Subject – Math(Standard Level)
Topic - Algebra
Year - Nov 2011 – Nov 2019

Question -1

attempt to expand $\left(1+\frac{2}{3}x\right)^n$ ***(M1)***

e.g. Pascal's triangle, $\left(1+\frac{2}{3}x\right)^n = 1+\frac{2}{3}nx+\ldots$

correct first two terms of $\left(1+\frac{2}{3}x\right)^n$ (seen anywhere) ***(A1)***

e.g. $1+\frac{2}{3}nx$

correct first two terms of quadratic (seen anywhere) ***(A1)***

e.g. 9, $6nx$; $(9+6nx+n^2x^2)$

correct calculation for the x-term ***A2***

e.g. $\frac{2}{3}nx\times 9+6nx$; $6n+6n$, $12n$

correct equation ***A1***

e.g. $6n+6n=84$, $12nx=84x$

$n=7$ ***A1*** ***N1***

[7 marks]

Question -2

(a) evidence of correct formula *(M1)*

eg $\log a - \log b = \log \frac{a}{b}$, $\log\left(\frac{40}{5}\right)$, $\log 8 + \log 5 - \log 5$

Note: Ignore missing or incorrect base.

correct working *(A1)*

eg $\log_2 8$, $2^3 = 8$

$\log_2 40 - \log_2 5 = 3$ ***A1*** ***N2***

[3 marks]

(b) attempt to write 8 as a power of 2 (seen anywhere) *(M1)*

eg $(2^3)^{\log_2 5}$, $2^3 = 8$, 2^a

multiplying powers *(M1)*

eg $2^{3\log_2 5}$, $a\log_2 5$

correct working *(A1)*

eg $2^{\log_2 125}$, $\log_2 5^3$, $\left(2^{\log_2 5}\right)^3$

$8^{\log_2 5} = 125$ ***A1*** ***N3***

[4 marks]

Question -3

(a) **METHOD 1**

evidence of correct formula *(M1)*

eg $\log u^n = n\log u$, $2\log_3 p$

$\log_3(p^2) = 12$ *A1* *N2*

METHOD 2

valid method using $p = 3^6$ *(M1)*

eg $\log_3(3^6)^2$, $\log 3^{12}$, $12\log_3 3$

$\log_3(p^2) = 12$ *A1* *N2*

[2 marks]

(b) **METHOD 1**

evidence of correct formula *(M1)*

eg $\log\left(\frac{p}{q}\right) = \log p - \log q$, $6 - 7$

$\log_3\left(\frac{p}{q}\right) = -1$ *A1* *N2*

METHOD 2

valid method using $p = 3^6$ and $q = 3^7$ *(M1)*

eg $\log_3\left(\frac{3^6}{3^7}\right)$, $\log 3^{-1}$, $-\log_3 3$

$\log_3\left(\frac{p}{q}\right) = -1$ *A1* *N2*

[2 marks]

(c) **METHOD 1**

evidence of correct formula *(M1)*

eg $\log_3 uv = \log_3 u + \log_3 v$, $\log 9 + \log p$

$\log_3 9 = 2$ (may be seen in expression) *A1*

eg $2 + \log p$

$\log_3(9p) = 8$ *A1* *N2*

METHOD 2

valid method using $p = 3^6$ *(M1)*

eg $\log_3(9 \times 3^6)$, $\log_3(3^2 \times 3^6)$

correct working *A1*

eg $\log_3 9 + \log_3 3^6$, $\log_3 3^8$

$\log_3(9p) = 8$ *A1* *N2*

[3 marks]

Total [7 marks]

Question -4

(a) (i) correct expression for r *A1* *N1*

eg $r = \frac{6}{m-1}, \frac{m+4}{6}$

(ii) correct equation *A1*

eg $\frac{6}{m-1} = \frac{m+4}{6}, \frac{6}{m+4} = \frac{m-1}{6}$

correct working *(A1)*

eg $(m+4)(m-1) = 36$

correct working *A1*

eg $m^2 - m + 4m - 4 = 36$, $m^2 + 3m - 4 = 36$

$m^2 + 3m - 40 = 0$ *AG* *N0*

[4 marks]

(b) (i) valid attempt to solve *(M1)*

eg $(m+8)(m-5) = 0$, $m = \frac{-3 \pm \sqrt{9 + 4 \times 40}}{2}$

$m = -8$, $m = 5$ *A1A1* *N3*

(ii) attempt to substitute **any** value of m to find r *(M1)*

eg $\frac{6}{-8-1}, \frac{5+4}{6}$

$r = \frac{3}{2}, r = -\frac{2}{3}$ *A1A1* *N3*

[6 marks]

(c) (i) $r = -\frac{2}{3}$ (may be seen in justification) *A1*

valid reason *R1* *N0*

eg $|r| < 1$, $-1 < \frac{-2}{3} < 1$

Notes: Award ***R1*** for $|r| < 1$ only if ***A1*** awarded.

finding the first term of the sequence which has $|r| < 1$ *(A1)*

eg $-8-1$, $6 \div \frac{-2}{3}$

$u_1 = -9$ (may be seen in formula) *(A1)*

correct substitution of u_1 and **their** r into $\frac{u_1}{1-r}$, as long as $|r| < 1$ *A1*

eg $S_\infty = \frac{-9}{1 - \left(-\frac{2}{3}\right)}, \frac{-9}{\frac{5}{3}}$

$S_\infty = -\frac{27}{5}$ $(= -5.4)$ *A1* *N3*

[6 marks]

Total [16 marks]

Question -5

(a) attempt to find d *(M1)*

eg $\frac{16-10}{2}$, $10-2d=16-4d$, $2d=6$, $d=6$

$d=3$ ***A1*** ***N2***

[2 marks]

(b) correct approach *(A1)*

eg $10=u_1+2\times 3, 10-3-3$

$u_1=4$ ***A1*** ***N2***

[2 marks]

