Subject – Math (Standard Level)

Topic : Algebra

1.	(a)	attempt to substitute into sum formula for AP (accept term formula)	(M1)	
		e.g. $S_{20} = \frac{20}{2}\{2(-7)+19d\}, \left(\text{or } \frac{20}{2}(-7+u_{20})\right)$		
		setting up correct equation using sum formula	A1	
		e.g. $\frac{20}{2}\{2(-7) + 19d\} = 620$	A1	N2
	(b)	correct substitution $u_{78} = -7 + 77(4)$	(A1)	
		$= 301$	A1	N2

[5]

2.	(a)	evidence of expanding	M1	
		e.g. $(x-2)^4 = x^4 + 4x^3(-2) + 6x^2(-2)^2 + 4x(-2)^3 + (-2)^4$		
		$(x-2)^4 = x^4 - 8x^3 + 24x^2 - 32x + 16$	A2	N2
	(b)	finding coefficients, 3×24 (= 72),$4 \times (-8)(= -32)$	(A1)(A1)	
		term is $40x^3$	A1	N3

[6]

3.	(a)	evidence of dividing two terms	(M1)	
		e.g. $-\frac{1800}{3000}, -\frac{1800}{1080}$		
		$r = -0.6$	A1	N2
	(b)	evidence of substituting into the formula for the 10th term	(M1)	
		e.g. $u_{10} = 3000(-0.6)^9$		
		$u_{10} = -30.2$ (accept the exact value -30.233088)	A1	N2
	(c)	evidence of substituting into the formula for the infinite sum	(M1)	
		e.g. $S = \frac{3000}{1.6}$		
		$S = 1875$	A1	N2

[6]

4. (a) 7 terms A1 N1

(b) A valid approach (M1)

Correct term **chosen** $\binom{6}{3}(x^3)^3(-3x)^3$ A1

Calculating $\binom{6}{3}=20, (-3)^3=-27$ (A1)(A1)

Term is $-540x^{12}$ A1 N3

[6]

5. (a) Ashley

AP 12 + 14 + 16 + ... to 15 terms (M1)

$S_{15} = \frac{15}{2}[2(12) + 14(2)]$ (M1)

$= 15 \times 26$

$= 390$ hours (A1) 3

(b) Billie

GP 12, 12(1.1), $12(1.1)^2$… (M1)

(i) In week 3, $12(1.1)^2$ (A1)

$= 14.52$ hours (AG)

(ii) $S_{15} = \frac{12[(1.1)^{15} - 1]}{1.1 - 1}$ (M1)

$= 381$ hours (3 sf) (A1) 4

(c) $12\,(1.1)^{n-1} > 50$ (M1)

$(1.1)^{n-1} > \dfrac{50}{12}$ (A1)

$(n-1)\ln 1.1 > \ln \dfrac{50}{12}$

$n - 1 > \dfrac{\ln \frac{50}{12}}{\ln 1.1}$ (A1)

$n - 1 > 14.97$

$n > 15.97$

$\Rightarrow$ Week 16 (A1)

OR

$12(1.1)^{n-1} > 50$ (M1)

By trial and error

$12(1.1)^{14} = 45.6$, $12(1.1)^{15} = 50.1$ (A1)

$\Rightarrow n - 1 = 15$ (A1)

$\Rightarrow n = 16$ (Week 16) (A1) 4

[11]

6. **METHOD 1**

$\log_9 81 + \log_9\left(\dfrac{1}{9}\right) + \log_9 3 = 2 - 1 + \dfrac{1}{2}$ (M1)

$\Rightarrow \dfrac{3}{2} = \log_9 x$ (A1)

$\Rightarrow x = 9^{\frac{3}{2}}$ (M1)

$\Rightarrow x = 27$ (A1) (C4)

METHOD 2

$\log 81 + \log_9\left(\dfrac{1}{9}\right) + \log_9 3 = \log_9\left[81\left(\dfrac{1}{9}\right)3\right]$ (M2)

$= \log_9 27$ (A1)

$\Rightarrow x = 27$ (A1) (C4)

[4]

