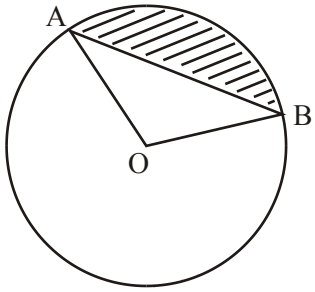
SATPREP

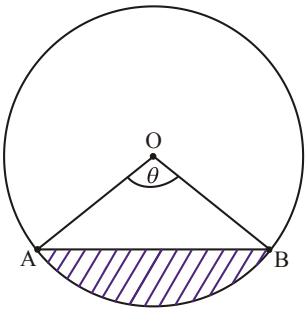
Assignment : Circular measure -2

1. The diagram below shows a circle centre O and radius OA = 5 cm. The angle $A\hat{O}B = 135°$.



Find the area of the shaded region.

2. The following diagram shows the points A and B on the circumference of a circle, centre O, and radius 4 cm, where $A\hat{O}B = \theta$. Points A and B are moving on the circumference so that θ is increasing at a constant rate.



Given that the rate of change of the length of the minor arc AB is numerically equal to the rate of change of the area of the shaded segment, find the acute value of θ.

3. The diagram below shows two straight lines intersecting at O and two circles, each with centre O. The outer circle has radius R and the inner circle has radius r.

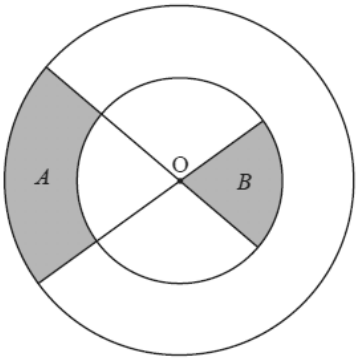
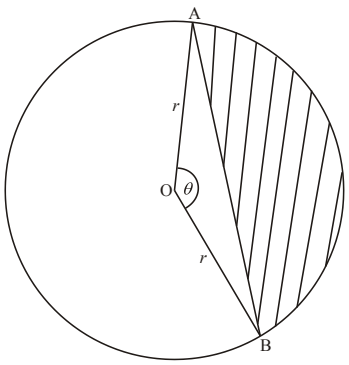


diagram not to scale

Consider the shaded regions with areas A and B. Given that $A : B = 2 : 1$, find the **exact** value of the ratio $R : r$

4. The following diagram shows a circle centre O, radius r. The angle $A\hat{O}B$ at the centre of the circle is θ radians. The chord AB divides the circle into a minor segment (the shaded region) and a major segment.



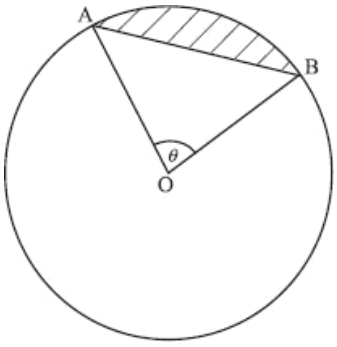
(a) Show that the area of the minor segment is $\frac{1}{2}r^2(\theta - \sin\theta)$.

(b) Find the area of the major segment.

(c) Given that the ratio of the areas of the two segments is 2:3, show that $\sin\theta = \theta - \frac{4\pi}{5}$.

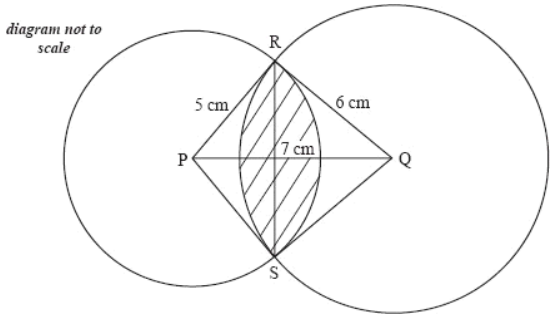
(d) Hence find the value of θ.

5. The diagram shows a circle centre O and radius 1, with $A\hat{O}B = \theta$, $\theta \neq 0$. The area of ΔAOB is three times the shaded area.



Find the value of θ.

6. The diagram below shows a pair of intersecting circles with centres at P and Q with radii of 5 cm and 6 cm respectively. RS is the common chord of both circles and PQ is 7 cm.



Find the area of the shaded region.

7. The interior of a circle of radius 2 cm is divided into an infinite number of sectors. The areas of these sectors form a geometric sequence with common ratio k. The angle of the first sector is θ radians.

(a) Show that $\theta = 2\pi(1 - k)$.

(b) The perimeter of the third sector is half the perimeter of the first sector.

Find the value of k and of θ.

8. The diagram below shows two concentric circles with centre O and radii 2 cm and 4 cm.

The points P and Q lie on the larger circle and $\text{P}\hat{\text{O}}\text{Q} = x$, where $0 < x < \frac{\pi}{2}$.

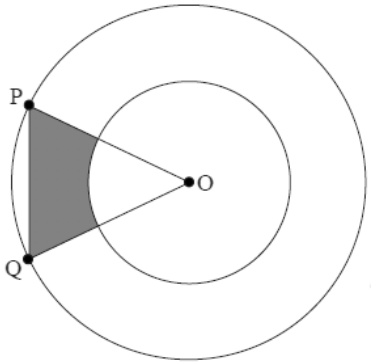


diagram not to scale

(a) Show that the area of the shaded region is $8 \sin x - 2x$.

(b) Find the maximum area of the shaded region.

Answer to assignment circular measure -2

1. Shaded area = area of sector OAB – area of ΔOAB = 20.6 (cm^2)

2. $\theta = \frac{\pi}{3}\ (\text{Accept } 60°)$

3. ratio $R : r$ is $\sqrt{3}$:1

4. (a) $\frac{1}{2}r^2(\theta - \sin\theta)$

(b) Area of the major segment = area of circle – shaded area $= \pi r^2 - \frac{1}{2}r^2(\theta - \sin\theta)$ $\left(= r^2\left(\pi - \frac{\theta}{2} + \frac{\sin\theta}{2}\right)\right)$

(c) Given ratio of segments is 3:2

$\Rightarrow \sin\theta = \theta - \frac{4\pi}{5}$

(d) $\theta = 2.82$ radians

5. $\theta = 1.28$ radians

6. 23.4 cm^2

7. $k = 0.456$ and then $\theta = 3.42$

8. (a) $8 \sin x - 2x$ *(b)* 5.11