(c) correct substitution into sum or term formula *(A1)*

eg $\frac{20}{2}(2\times 4+19\times 3)$, $u_{20}=4+19\times 3$

correct simplification *(A1)*

eg $8+57$, $4+61$

$S_{20}=650$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

Question -6

(a) (i) $\log_3 27=3$ ***A1*** ***N1***

(ii) $\log_8 \frac{1}{8}=-1$ ***A1*** ***N1***

(iii) $\log_{16} 4=\frac{1}{2}$ ***A1*** ***N1***

[3 marks]

(b) correct equation with **their** three values *(A1)*

eg $\frac{3}{2}=\log_4 x$, $3+(-1)-\frac{1}{2}=\log_4 x$

correct working involving powers *(A1)*

eg $x=4^{\frac{3}{2}}$, $4^{\frac{3}{2}}=4^{\log_4 x}$

$x=8$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

Question -7

(a) valid method *(M1)*

eg $u_2 = S_2 - S_1$, $1 + k + u_2 = 5 + 3k$

$u_2 = 4 + 2k$, $u_3 = 7 + 4k$, $u_4 = 10 + 8k$ ***A1A1A1*** ***N4***

[4 marks]

(b) correct AP **or** GP *(A1)*

eg finding common difference is 3, common ratio is 2

valid approach using arithmetic **and** geometric formulas *(M1)*

eg $1 + 3(n-1)$ **and** $r^{n-1}k$

$u_n = 3n - 2 + 2^{n-1}k$ ***A1A1*** ***N4***

Note: Award ***A1*** for $3n - 2$, ***A1*** for $2^{n-1}k$.

[4 marks]

Total [8 marks]

Question -8

(a) correct approach *(A1)*

eg $d = u_2 - u_1$, $5 - 2$

$d = 3$ ***A1*** ***N2***

[2 marks]

(b) correct approach *(A1)*

eg $u_8 = 2 + 7 \times 3$, listing terms

$u_8 = 23$ ***A1*** ***N2***

[2 marks]

(c) correct approach *(A1)*

eg $S_8 = \frac{8}{2}(2 + 23)$, listing terms, $\frac{8}{2}\left(2(2) + 7(3)\right)$

$S_8 = 100$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

Question -9

(a) correct application of $\ln a^b = b\ln a$ (seen anywhere) *(A1)*

eg $\ln 4 = 2\ln 2$, $3\ln 2 = \ln 2^3$, $3\log 2 = \log 8$

correct working *(A1)*

eg $3\ln 2 - 2\ln 2$, $\ln 8 - \ln 4$

$\ln 2$ (accept $k = 2$) ***A1*** ***N2***

[3 marks]

(b) **METHOD 1**

attempt to substitute **their** answer into the equation *(M1)*

eg $\ln 2 = -\ln x$

correct application of a log rule *(A1)*

eg $\ln\frac{1}{x}$, $\ln\frac{1}{2} = \ln x$, $\ln 2 + \ln x = \ln 2x\ (=0)$

$x = \frac{1}{2}$ ***A1*** ***N2***

METHOD 2

attempt to rearrange equation, with $3\ln 2$ written as $\ln 2^3$ or $\ln 8$ *(M1)*

eg $\ln x = \ln 4 - \ln 2^3$, $\ln 8 + \ln x = \ln 4$, $\ln 2^3 = \ln 4 - \ln x$

correct working applying $\ln a \pm \ln b$ *(A1)*

eg $\frac{4}{8}$, $8x = 4$, $\ln 2^3 = \ln\frac{4}{x}$

$x = \frac{1}{2}$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

Question -10

(a) $m = 3$, $n = 4$ ***A1A1*** ***N2***

[2 marks]

(b) attempt to apply $\left(2^a\right)^b = 2^{ab}$ ***(M1)***

eg $6x+3$, $4(2x-3)$

equating **their** powers of 2 (seen anywhere) ***M1***

eg $3(2x+1) = 8x-12$

correct working ***A1***

eg $8x-12 = 6x+3$, $2x = 15$

$x = \frac{15}{2}\ (7.5)$ ***A1*** ***N2***

[4 marks]

Total [6 marks]

Question -11

evidence of valid binomial expansion with binomial coefficients **(M1)**

eg $\binom{n}{r}(3x)^r(1)^{n-r}$, $(3x)^n + n(3x)^{n-1} + \binom{n}{2}(3x)^{n-2} + \ldots$, $\binom{n}{r}(1)^{n-r}(3x)^r$

attempt to identify correct term **(M1)**

eg $\binom{n}{n-2}$, $(3x)^2$, $n-r=2$

setting **correct** coefficient or term equal to $135n$ (may be seen later) **A1**

eg $9\binom{n}{2}=135n$, $\frac{9n(n-1)}{2}x^2=135nx^2$

correct working for binomial coefficient (using ${}_nC_r$ formula) **(A1)**

eg $\frac{n(n-1)(n-2)(n-3)\ldots}{2\times1\times(n-2)(n-3)(n-4)\ldots}$, $\frac{n(n-1)}{2}$

EITHER

evidence of correct working (with linear equation in n) **(A1)**

eg $\frac{9(n-1)}{2}=135$, $\frac{9(n-1)}{2}x^2=135x^2$

correct simplification **(A1)**

eg $n-1=\frac{135\times2}{9}$, $\frac{(n-1)}{2}=15$

$n=31$ **A1** **N2**

OR

evidence of correct working (with quadratic equation in n) **(A1)**

eg $9n^2-279n=0$, $n^2-n=30n$, $(9n^2-9n)x^2=270nx^2$

evidence of solving **(A1)**

eg $9n(n-31)=0$, $9n^2=279n$

$n=31$ **A1** **N2**

[7 marks]

Question -12

Note: There are many approaches to this question, and the steps may be done in any order. There are 3 relationships they may need to apply at some stage, for the 3rd, 4th and 5th marks. These are

equating bases *eg* recognising 9 is 3^2

log rules: $\ln b + \ln c = \ln(bc)$, $\ln b - \ln c = \ln\left(\frac{b}{c}\right)$,

exponent rule: $\ln b^n = n \ln b$.