7. (a) **METHOD 1**

recognizing that $f(8) = 1$ (M1)

e.g. $1 = k \log_2 8$

recognizing that $\log_2 8 = 3$ (A1)

e.g. $1 = 3k$

$k = \frac{1}{3}$ A1 N2

METHOD 2

attempt to find the inverse of $f(x) = k \log_2 x$ (M1)

e.g. $x = k \log_2 y$, $y = 2^{\frac{x}{k}}$

substituting 1 and 8 (M1)

e.g. $1 = k \log_2 8$, $2^{\frac{1}{k}} = 8$

$k = \frac{1}{\log_2 8}$ $\left(k = \frac{1}{3}\right)$ A1 N2

(b) **METHOD 1**

recognizing that $f(x) = \frac{2}{3}$ (M1)

e.g. $\frac{2}{3} = \frac{1}{3}\log_2 x$

$\log_2 x = 2$ (A1)

$f^{-1}\left(\frac{2}{3}\right) = 4$ (accept $x = 4$) A2 N3

METHOD 2

attempt to find inverse of $f(x) = \frac{1}{3}\log_2 x$ (M1)

e.g. interchanging x and y, substituting $k = \frac{1}{3}$ into $y = 2^{\frac{x}{k}}$

correct inverse (A1)

e.g. $f^{-1}(x) = 2^{3x}$, 2^{3x}

$f^{-1}\left(\frac{2}{3}\right) = 4$ A2 N3

[7]

8. (a) 5 A1 N1

(b) **METHOD 1**

$\log_2\left(\frac{32^x}{8^y}\right) = \log_2 32^x - \log_2 8^y$ (A1)

$= x \log_2 32 - y \log_2 8$ (A1)

$\log_2 8 = 3$ (A1)

$p = 5$, $q = -3$ (accept $5x - 3y$) A1 N3

METHOD 2

$\frac{32^x}{8^y} = \frac{(2^5)^x}{(2^3)^y}$ (A1)

$= \frac{2^{5x}}{2^{3y}}$ (A1)

$= 2^{5x-3y}$ (A1)

$\log_2 (2^{5x-3y}) = 5x - 3y$

$p = 5$, $q = -3$ (accept $5x - 3y$) A1 N3

[5]

9. $\log_{27} (x(x - 0.4)) = 1$ (M1)(A1)

$x^2 - 0.4x = 27$ (M1)

$x = 5.4$ or $x = -5$ (G2)

$x = 5.4$ (A1) (C6)

***Note**: Award (C5) for giving both roots.*

[6]

10. (a) 10 (A2) (C2)

(b) $(3x^2)^3 \left(-\frac{1}{x}\right)^6$ [for correct exponents] (M1)(A1)

$\binom{9}{6} 3^3 x^6 \frac{1}{x^6} \left(\text{or } 84 \times 3^3 x^6 \frac{1}{x^6}\right)$ (A1)

constant = 2268 (A1) (C4)

[6]

11. $\binom{10}{3}2^7(ax)^3$ $\left(\text{accept}\binom{10}{7}\right)$ (A1)(A1)(A1)

$\binom{10}{3} = 120$ (A1)

$120 \times 2^7 a^3 = 414\ 720$ (M1)

$a^3 = 27$

$a = 3$ (A1) (C6)

Note: *Award (A1)(A1)(A0) for* $\binom{10}{3}2^7 ax^3$*. If this leads to the answer a = 27, do not award the final (A1).*

[6]

12. Selecting one term (may be implied) (M1)

$\binom{7}{2}5^2(2x^2)^5$ (A1)(A1)(A1)

$= 16800x^{10}$ (A1)(A1) (C6)

Note: *Award C5 for 16800.*

[6]

13. For using $u_3 = u_1r^2 = 8$ (M1)

$8 = 18r^2$ (A1)

$r^2 = \frac{8}{18}\left(=\frac{4}{9}\right)$

$r = \pm\frac{2}{3}$ (A1)(A1)

$S_\infty = \frac{u_1}{1-r}$,

$S_\infty = 54, \frac{54}{5}(=10.8)$ (A1)(A1)(C3)(C3)

[6]

14. (a) $\frac{1}{5}$ (0.2) A1 N1

(b) (i) $u_{10} = 25\left(\frac{1}{5}\right)^9$ (M1)

$= 0.0000128 \left(\left(\frac{1}{5}\right)^7, 1.28\times10^{-5}, \frac{1}{78125}\right)$ A1 N2

