 2\sin^2 x$ — Decompose into sine and cosine

$\frac{\sin x}{\cos x} + 2\sin^2 x$ — Simplify

$\frac{\sin x \cdot (2\sin x\cos x + 1)}{\cos x}$ — Use $\sin 2x = 2\sin x\cos x$

$\frac{\sin x \cdot (1 + \sin 2x)}{\cos x}$ — Use $\cot x = \frac{\cos x}{\sin x}$

$\frac{1 + \sin 2x}{\cot x}$ ■

7) $\frac{\tan 2x}{2\tan^2 x\sin x\cos x}$ — Use $\sin 2x = 2\sin x\cos x$

$\frac{\tan 2x}{\tan^2 x\sin 2x}$ — Use $\tan 2x = \frac{\sin 2x}{\cos 2x}$

$\frac{\sin 2x}{\tan^2 x\cos 2x\sin 2x}$ — Cancel common factors

$\frac{1}{\cos 2x\tan^2 x}$ — Use $\cot x = \frac{1}{\tan x}$

$\frac{\cot^2 x}{\cos 2x}$ ■

8) $\frac{2(1 + \cos 2x)}{\cot^2 x}$ — Use $\cos 2x = 2\cos^2 x - 1$

$\frac{4\cos^2 x}{\cot^2 x}$ — Use $\cot x = \frac{\cos x}{\sin x}$

$\frac{4\sin^2 x\cos^2 x}{\cos^2 x}$ — Cancel common factors

$4\sin^2 x$ — Use $\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x + 1 - \cos 2x$ ■

9) $\left\{0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}\right\}$

10) $\left\{\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}\right\}$

11) $\left\{\frac{\pi}{2}\right\}$

12) $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$