

1. Find the domain and range of each of the following, where y is a function of x .

(a) $y = 5x + 3$

(h) $y = -1 + 2x - x^2$

(o) $y = \sqrt{\frac{1}{x-2}}$

(b) $y = -7x - 4$

(i) $y = \sqrt{x+9}$

(p) $y = \frac{2}{\sqrt{2x-5}}$

(c) $y = 7$

(j) $y = \sqrt{3x-4}$

(q) $y = \frac{x}{\sqrt{x+1}}$

(d) $y = x^2$

(k) $y = \sqrt{x^2-4}$

(e) $y = 2x^2 + 1$

(l) $y = \sqrt{4-x^2}$

(f) $y = x^2 - 2x + 5$

(m) $y = \frac{1}{7-x}$

(r)* $y = \sqrt{x-2} + \frac{1}{x-5}$

(g) $y = 4 - x^2$

(n) $y = \frac{6}{x+2}$

(s) $y = \frac{1}{1 - \frac{1}{x-2}}$

2. For each of the following pairs of functions, find

1. $(f \circ g)(x)$

2. $(g \circ f)(x)$

3. The domain of $(f \circ g)(x)$

4. The range of $(f \circ g)(x)$

(a) $f(x) = 2x + 4, \quad g(x) = 3x - 5$

(e) $f(x) = \frac{1}{x}, \quad g(x) = \frac{1}{x}$

(b) $f(x) = x^2, \quad g(x) = x - 3$

(f) $f(x) = x^2 + 2, \quad g(x) = \sqrt{x-4}$

(c) $f(x) = \sqrt{x+2}, \quad g(x) = 2x - 4$

(g)* $f(x) = \frac{1}{1-x}, \quad g(x) = \sqrt{5-x}$

(d) $f(x) = 3x - 4, \quad g(x) = \sqrt{x+2}$

(h)* $f(x) = \sqrt{x-2}, \quad g(x) = \frac{3}{x-9}$

* The ranges of $(f \circ g)(x)$ in (g) and (h) are hard to find because the functions produce graphs you may not have covered previously. See if you can work them out with some intelligent guesswork.

Answer

1

(a) $D = \mathbb{R}, R = \mathbb{R}$

(b) $D = \mathbb{R}, R = \mathbb{R}$

(c) $D = \mathbb{R}, R = 7$

(d) $D = \mathbb{R}, R = \{y \in \mathbb{R} : y \geq 0\}$

(e) $D = \mathbb{R}, R = \{y \in \mathbb{R} : y \geq 1\}$

(f) $D = \mathbb{R}, R = \{y \in \mathbb{R} : y \geq 4\}$

(g) $D = \mathbb{R}, R = \{y \in \mathbb{R} : y \leq 4\}$

(h) $D = \mathbb{R}, R = \{y \in \mathbb{R} : y \leq 0\}$

(i) $D = \{x \in \mathbb{R} : x \geq -9\}, R = \{y \in \mathbb{R} : y \geq 0\}$

(j) $D = \{x \in \mathbb{R} : x \geq 4/3\}, R = \{y \in \mathbb{R} : y \geq 0\}$

(k) $D = \{x \in \mathbb{R} : x \leq -2 \text{ or } x \geq 2\}, R = \{y \in \mathbb{R} : y \geq 0\}$

(l) $D = \{x \in \mathbb{R} : -2 \leq x \leq 2\}, R = \{y \in \mathbb{R} : 0 \leq y \leq 2\}$

(m) $D = \{x \in \mathbb{R} : x \neq 7\}, R = \{y \in \mathbb{R} : y \neq 0\}$

(n) $D = \{x \in \mathbb{R} : x \neq -2\}, R = \{y \in \mathbb{R} : y \neq 0\}$

(o) $D = \{x \in \mathbb{R} : x > 2\}, R = \{y \in \mathbb{R} : y > 0\}$

(p) $D = \{x \in \mathbb{R} : x > 5/2\}, R = \{y \in \mathbb{R} : y > 0\}$

(q) $D = \{x \in \mathbb{R} : x > -1\}, R = \mathbb{R}$

(r) $D = \{x \in \mathbb{R} : x \geq 2 \text{ and } x \neq 5\}$

(s) $D = \{x \in \mathbb{R} : x \neq 2 \text{ and } x \neq 3\}, R = \{y \in \mathbb{R} : y \neq 0 \text{ and } y \neq 1\}$

(a)

1. $(f \circ g)(x) = 6x - 6$
2. $(g \circ f)(x) = 6x + 7$
3. $D = \mathbb{R}$
4. $R = \mathbb{R}$

(d)

1. $(f \circ g)(x) = 3\sqrt{x+2} - 4$
2. $(g \circ f)(x) = \sqrt{3x-2}$
3. $D = \{x \in \mathbb{R} : x \geq -2\}$
4. $R = \{y \in \mathbb{R} : y \geq -4\}$

(b)

1. $(f \circ g)(x) = (x-3)^2$
2. $(g \circ f)(x) = x^2 - 3$
3. $D = \mathbb{R}$
4. $R = \{y \in \mathbb{R} : y \geq 0\}$

(e)

1. $(f \circ g)(x) = x$
2. $(g \circ f)(x) = x$
3. $D = \{x \in \mathbb{R} : x \neq 0\}$
4. $R = \{y \in \mathbb{R} : y \neq 0\}$

(c)

1. $(f \circ g)(x) = \sqrt{2x-2}$
2. $(g \circ f)(x) = 2\sqrt{x+2} - 4$
3. $D = \{x \in \mathbb{R} : x \geq 1\}$
4. $R = \{y \in \mathbb{R} : y \geq 0\}$

(f)

1. $(f \circ g)(x) = x - 2$
2. $(g \circ f)(x) = \sqrt{x^2 - 2}$
3. $D = \{x \in \mathbb{R} : x \geq 4\}$
4. $R = \{y \in \mathbb{R} : y \geq 2\}$

(g)*

1. $(f \circ g)(x) = \frac{1}{1 - \sqrt{5-x}}$

2. $(g \circ f)(x) = \sqrt{5 - \frac{1}{1-x}}$

3. $D = \{x \in \mathbb{R} : x \leq 5 \text{ and } x \neq 4\}$

4. $R = \{y \in \mathbb{R} : y \geq 1 \text{ or } y < 0\}$

(h)*

1. $(f \circ g)(x) = \sqrt{\frac{3}{x-9} - 2}$

2. $(g \circ f)(x) = \frac{3}{\sqrt{x-2} - 9}$

3. $D = \{x \in \mathbb{R} : 9 < x \leq \frac{21}{2}\}$

4. $R = \{y \in \mathbb{R} : y \geq 0\}$