(ii) $u_n = 25\left(\frac{1}{5}\right)^{n-1}$ A1 N1

(c) For attempting to use infinite sum formula for a GP $\left(\frac{25}{1-\left(\frac{1}{5}\right)}\right)$ (M1)

$S = \frac{125}{4} = 31.25 \ (= 31.3 \text{ to } 3\,s\,f)$ A1 N2

[6]

15. evidence of using binomial expansion (M1)

e.g. selecting correct term, $a^8b^0 + \binom{8}{1}a^7b + \binom{8}{2}a^6b^2 + ...$

evidence of calculating the factors, in any order A1A1A1

e.g. 56, $\frac{2^3}{3^3}, -3^5, \binom{8}{5}\left(\frac{2}{3}x\right)^3(-3)^5$

$-4032x^3$ (accept $= -4030x^3$ to 3 s.f.) A1 N2

[5]

16. (a) $\log_3 x - \log_3(x-5) = \log_3\left(\frac{x}{x-5}\right)$ (A1)

$A = \frac{x}{x-5}$ (A1) (C2)

Note: *If candidates have an incorrect or no answer to part (a) award (A1)(A0)*
if $\log\left(\frac{x}{x-5}\right)$ *seen in part (b).*

(b) **EITHER**

$$\log_3\left(\frac{x}{x-5}\right)=1$$

$$\frac{x}{x-5}=3^1\,(=3)$$ (M1)(A1)(A1)

$$x=3x-15$$

$$-2x=-15$$

$$x=\frac{15}{2}$$ (A1) (C4)

OR

$$\frac{\log_{10}\left(\frac{x}{x-5}\right)}{\log_{10}3}=1$$ (M1)(A1)

$$\log_{10}\left(\frac{x}{x-5}\right)=\log_{10}3$$ (A1)

$$x=7.5$$ (A1) (C4)

[6]

17. (a) $d = 3$ (A1)

evidence of substitution into $u_n = a + (n - 1)\,d$ (M1)

e.g. $u_{101} = 2 + 100 \times 3$

$u_{101} = 302$ A1 N3

(b) correct approach (M1)

e.g. $152 = 2 + (n - 1) \times 3$

correct simplification (A1)

e.g. $150 = (n - 1) \times 3$, $50 = n - 1$, $152 = -1 + 3n$

$n = 51$ A1 N2

[6]

18. (a) $u_1 = S_1 = 7$ (A1) (C1)

(b) $u_2 = S_2 - u_1 = 18 - 7$

$= 11$ (A1)

$d = 11 - 7$ (M1)

$= 4$ (A1) (C3)

(c) $u_4 = u_1 + (n-1)d = 7 + 3(4)$ (M1)

$u_4 = 19$ (A1) (C2)

[6]

19. **METHOD 1**

$9 = 3^2,\ 27 = 3^3$ (A1)(A1)

expressing as a power of 3, $(3^2)^{2x} = (3^3)^{1-x}$ (M1)

$3^{4x} = 3^{3-3x}$ (A1)

$4x = 3 - 3x$ (A1)

$7x = 3$

$\Rightarrow x = \frac{3}{7}$ (A1) (C6)

METHOD 2

$2x \log 9 = (1 - x) \log 27$ (M1)(A1)(A1)

$\frac{2x}{1-x} = \frac{\log 27}{\log 9} \left(= \frac{3}{2} \right)$ (A1)

$4x = 3 - 3x$ (A1)

$7x = 3$

$\Rightarrow x = \frac{3}{7}$ (A1) (C6)

Notes: *Candidates may use a graphical method.*
Award (M1)(A1)(A1) for a sketch, (A1) for showing the point of intersection, (A1) for 0.4285…., and (A1) for $\frac{3}{7}$.