correct substitution into u_{13} formula **(A1)**

eg $\ln a + (13-1)\ln 3$

set up equation for u_{13} in any form (seen anywhere) **(M1)**

eg $\ln a + 12\ln 3 = 8\ln 9$

correct application of relationships (examples below) **(A1)(A1)(A1)**

$a = 81$ **A1 N3**

[6 marks]

Examples of application of relationships

Example 1

correct application of exponent rule for logs **(A1)**

eg $\ln a + \ln 3^{12} = \ln 9^8$

correct application of addition rule for logs **(A1)**

eg $\ln\left(a\,3^{12}\right) = \ln 9^8$

substituting for 9 or 3 in ln expression in equation **(A1)**

eg $\ln\left(a\,3^{12}\right) = \ln 3^{16}$, $\ln\left(a\,9^{6}\right) = \ln 9^8$

Example 2

recognising $9 = 3^2$ **(A1)**

eg $\ln a + 12\ln 3 = 8\ln 3^2$, $\ln a + 12\ln 9^{\frac{1}{2}} = 8\ln 9$

one correct application of exponent rule for logs relating $\ln 9$ to $\ln 3$ **(A1)**

eg $\ln a + 12\ln 3 = 16\ln 3$, $\ln a + 6\ln 9 = 8\ln 9$

another correct application of exponent rule for logs **(A1)**

eg $\ln a = \ln 3^4$, $\ln a = \ln 9^2$

Question -13

Total [6 marks]

(a) correct approach *(A1)*

eg $\ln 5 - \ln 3$

$\ln\left(\frac{5}{3}\right) = y - x$ ***A1*** ***N2***

[2 marks]

(b) recognizing factors of 45 (may be seen in log expansion) *(M1)*

eg $\ln(9 \times 5),\ 3 \times 3 \times 5,\ \log 3^2 \times \log 5$

correct application of $\log(ab) = \log a + \log b$ *(A1)*

eg $\ln 9 + \ln 5,\ \ln 3 + \ln 3 + \ln 5,\ \ln 3^2 + \ln 5$

correct working *(A1)*

eg $2\ln 3 + \ln 5,\ x + x + y$

$\ln 45 = 2x + y$ ***A1*** ***N3***

[4 marks]

Total [6 marks]

Question -14

METHOD 1

valid approach *(M1)*

eg $r = \frac{6}{x-3}$, $(x-3)\times r = 6$, $(x-3)r^2 = x+2$

correct equation in terms of x only ***A1***

eg $\frac{6}{x-3} = \frac{x+2}{6}$, $(x-3)(x+2) = 6^2$, $36 = x^2 - x - 6$

correct working *(A1)*

eg $x^2 - x - 42$, $x^2 - x = 42$

valid attempt to solve **their** quadratic equation *(M1)*

eg factorizing, formula, completing the square

evidence of correct working *(A1)*

eg $(x-7)(x+6)$, $\frac{1 \pm \sqrt{169}}{2}$

$x = 7$, $x = -6$ ***A1*** ***N4***

METHOD 2 (finding r first)

valid approach *(M1)*

eg $r = \frac{6}{x-3}$, $6r = x+2$, $(x-3)r^2 = x+2$

correct equation in terms of r only ***A1***

eg $\frac{6}{r} + 3 = 6r - 2$, $6 + 3r = 6r^2 - 2r$, $6r^2 - 5r - 6 = 0$

evidence of correct working *(A1)*

eg $(3r+2)(2r-3)$, $\frac{5 \pm \sqrt{25+144}}{12}$

$r = -\frac{2}{3}$, $r = \frac{3}{2}$ ***A1***

substituting their values of r to find x *(M1)*

eg $(x-3)\left(\frac{2}{3}\right) = 6$, $x = 6\left(\frac{3}{2}\right) - 2$

$x = 7$, $x = -6$ ***A1*** ***N4***

Question -15

(a) evidence of dividing terms (in any order) *(M1)*

eg $\frac{u_2}{u_1}$, $\frac{2\log_2 x}{\log_2 x}$

$r = \frac{1}{2}$ ***A1*** ***N2***

[2 marks]

(b) correct substitution *(A1)*

eg $\frac{2\log_2 x}{1-\frac{1}{2}}$

correct working ***A1***

eg $\frac{2\log_2 x}{\frac{1}{2}}$

$S_\infty = 4\log_2 x$ ***AG*** ***N0***

[2 marks]

(c) evidence of subtracting two terms (in any order) *(M1)*

eg $u_3 - u_2$, $\log_2 x - \log_2 \frac{x}{2}$

correct application of the properties of logs *(A1)*

eg $\log_2\left(\frac{\frac{x}{2}}{x}\right)$, $\log_2\left(\frac{x}{2}\times\frac{1}{x}\right)$, $(\log_2 x - \log_2 2) - \log_2 x$

correct working *(A1)*

eg $\log_2 \frac{1}{2}$, $-\log_2 2$

$d = -1$ ***A1*** ***N3***

[4 marks]

(d) correct substitution into the formula for the sum of an arithmetic sequence *(A1)*

eg $\frac{12}{2}(2\log_2 x + (12-1)(-1))$

correct working ***A1***

eg $6(2\log_2 x - 11)$, $\frac{12}{2}(2\log_2 x - 11)$

$12\log_2 x - 66$ ***AG*** ***N0***

[2 marks]

(e) correct equation *(A1)*

eg $12\log_2 x - 66 = 2\log_2 x$

correct working *(A1)*

eg $10\log_2 x = 66$, $\log_2 x = 6.6$, $2^{66} = x^{10}$, $\log_2\left(\frac{x^{12}}{x^2}\right) = 66$

$x = 2^{6.6}$ (accept $p = \frac{66}{10}$) ***A1*** ***N2***

[3 marks]

[Total 13 marks]

