SATPREP **Name** _____

Assignment : Function and Equation **Date** _____

1. Consider the functions given below.

$$f(x) = 2x + 3$$
$$g(x) = \frac{1}{x},\ x \neq 0$$

(a) (i) Find $(g \circ f)(x)$ and write down the domain of the function.

(ii) Find $(f \circ g)(x)$ and write down the domain of the function.

(b) Find the coordinates of the point where the graph of $y = f(x)$ and the graph of $y = (g^{-1} \circ f \circ g)(x)$ intersect.

2. The quadratic function $f(x) = p + qx - x^2$ has a maximum value of 5 when $x = 3$.

(a) Find the value of p and the value of q.

(b) The graph of $f(x)$ is translated 3 units in the positive direction parallel to the x-axis. Determine the equation of the new graph.

3. Write $\ln(x^2 - 1) - 2\ln(x + 1) + \ln(x^2 + x)$ as a single logarithm, in its simplest form.

4. Consider the function f, where $f(x) = \arcsin(\ln x)$.

(a) Find the domain of f.

(b) Find $f^{-1}(x)$.

5. Solve the equation $4^{x-1} = 2^x + 8$.

6. Let $f(x) = \frac{1-x}{1+x}$ and $g(x) = \sqrt{x+1}$, $x > -1$.

Find the set of values of x for which $f'(x) \leq f(x) \leq g(x)$.

7. A function f is defined by $f(x) = \frac{2x-3}{x-1}$, $x \neq 1$.

(a) Find an expression for $f^{-1}(x)$.

(b) Solve the equation $\left|f^{-1}(x)\right| = 1 + f^{-1}(x)$.

8. Consider the function $f: x \to \sqrt{\frac{\pi}{4} - \arccos x}$.

(a) Find the largest possible domain of f.

(b) Determine an expression for the inverse function, f^{-1}, and write down its domain.

9. Let $f(x) = \frac{4-x^2}{4-\sqrt{x}}$. State the largest possible domain for f.

10. Find the solution of the equation $\ln 2^{4x-1} = \ln 8^{x+5} + \log_2 16^{1-2x}$, expressing your answer in terms of $\ln 2$.

Answer to Assignment Function and Equation

1. (a) (i) $\frac{1}{2x+3}, x \neq -\frac{3}{2}$ (or equivalent)

(ii) $\frac{2}{x} + 3, x \neq 0$ (or equivalent)

(b) $\frac{1}{2x+3} = \frac{2}{x} + 3$

THEN

$6x^2 + 12x + 6 = 0$ (or equivalent)
$x = -1, y = 1$ (coordinates are $(-1, 1)$)

2. (a) $q = 6, p = -4$

(b) $g(x) = -4 + 6(x - 3) - (x - 3)^2$ $(= -31 + 12x - x^2)$

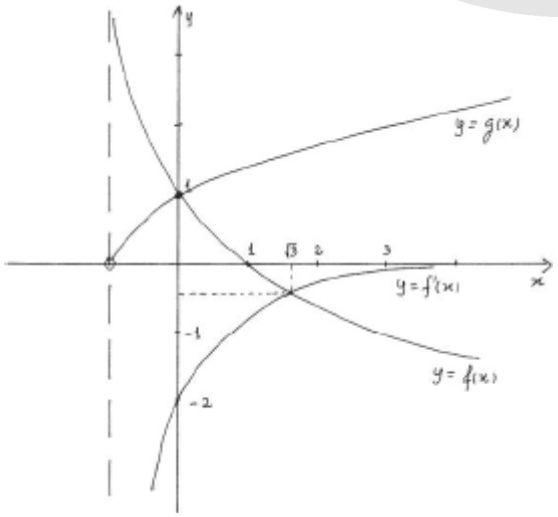
3. $\ln x(x - 1)$

4. (a) $-1 \leq \ln x \leq 1$

(b) $f^{-1}(x) = e^{\sin x}$

5. $x = 3$

6.



$f'(x) = \frac{-2}{(1+x)^2}$

7. (a) $\Rightarrow f^{-1}(x) = \dfrac{x-3}{x-2}$ $(x \neq 2)$

(b) $x = \dfrac{8}{3}$

8. (a)

$\Rightarrow \dfrac{\sqrt{2}}{2} \leq x \leq 1$ $\left(\text{accept } \dfrac{1}{\sqrt{2}} \leq x \leq 1\right)$

(b) $0 \leq x \leq \sqrt{\dfrac{\pi}{4}}$

9. (a) $x \geq 0$ and $x \neq 16$

(b) $x = 0$ or $x = 1$
$0 \leq x \leq 1$ or $x > 16$

10. (a) $x = \dfrac{4 + 16\ln 2}{8 + \ln 2}$

(b) $x = a^2$

$a = 1.318$