[6]

20. (a) interchanging x and y (seen anywhere) (M1)

e.g. $x = \log \sqrt{y}$ (accept any base)

evidence of correct manipulation A1

e.g. $3^x = \sqrt{y}, 3^y = x^{\frac{1}{2}}, x = \frac{1}{2} \log_3 y, 2y = \log_3 x$

$f^{-1}(x) = 3^{2x}$ AG N0

(b) $y > 0, f^{-1}(x) > 0$ A1 N1

(c) **METHOD 1**

finding $g(2) = \log_3 2$ (seen anywhere) A1

attempt to substitute (M1)
e.g. $(f^{-1} \circ g)(2) = 3^{\log_3 2}$

evidence of using log or index rule (A1)
e.g. $(f^{-1} \circ g)(2) = 3^{\log_3 4}, 3^{\log_3 2^2}$

$(f^{-1} \circ g)(2) = 4$ A1 N1

METHOD 2

attempt to form composite (in any order) (M1)
e.g. $(f^{-1} \circ g)(x) = 3^{2\log_3 x}$

evidence of using log or index rule (A1)
e.g. $(f^{-1} \circ g)(x) = 3^{\log_3 x^2}, 3^{\log_3 x^2}$

$(f^{-1} \circ g)(x) = x^2$ A1

$(f^{-1} \circ g)(2) = 4$ A1 N1

[7]

21. (a) (i) attempt to set up equations (M1)
$-37 = u_1 + 20d$ and $-3 = u_1 + 3d$ A1
$-34 = 17d$
$d = -2$ A1 N2

(ii) $-3 = u_1 - 6 \Rightarrow u_1 = 3$ A1 N1

(b) $u_{10} = 3 + 9 \times -2 = -15$ (A1)

$S_{10} = \frac{10}{2}(3 + (-15))$ M1

$= -60$ A1 N2

[7]

22. (a) combining 2 terms (A1)

e.g. $\log_3 8x - \log_3 4$, $\log_3 \frac{1}{2}x + \log_3 4$

expression which clearly leads to answer given A1

e.g. $\log_3 \frac{8x}{3}$, $\log_3 \frac{4x}{2}$

$f(x) = \log_3 2x$ AG N0 2

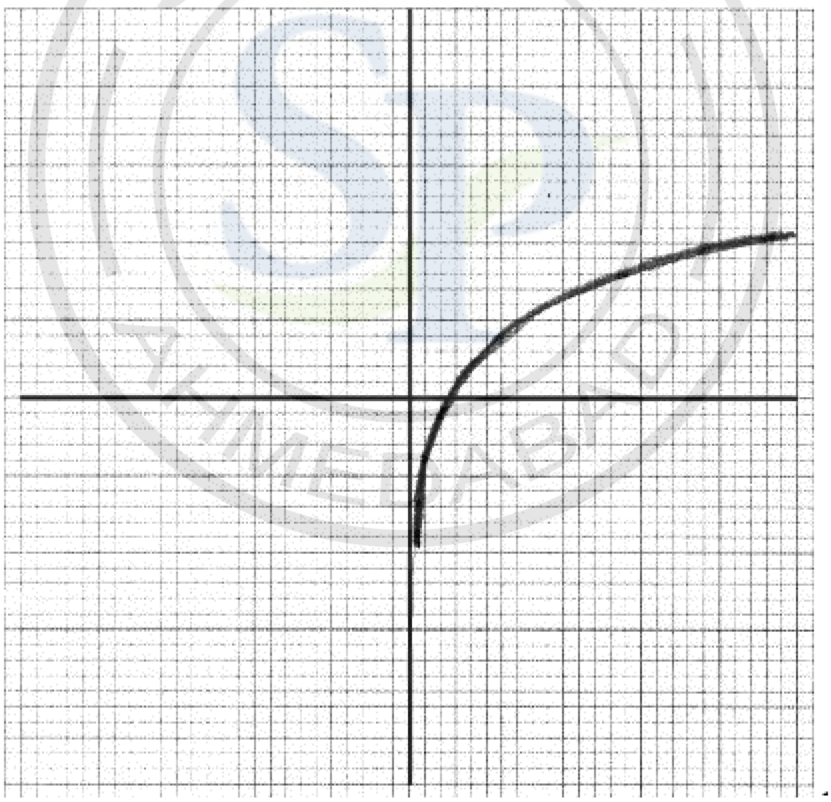
(b) attempt to substitute either value into f (M1)

e.g. $\log_3 1$, $\log_3 9$

$f(0.5) = 0$, $f(4.5) = 2$ A1A1 N3 3

(c) (i) $a = 2$, $b = 3$ A1A1 N1N1

(ii)