Question -16

(a) correct use $\log x^n = n\log x$ ***A1***

eg $16\ln x$

valid approach to find r *(M1)*

eg $\frac{u_{n+1}}{u_n}$, $\frac{\ln x^8}{\ln x^{16}}$, $\frac{4\ln x}{8\ln x}$, $\ln x^4 = \ln x^{16} \times r^2$

$r = \frac{1}{2}$ ***A1*** ***N2***

[3 marks]

(b) recognizing a sum (finite or infinite) *(M1)*

eg $2^4\ln x + 2^3\ln x$, $\frac{a}{1-r}$, S_∞, $16\ln x + \ldots$

valid approach (seen anywhere) *(M1)*

eg recognizing GP is the same as part (a), using **their** r value from part (a), $r = \frac{1}{2}$

correct substitution into infinite sum (only if $|r|$ is a constant and less than 1) ***A1***

eg $\frac{2^4\ln x}{1-\frac{1}{2}}$, $\frac{\ln x^{16}}{\frac{1}{2}}$, $32\ln x$

correct working *(A1)*

eg $\ln x = 2$

$x = e^2$ ***A1*** ***N3***

[5 marks]

Total [8 marks]

Question -17

(a) attempt to subtract terms *(M1)*

eg $d = u_2 - u_1$, $7-3$

$d = 4$ **A1** **N2**

[2 marks]

(b) correct approach *(A1)*

eg $u_{10} = 3 + 9(4)$

$u_{10} = 39$ **A1** **N2**

[2 marks]

(c) correct substitution into sum *(A1)*

eg $S_{10} = 5(3+39)$, $S_{10} = \frac{10}{2}(2\times 3 + 9\times 4)$

$S_{10} = 210$ **A1** **N2**

[2 marks]

[Total 6 marks]

Question -18

(a) (i) $p = 6$ **A1** **N1**

(ii) $q = 5$ **A1** **N1**

[2 marks]

(b) correct approach *(A1)*

eg $p \times q$, 5×6

$k = 30$ **A1** **N2**

[2 marks]

(c) correct approach *(A1)*

eg rows $= n+1$, columns $= n$

$A(n) = n(n+1)$ $(= n^2 + n)$ (cm^2) **A1** **N2**

[2 marks]

[Total 6 marks]

Question -19

correct application of $\log a + \log b = \log ab$ *(A1)*

eg $\log_2(2\sin x\cos x)$, $\log 2 + \log(\sin x) + \log(\cos x)$

correct equation without logs *A1*

eg $2\sin x\cos x = 2^{-1}$, $\sin x\cos x = \frac{1}{4}$, $\sin 2x = \frac{1}{2}$

recognizing double-angle identity (seen anywhere) *A1*

eg $\log(\sin 2x)$, $2\sin x\cos x = \sin 2x$, $\sin 2x = \frac{1}{2}$

evaluating $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ (30°) *(A1)*

correct working *A1*

eg $x = \frac{\pi}{12} + 2\pi$, $2x = \frac{25\pi}{6}$, $\frac{29\pi}{6}$, 750°, 870°, $x = \frac{\pi}{12}$ **and** $x = \frac{5\pi}{12}$, one correct final answer

$x = \frac{25\pi}{12}, \frac{29\pi}{12}$ (do not accept additional values) *A2* *N0*

[7 marks]

Question -20

(a) subtracting terms *(M1)*

eg $5-8$, $u_2 - u_1$

$d = -3$ *A1* *N2*

[2 marks]

(b) correct substitution into formula *(A1)*

eg $u_{10} = 8 + (10-1)(-3)$, $8-27$, $-3(10)+11$

$u_{10} = -19$ *A1* *N2*

[2 marks]

(c) correct substitution into formula for sum *(A1)*

eg $S_{10} = \frac{10}{2}(8-19)$, $5\left(2(8)+(10-1)(-3)\right)$

$S_{10} = -55$ *A1* *N2*

[2 marks]

Total [6 marks]

Question -21

(a) infinite sum of segments is 2 (seen anywhere) ***(A1)***

eg $p+p^2+p^3+\ldots=2$, $\frac{u_1}{1-r}=2$

recognizing GP ***(M1)***

eg ratio is p, $\frac{u_1}{1-r}$, $u_n=u_1\times r^{n-1}$, $\frac{u_1\left(r^n-1\right)}{r-1}$

correct substitution into S_∞ formula (may be seen in equation) ***A1***

eg $\frac{p}{1-p}$

correct equation ***(A1)***

eg $\frac{p}{1-p}=2$, $p=2-2p$

correct working leading to answer ***A1***

eg $3p=2$, $2-3p=0$

$p=\frac{2}{3}$ (cm) ***AG*** ***N0***

[5 marks]

(b) recognizing infinite geometric series with squares ***(M1)***

eg $k^2+k^4+k^6+\ldots$, $\frac{k^2}{1-k^2}$

correct substitution into $S_\infty=\frac{9}{16}$ (must substitute into formula) ***(A2)***

eg $\frac{k^2}{1-k^2}=\frac{9}{16}$

correct working ***(A1)***

eg $16k^2=9-9k^2$, $25k^2=9$, $k^2=\frac{9}{25}$

$k=\frac{3}{5}$ (seen anywhere) ***A1***

valid approach with segments and CD (may be seen earlier) ***(M1)***

eg $r=k$, $S_\infty=b$

correct expression for b in terms of k (may be seen earlier) **(A1)**

eg $b = \dfrac{k}{1-k}$, $b = \sum_{n=1}^{\infty} k^n$, $b = k + k^2 + k^3 + \ldots$

substituting **their** value of k into **their** formula for b **(M1)**

eg $\dfrac{\frac{3}{5}}{1-\frac{3}{5}}$, $\dfrac{\left(\frac{3}{5}\right)}{\left(\frac{2}{5}\right)}$

$b = \dfrac{3}{2}$ **A1** **N3**

[9 marks]

Total [14 marks]

