SATPREP

PARAMETRICS AND CALCULUS

On problems 1 – 5, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

1. $x = t^2,\ y = t^2 + 6t + 5$
2. $x = t^2 + 1,\ y = 2t^3 - t^2$
3. $x = \sqrt{t},\ y = 3t^2 + 2t$
4. $x = \ln t,\ y = t^2 + t$
5. $x = 3\sin t + 2,\ y = 4\cos t - 1$

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6. A curve C is defined by the parametric equations $x = t^2 + t - 1,\ y = t^3 - t^2$.

(a) Find $\frac{dy}{dx}$ in terms of t.

(b) Find an equation of the tangent line to C at the point where $t = 2$.

7. A curve C is defined by the parametric equations $x = 2\cos t,\ y = 3\sin t$.

(a) Find $\frac{dy}{dx}$ in terms of t.

(b) Find an equation of the tangent line to C at the point where $t = \frac{\pi}{4}$.

On problems 8 – 10, find:

(a) $\frac{dy}{dx}$ in terms of t.

(b) all points of horizontal and vertical tangency

8. $x = t + 5,\ y = t^2 - 4t$
9. $x = t^2 - t + 1,\ y = t^3 - 3t$
10. $x = 3 + 2\cos t,\ y = -1 + 4\sin t$

On problems 11 - 12, a curve C is defined by the parametric equations given. For each problem, write an integral expression that represents the length of the arc of the curve over the given interval.

11. $x = t^2,\ y = t^3,\ 0 \le t \le 2$
12. $x = e^{2t} + 1,\ y = 3t - 1,\ -2 \le t \le 2$

Answers to Worksheet on Parametrics and Calculus

1. $\dfrac{dy}{dx}=\dfrac{2t+6}{2t}=1+\dfrac{3}{t};\quad \dfrac{d^2y}{dx^2}=\dfrac{-\dfrac{3}{t^2}}{2t}=-\dfrac{3}{2t^3}$

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2. $\dfrac{dy}{dt}=3t-1;\quad \dfrac{d^2y}{dx^2}=\dfrac{3}{2t}$

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3. $\dfrac{dy}{dx}=\dfrac{6t+2}{\frac{1}{2}t^{-\frac{1}{2}}}=12t^{\frac{3}{2}}+4t^{\frac{1}{2}};\quad \dfrac{d^2y}{dx^2}=\dfrac{18t^{\frac{1}{2}}+2t^{-\frac{1}{2}}}{\frac{1}{2}t^{-\frac{1}{2}}}=36t+4$

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4. $\dfrac{dy}{dx}=\dfrac{2t+1}{\frac{1}{t}}=2t^2+t;\quad \dfrac{d^2y}{dx^2}=\dfrac{4t+1}{\frac{1}{t}}=4t^2+t$

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5. $\dfrac{dy}{dx}=\dfrac{-4\sin t}{3\cos t}=-\dfrac{4}{3}\tan t;\quad \dfrac{d^2y}{dx^2}=\dfrac{-\frac{4}{3}\sec^2 t}{3\cos t}=-\dfrac{4}{9}\sec^3 t$

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6. (a) $\dfrac{dy}{dx}=\dfrac{3t^2-2t}{2t+1}$

(b) When $t=1$, $\dfrac{dy}{dx}=\dfrac{3\cdot 1^2-2\cdot 1}{2\cdot 1+1}=\dfrac{8}{5}$, $x=5$, $y=4$ so the tangent line equation is

$y-4=\dfrac{8}{5}(x-5)$

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7. (a) $\dfrac{dy}{dx}=\dfrac{3\cos t}{-2\sin t}=-\dfrac{3}{2}\cot t$

(b) When $t=\dfrac{\pi}{4}$, $\dfrac{dy}{dx}=-\dfrac{3}{2}\cot\dfrac{\pi}{4}=-\dfrac{3}{2}$, $x=\sqrt{2}$, $y=\dfrac{3\sqrt{2}}{2}$ so the tangent line equation is

$y-\dfrac{3\sqrt{2}}{2}=-\dfrac{3}{2}\left(x-\sqrt{2}\right)$

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8. (a) $\dfrac{dy}{dx}=\dfrac{2t-4}{1}$

(b) A horizontal tangent occurs when $\dfrac{dy}{dt}=0$ and $\dfrac{dx}{dt}\neq 0$ so a horizontal tangent occurs when $2t-4=0$ which is at $t=2$. When $t=2$, $x=7$ and $y=-4$ so a horizontal tangent occurs at the point $(7,-4)$. A vertical tangent occurs when $\dfrac{dx}{dt}=0$ and $\dfrac{dy}{dt}\neq 0$

Since $1\neq 0$, there is no point of vertical tangency on this curve.

9. (a) $\frac{dy}{dx}=\frac{3t^2-3}{2t-1}$

(b) A horizontal tangent occurs when $\frac{dy}{dt}=0$ and $\frac{dx}{dt}\neq 0$ so a horizontal tangent occurs when $3t^2-3=0$ which is at $t=\pm 1$. When $t=1$, $x=1$ and $y=-2$, and when $t=-1$, $x=3$ and $y=2$ so a horizontal tangent occurs at the points $(1,-2)$ and $(3,2)$

A vertical tangent occurs when $\frac{dx}{dt}=0$ and $\frac{dy}{dt}\neq 0$ so a vertical tangent occurs when $2t-1=0$ so $t=\frac{1}{2}$. When $t=\frac{1}{2}$, $x=\frac{3}{4}$ and $y=-\frac{11}{8}$ so a vertical tangent occurs at the point $\left(\frac{3}{4},-\frac{11}{8}\right)$.

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10. (a) $\frac{dy}{dx}=\frac{4\cos t}{-2\sin t}$

(b) A horizontal tangent occurs when $\frac{dy}{dt}=0$ and $\frac{dx}{dt}\neq 0$ so a horizontal tangent occurs when $4\cos t=0$ which is at $t=\frac{\pi}{2}$ and $\frac{3\pi}{2}$. When $t=\frac{\pi}{2}$, $x=3$ and $y=3$, and when $t=\frac{3\pi}{2}$, $x=3$ and $y=-5$ so a horizontal tangent occurs at the points $(3,3)$ and $(3,-5)$.

A vertical tangent occurs when $\frac{dx}{dt}=0$ and $\frac{dy}{dt}\neq 0$ so a vertical tangent occurs when $-2\sin t=0$ so $t=0$ and π. When $t=0$, $x=5$ and $y=-1$ and when $t=\pi$, $x=1$ and $y=-1$ so a vertical tangent occurs at the points $(5,-1)$ and $(1,-1)$.

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11. $s=\int_a^b\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt=\int_0^2\sqrt{(2t)^2+(3t^2)^2}\,dt$ or $\int_0^2\sqrt{4t^2+9t^4}\,dt$

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12. $s=\int_a^b\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt=\int_{-2}^2\sqrt{(2e^{2t})^2+(3)^2}\,dt$ or $\int_{-2}^2\sqrt{4e^{4t}+9}\,dt$