A1A1A1 N3

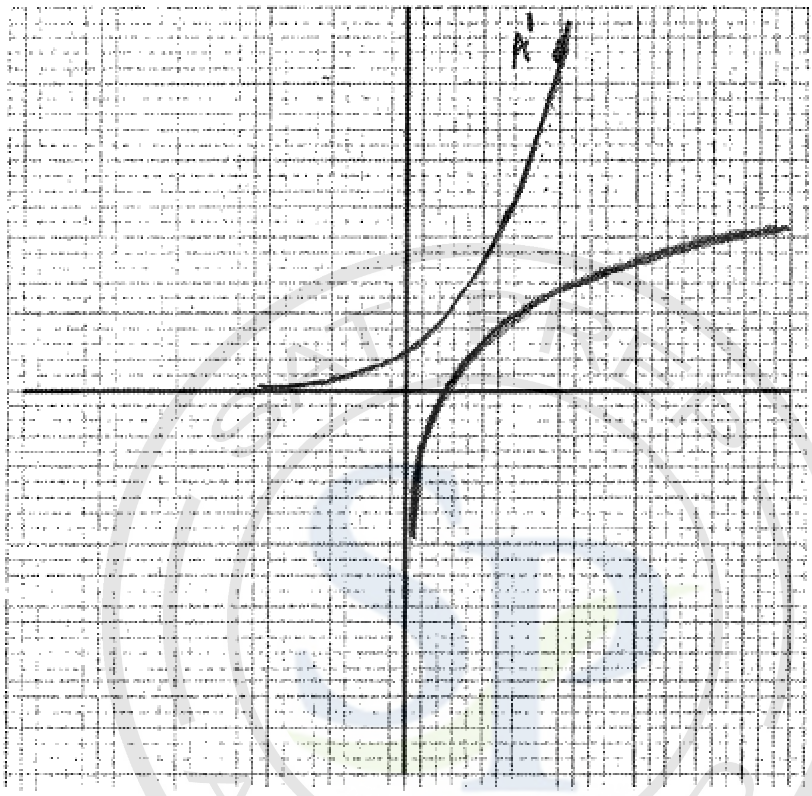
Note: *Award A1 for sketch approximately through*
(0.5 ± 0.1, 0 ± 0.1)
A1 for approximately correct shape,
A1 for sketch asymptotic to the y-axis.

(iii) $x = 0$ (must be an equation) A1 N1

[6]

(d) $f^{-1}(0) = 0.5$ A1 N1 1

(e)



A1A1A1A1 N4 4

Note: *Award A1 for sketch approximately through (0 ± 0.1, 0.5 ± 0.1),*
A1 for approximately correct shape of the graph reflected over y = x,
A1 for sketch asymptotic to x-axis,
A1 for point (2 ± 0.1, 4.5 ± 0.1) clearly marked ***and*** *on curve.*

[16]

23. (a) Recognizing an AP (M1)

$u_1 = 15 \quad d = 2 \quad n = 20$ (A1) 4

substituting into $u_{20} = 15 + (20 - 1) \times 2$ M1

$= 53$ (that is, 53 seats in the 20th row) A1

(b) Substituting into $S_{20} = \frac{20}{2}(2(15) + (20-1)2)$ (or into $\frac{20}{2}(15 + 53)$) M1

$= 680$ (that is, 680 seats in total) A1 2

[6]

24. (a) (i) $r = -2$ A1 N1

(ii) $u_{15} = -3\,(-2)^{14}$ (A1)

$= -49152$ (accept -49200) A1 N2

(b) (i) 2, 6, 18 A1 N1

(ii) $r = 3$ A1 N1

(c) Setting up equation (or a sketch) M1

$\frac{x+1}{x-3} = \frac{2x+8}{x+1}$ (or correct sketch with relevant information) A1

$x^2 + 2x + 1 = 2x^2 + 2x - 24$ (A1)

$x^2 = 25$

$x = 5$ or $x = -5$

$x = -5$ A1 N2

***Notes**: If "trial and error" is used, work must be documented with several trials shown.*
Award full marks for a correct answer with this approach.
*If the work is **not** documented, award N2 for a correct answer.*

(d) (i) $r = \frac{1}{2}$ A1 N1

(ii) For attempting to use infinite sum formula for a GP (M1)

$S = \frac{-8}{1-\frac{1}{2}}$

$S = -16$ A1 N2

***Note:** Award M0A0 if candidates use a value of r where r > 1, or r < –1.*

[12]