Question -22

METHOD 1 – using discriminant

correct equation without logs **(A1)**
eg $6x - 3x^2 = k^2$

valid approach **(M1)**
eg $-3x^2 + 6x - k^2 = 0$, $3x^2 - 6x + k^2 = 0$

recognizing discriminant must be zero (seen anywhere) **M1**
eg $\Delta = 0$

correct discriminant **(A1)**
eg $6^2 - 4(-3)(-k^2)$, $36 - 12k^2 = 0$

correct working **(A1)**
eg $12k^2 = 36$, $k^2 = 3$

$k = \sqrt{3}$ **A2** **N2**

METHOD 2 – completing the square

correct equation without logs *(A1)*

eg $6x-3x^2=k^2$

valid approach to complete the square *(M1)*

eg $3\left(x^2-2x+1\right)=-k^2+3,\ x^2-2x+1-1+\dfrac{k^2}{3}=0$

correct working *(A1)*

eg $3(x-1)^2=-k^2+3,\ (x-1)^2-1+\dfrac{k^2}{3}=0$

recognizing conditions for one solution ***M1***

eg $(x-1)^2=0,\ -1+\dfrac{k^2}{3}=0$

correct working *(A1)*

eg $\dfrac{k^2}{3}=1,\ k^2=3$

$k=\sqrt{3}$ ***A2*** ***N2***

[7 marks]

Question -23

(a) (i) valid approach *(M1)*

eg $\dfrac{u_2}{u_1},\ \dfrac{u_1}{u_2}$

$r=\dfrac{12\sin^2\theta}{18}\left(=\dfrac{2\sin^2\theta}{3}\right)$ ***A1*** ***N2***

(ii) recognizing that $\sin\theta$ is bounded *(M1)*

eg $0\le\sin^2\theta\le1,\ -1\le\sin\theta\le1,\ -1<\sin\theta<1$

$0<r\le\dfrac{2}{3}$ ***A2*** ***N3***

Note:	If working shown, award ***M1A1*** for correct values with incorrect inequality sign(s). If no working shown, award ***N1*** for correct values with incorrect inequality sign(s).

[5 marks]

(b) correct substitution into formula for infinite sum **A1**

eg $\frac{18}{1-\frac{2\sin^2\theta}{3}}$

evidence of choosing an appropriate rule for $\cos 2\theta$ (seen anywhere) ***(M1)***

eg $\cos 2\theta = 1-2\sin^2\theta$

correct substitution of identity/working (seen anywhere) ***(A1)***

eg $\frac{18}{1-\frac{2}{3}\left(\frac{1-\cos 2\theta}{2}\right)}$, $\frac{54}{3-2\left(\frac{1-\cos 2\theta}{2}\right)}$, $\frac{18}{\frac{3-2\sin^2\theta}{3}}$

correct working that clearly leads to the given answer ***A1***

eg $\frac{18\times 3}{2+(1-2\sin^2\theta)}$, $\frac{54}{3-(1-\cos 2\theta)}$

$\frac{54}{2+\cos(2\theta)}$ ***AG*** ***N0***

[4 marks]

(c) **METHOD 1** (using differentiation)

recognizing $\frac{dS_\infty}{d\theta}=0$ (seen anywhere) ***(M1)***

finding any correct expression for $\frac{dS_\infty}{d\theta}$ ***(A1)***

eg $\frac{0-54\times(-2\sin 2\theta)}{(2+\cos 2\theta)^2}$, $-54(2+\cos 2\theta)^{-2}(-2\sin 2\theta)$

correct working ***(A1)***
eg $\sin 2\theta = 0$

any correct value for $\sin^{-1}(0)$ (seen anywhere) ***(A1)***
eg $0, \pi, \ldots$, sketch of sine curve with x-intercept(s) marked

both correct values for 2θ (ignore additional values) ***(A1)***
$2\theta = \pi, 3\pi$ (accept values in degrees)

both correct answers $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ ***A1*** ***N4***

Note: Award ***A0*** if either or both correct answers are given in degrees. Award ***A0*** if additional values are given.

Question -24

METHOD 1

evidence of discriminant *eg* $b^2 - 4ac$, Δ	***(M1)***	
correct substitution into discriminant *eg* $q^2 - 4p(-4p)$	***(A1)***	
correct discriminant *eg* $q^2 + 16p^2$	***A1***	
$16p^2 > 0$ (accept $p^2 > 0$)	***A1***	
$q^2 \geq 0$ (do not accept $q^2 > 0$)	***A1***	
$q^2 + 16p^2 > 0$	***A1***	
f has 2 roots	***A1***	***N0***

METHOD 2

y-intercept $= -4p$ (seen anywhere)	***A1***	
if p is positive, then the y-intercept will be negative	***A1***	
an upward-opening parabola with a negative y-intercept *eg* sketch that must indicate $p > 0$.	***R1***	
if p is negative, then the y-intercept will be positive	***A1***	
a downward-opening parabola with a positive y-intercept *eg* sketch that must indicate $p < 0$.	***R1***	
f has 2 roots	***A2***	***N0***

[7 marks]

Question -25

(a) valid approach involving addition or subtraction *M1*

eg $u_2 = \log_c p + d$, $u_1 - u_2$

correct application of log law *A1*

eg $\log_c(pq) = \log_c p + \log_c q$, $\log_c\left(\frac{pq}{p}\right)$

$d = \log_c q$ *AG* *N0*

[2 marks]

(b) **METHOD 1** (finding u_1 and d)

recognizing $\Sigma = S_{20}$ (seen anywhere) *(A1)*

attempt to find u_1 or d using $\log_c c^k = k$ *(M1)*

eg $2\log_c c$, $3\log_c c$, correct value of u_1 or d

$u_1 = 2$, $d = 3$ (seen anywhere) *(A1)(A1)*

correct working *(A1)*

eg $S_{20} = \frac{20}{2}(2\times 2 + 19\times 3)$, $S_{20} = \frac{20}{2}(2+59)$, $10(61)$

$\sum_{n=1}^{20} u_n = 610$ *A1* *N2*

METHOD 2 (expressing S in terms of c)

recognizing $\Sigma = S_{20}$ (seen anywhere) *(A1)*

correct expression for S in terms of c *(A1)*

eg $10(2\log_c c^2 + 19\log_c c^3)$

$\log_c c^2 = 2$, $\log_c c^3 = 3$ (seen anywhere) *(A1)(A1)*

correct working *(A1)*

eg $S_{20} = \frac{20}{2}(2\times 2 + 19\times 3)$, $S_{20} = \frac{20}{2}(2+59)$, $10(61)$

$\sum_{n=1}^{20} u_n = 610$ *A1* *N2*

METHOD 3 (expressing S in terms of c)

recognizing $\sum = S_{20}$ (seen anywhere) ***(A1)***

correct expression for S in terms of c ***(A1)***

eg $10(2\log_c c^2 + 19\log_c c^3)$

correct application of log law ***(A1)***

eg $2\log_c c^2 = \log_c c^4$, $19\log_c c^3 = \log_c c^{57}$, $10\left(\log_c\left(c^2\right)^2 + \log_c\left(c^3\right)^{19}\right)$,

$10\left(\log_c c^4 + \log_c c^{57}\right)$, $10\left(\log_c c^{61}\right)$

correct application of definition of log ***(A1)***

eg $\log_c c^{61} = 61$, $\log_c c^4 = 4$, $\log_c c^{57} = 57$

correct working ***(A1)***

eg $S_{20} = \frac{20}{2}(4+57)$, $10(61)$

$\sum_{n=1}^{20} u_n = 610$ ***A1*** ***N2***

[6 marks]

[Total: 8 marks]

Question 26

(a) (i) valid approach ***(M1)***

eg $\frac{u_2}{u_1}$, $\frac{u_1}{u_2}$

$r = \frac{12\sin^2\theta}{18}\left(= \frac{2\sin^2\theta}{3}\right)$ ***A1*** ***N2***

(ii) recognizing that $\sin\theta$ is bounded ***(M1)***

eg $0 \le \sin^2\theta \le 1$, $-1 \le \sin\theta \le 1$, $-1 < \sin\theta < 1$

$0 < r \le \frac{2}{3}$ ***A2*** ***N3***

[5 marks]

(b) correct substitution into formula for infinite sum *A1*

eg $\dfrac{18}{1-\dfrac{2\sin^2\theta}{3}}$

evidence of choosing an appropriate rule for $\cos 2\theta$ (seen anywhere) *(M1)*

eg $\cos 2\theta = 1 - 2\sin^2\theta$

correct substitution of identity/working (seen anywhere) *(A1)*

eg $\dfrac{18}{1-\dfrac{2}{3}\left(\dfrac{1-\cos 2\theta}{2}\right)}$, $\dfrac{54}{3-2\left(\dfrac{1-\cos 2\theta}{2}\right)}$, $\dfrac{18}{\dfrac{3-2\sin^2\theta}{3}}$

correct working that clearly leads to the given answer *A1*

eg $\dfrac{18\times 3}{2+(1-2\sin^2\theta)}$, $\dfrac{54}{3-(1-\cos 2\theta)}$

$\dfrac{54}{2+\cos(2\theta)}$ *AG* *N0*

[4 marks]

(c) **METHOD 1** (using differentiation)

recognizing $\dfrac{\mathrm{d}S_\infty}{\mathrm{d}\theta} = 0$ (seen anywhere) *(M1)*

finding any correct expression for $\dfrac{\mathrm{d}S_\infty}{\mathrm{d}\theta}$ *(A1)*

eg $\dfrac{0-54\times(-2\sin 2\theta)}{(2+\cos 2\theta)^2}$, $-54(2+\cos 2\theta)^{-2}(-2\sin 2\theta)$

correct working *(A1)*

eg $\sin 2\theta = 0$

any correct value for $\sin^{-1}(0)$ (seen anywhere) *(A1)*

eg $0, \pi, \ldots$, sketch of sine curve with x-intercept(s) marked

both correct values for 2θ (ignore additional values) *(A1)*

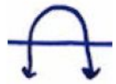
$2\theta = \pi, 3\pi$ (accept values in degrees)

both correct answers $\theta = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ *A1* *N4*

Question 27

(a) **METHOD 1** (using symmetry to find p)

(i) valid approach **(M1)**

eg $\frac{-1+3}{2}$, , 

$p=1$ **A1** **N2**

Note: Award no marks if they work backwards by substituting $a=2$ into $-\frac{b}{2a}$ to find p. Do not accept $p=\frac{2}{a}$.

(ii) valid approach **M1**

eg $-\frac{b}{2a}$, $\frac{4}{2a}$ (might be seen in (i)), $f'(1)=0$

correct equation **A1**

eg $\frac{4}{2a}=1$, $2a(1)-4=0$

$a=2$ **AG** **N0**

METHOD 2 (calculating a first)

(i) & (ii) valid approach to calculate a **M1**

eg $a+4-c=a(3^2)-4(3)-c$, $f(-1)=f(3)$

correct working **A1**

eg $8a=16$

$a=2$ **AG** **N0**

valid approach to find p **(M1)**

eg $-\frac{b}{2a}$, $\frac{4}{2(2)}$

$p=1$ **A1** **N2**

[4 marks]

(b) valid approach **(M1)**

eg $f(-1)=5$, $f(3)=5$

correct working **(A1)**

eg $2+4-c=5$, $18-12-c=5$

$c=1$ **A1** **N2**

[3 marks]

Total [7 marks]

Question 28

(a) valid approach **(M1)**

eg $f(x)=0$, $x^2-4x-5=0$

valid attempt to solve quadratic equation **(M1)**

eg factorizing, formula, completing the square

evidence of correct working **(A1)**

eg $(x-5)(x+1)$, $x=\frac{4\pm\sqrt{16-4(-5)}}{2}$

$x=-1$, $x=5$ (accept $(-1, 0)$, $(5, 0)$) **A1A1** **N3**

[5 marks]

(b) correct working **(A1)**

eg $\frac{-(-4)}{2(1)}$, $\frac{-1+5}{2}$

$x=2$ (must be an equation with $x=$) **A1** **N2**

[2 marks]

(c) (i) $h=2$ **A1** **N1**

(ii) **METHOD 1**

valid approach **(M1)**

eg $f(2)$

correct substitution **(A1)**

eg $(2)^2-4(2)-5$

$k=-9$ **A1** **N2**

METHOD 2

valid attempt to complete the square **(M1)**

eg x^2-4x+4

correct working **(A1)**

eg $(x^2-4x+4)-4-5$, $(x-2)^2-9$

$k=-9$ **A1** **N2**

[4 marks]

(d) **METHOD 1** (working with vertex)

vertex of f is at $(2, -9)$ *(A1)*

correct horizontal reflection *(A1)*

eg $x = -2$, $(-2, -9)$

valid approach for translation of **their** x **or** y value *(M1)*

eg $x - 3$, $y + 6$, $\begin{pmatrix} -2 \\ -9 \end{pmatrix} + \begin{pmatrix} -3 \\ 6 \end{pmatrix}$, one correct coordinate for vertex

vertex of g is $(-5, -3)$ (accept $x = -5$, $y = -3$) ***A1A1*** ***N1N1***

METHOD 2 (working with function)

correct approach for horizontal reflection *(A1)*

eg $f(-x)$

correct horizontal reflection *(A1)*

eg $(-x)^2 - 4(-x) - 5$, $x^2 + 4x - 5$, $(-x - 2)^2 - 9$

valid approach for translation of **their** x **or** y value *(M1)*

eg $(x+3)^2 + 4(x+3) - 5 + 6$, $x^2 + 10x + 22$, $(x+5)^2 - 3$, one correct coordinate for vertex

vertex of g is $(-5, -3)$ (accept $x = -5$, $y = -3$) ***A1A1*** ***N1N1***

[5 marks]

Total [16 marks]

Question 29

(a) correct approach (A1)

eg $3\log_2 a$

$\log_2 a^3 = 3b$ **A1** **N2**

[2 marks]

(b) correct working (A1)

eg $\log_2 8 + \log_2 a$, $\log_2 8 = 3$

$\log_2 8a = 3 + b$ **A1** **N2**

[2 marks]

(c) correct working (A1)

eg $\frac{\log_2 a}{\log_2 8}$, $\frac{1}{3}\log_2 a$, $b\log_8 2$

$\log_8 a = \frac{b}{3}$ **A1** **N2**

[2 marks]

Total [6 marks]

Question 30

(a) correct working (A1)

eg $-5+(8-1)(3)$

$u_8 = 16$ **A1** **N2**

[2 marks]

(b) correct substitution into u_n formula (A1)

eg $-5+3(n-1)$, $3n-8$

correct equation (A1)

eg $-5+3(n-1)=67$, $3n-8=67$, $3(n-1)=72$

correct working (A1)

eg $3n=75$, $n-1=24$

$n = 25$ **A1** **N3**

[4 marks]

Total [6 marks]

Question 31

correct application of change of base (accept any base) **(A1)**

eg $\frac{\log_4(13-4x)}{\log_4 16}$, $\frac{\log_{16}(2-x)}{\log_{16} 4}$, $\frac{\log_2(2-x)}{\log_2 4}$, $\frac{\log(13-4x)}{\log 16}$

correct numerical value **(A1)**

eg $\log_4 16 = 2$, $\log_{16} 4 = \frac{1}{2}$

correct application of $r\log_c a = \log_c a^r$ **(A1)**

eg $\log_4(2-x)^2$

correct equation without logs **A1**

eg $(2-x)^2 = 13-4x$, $(2-x)^4 = (13-4x)^2$, $4-4x+x^2 = 13-4x$

correct working **A1**

eg $x^2 = 9$

$x = -3$ **A2** **N2**

[7 marks]

Question 32

(a) correct working **(A1)**

eg $\sin\left(\frac{\pi}{4}x\right) = 1$, $\sqrt{x}\left(1-\sin\left(\frac{\pi}{4}x\right)\right) = 0$

$\sin\left(\frac{\pi}{2}\right) = 1$ (seen anywhere) **(A1)**

correct working (ignore additional values) **(A1)**

eg $\frac{\pi}{4}x = \frac{\pi}{2}$, $\frac{\pi}{4}x = \frac{\pi}{2} + 2\pi$

$x = 2, 10$ **A1A1** **N1N1**

[5 marks]

(b) correct working **(A1)**

eg $d = 10-2$, $a+b = 2$, $a+2b = 10$

valid approach **(M1)**

eg $2+(n-1)8$, $a+2(2-a) = 10$, b = common difference

$a = -6, b = 8$ (accept $-6+8n$) **A1A1** **N2N2**

[4 marks]

(c) valid approach *(M1)*

eg first intersection at $x = 0$, $n = 20$

correct working **A1**

eg $-6 + 8 \times 20$, $2 + (20 - 1) \times 8$, $u_{20} = 154$

$\mathrm{P}\left(154, \sqrt{154}\right)$ (accept $x = 154$ and $y = \sqrt{154}$) **A1A1** **N3**

[4 marks]

Question 33

(a) valid approach *(M1)*

eg $f(x) = 0$, $9 - x^2 = 0$, one correct solution

$x = -3, 3$ (accept $(3, 0), (-3, 0)$) **A1** **N2**

[2 marks]

(b) valid approach *(M1)*

eg height $= f(b)$, base $= 2(\mathrm{OP})$ or $2b$, $2b(9 - x^2)$, $2b \times f(b)$

correct working that clearly leads to given answer **A1**

eg $2b(9 - b^2)$

> **Note:** Do not accept sloppy notation eg $2b \times 9 - b^2$.

area $= 18b - 2b^3$ **AG** **N0**

[2 marks]

(c) setting derivative $= 0$ (seen anywhere) *(M1)*

eg $A' = 0$, $\left[18b - 2b^3\right]' = 0$

correct derivative (must be in terms of b only) (seen anywhere) **A2**

eg $18 - 6b^2$, $2b(-2b) + (9 - b^2) \times 2$

correct working *(A1)*

eg $6b^2 = 18$, $b = \pm\sqrt{3}$

$b = \sqrt{3}$ **A1** **N3**

[5 marks]

(d) valid approach **(M1)**

eg $f = g$, $9 - x^2 = (x-3)^2 + k$

correct working **(A1)**

eg $9 - x^2 = x^2 - 6x + 9 + k$, $9 - x^2 - x^2 + 6x - 9 - k = 0$

$2x^2 - 6x + k = 0$ **AG** **N0**

[2 marks]

(e) **METHOD 1 (discriminant)**

recognizing to use discriminant (seen anywhere) **(M1)**

eg $\Delta, b^2 - 4ac$

discriminant $= 0$ (seen anywhere) **M1**

correct substitution into discriminant (do not accept only in quadratic formula) **(A1)**

eg $(-6)^2 - 4(2)(k)$, $(6)^2 - 4(2)(k)$

correct working **(A1)**

eg $36 - 8k = 0$, $8k = 36$

$k = \frac{36}{8}\ \left(= \frac{9}{2},\ 4.5\right)$ **A1** **N2**

METHOD 2 (completing the square)

valid approach to complete the square **(M1)**

eg $2\left(x^2 - 3x + \frac{9}{4}\right) = -k + \frac{18}{4}$, $x^2 - 3x + \frac{9}{4} - \frac{9}{4} + \frac{k}{2} = 0$

correct working **(A1)**

eg $2\left(x - \frac{3}{2}\right)^2 = -k + \frac{18}{4}$, $\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{k}{2} = 0$

recognizing condition for one solution **M1**

eg $\left(x - \frac{3}{2}\right)^2 = 0$, $-\frac{9}{4} + \frac{k}{2} = 0$

correct working **(A1)**

eg $-k = -\frac{18}{4}$, $\frac{k}{2} = \frac{9}{4}$

$k = \frac{18}{4}\ \left(= \frac{9}{2},\ 4.5\right)$ **A1** **N2**

METHOD 3 (using vertex)

valid approach to find vertex (seen anywhere) ***M1***

eg $\left(2x^2-6x+k\right)'=0,\ -\frac{b}{2a}$

correct working ***(A1)***

eg $\left(2x^2-6x+k\right)'=4x-6,\ -\frac{(-6)}{2(2)}$

$x=\frac{6}{4}\ \left(=\frac{3}{2}\right)$ ***(A1)***

correct substitution ***(A1)***

eg $2\left(\frac{3}{2}\right)^2-6\left(\frac{3}{2}\right)+k=0$

$k=\frac{18}{4}\ \left(=\frac{9}{2},\ 4.5\right)$ ***A1*** ***N2***

[5 marks]

Total [16 marks]

Question 34

(a) correct substitution into $b^2 - 4ac$ *(A1)*

eg $(5k)^2 - 4(2)(3k^2 + 2)$, $(5k)^2 - 8(3k^2 + 2)$

correct expansion of each term ***A1***

eg $25k^2 - 24k^2 - 16$, $25k^2 - (24k^2 + 16)$

$k^2 - 16$ ***AG*** ***N0***

[2 marks]

(b) valid approach ***M1***

eg $f'(x) > 0$, $f'(x) \geq 0$

recognizing discriminant <0 or ≤ 0 ***M1***

eg $D < 0$, $k^2 - 16 \leq 0$, $k^2 < 16$

two correct values for *k*/endpoints (even if inequalities are incorrect) *(A1)*

eg $k = \pm 4$, $k < -4$ and $k > 4$, $|k| < 4$

correct interval ***A1*** ***N2***

eg $-4 < k < 4$, $-4 \leq k \leq 4$

[4 marks]

Total [6 marks]

Question 35

(a) valid approach *(M1)*

eg $11-a=9$, $\frac{11!}{9!(11-9)!}$

$a=2$ ***A1*** ***N2***

[2 marks]

(b) valid approach for expansion using $n=11$ *(M1)*

eg $\binom{11}{r}x^{11-r}3^r$, $a^{11}b^0+\binom{11}{1}a^{10}b^1+\binom{11}{2}a^9b^2+\ldots$

evidence of choosing correct term ***A1***

eg $\binom{11}{2}3^2$, $\binom{11}{2}x^9 3^2$, $\binom{11}{9}3^2$

correct working for binomial coefficient (seen anywhere, do not accept factorials) ***A1***

eg 55, $\binom{11}{2}=55$, 55×3^2, $(55\times9)x^9$, $\frac{11\times10}{2}\times9$

495 ***A1*** ***N2***

Note: If there is clear evidence of adding instead of multiplying, award ***A1*** for the correct working for binomial coefficient, but no other marks. For example, $55x^9+3^2$ would earn ***M0A0A1A0***.

Do not award final ***A1*** for a final answer of $495x^9$, even if 495 is seen previously. If no working shown, award ***N1*** for $495x^9$.

[4 marks]

Total [6 marks]

Question 37

(a) valid approach *(M1)*

eg $11-5$, $11=5+d$

$d=6$ ***A1*** ***N2***

[2 marks]

(b) valid approach *(M1)*

eg u_2-d, $5-6$, $u_1+(3-1)(6)=11$

$u_1=-1$ ***A1*** ***N2***

[2 marks]

(c) correct substitution into sum formula

eg $\frac{20}{2}(2(-1)+19(6))$, $\frac{20}{2}(-1+113)$ *(A1)*

$S_{20}=1120$ ***A1*** ***N2***

[2 marks]

Total [6 marks]