

Subject – Math (Standard Level)

Topic : Algebra

1. In an arithmetic series, the first term is -7 and the sum of the first 20 terms is 620.

(a) Find the common difference.

(3)

(b) Find the value of the 78th term.

(2)

(Total 5 marks)

2. (a) Expand $(x - 2)^4$ and simplify your result.

(3)

(b) Find the term in x^3 in $(3x + 4)(x - 2)^4$.

(3)

(Total 6 marks)

3. Consider the infinite geometric sequence 3000, -1800 , 1080, -648 , ...

(a) Find the common ratio.

(2)

(b) Find the 10th term.

(2)

(c) Find the **exact** sum of the infinite sequence.

(2)

(Total 6 marks)

4. Consider the expansion of the expression $(x^3 - 3x)^6$.

(a) Write down the number of terms in this expansion.

(b) Find the term in x^{12} .

(Total 6 marks)

5. Ashley and Billie are swimmers training for a competition.

(a) Ashley trains for 12 hours in the first week. She decides to increase the amount of time she spends training by 2 hours each week. Find the total number of hours she spends training during the first 15 weeks.

(3)

(b) Billie also trains for 12 hours in the first week. She decides to train for 10% longer each week than the previous week.

(i) Show that in the third week she trains for 14.52 hours.

(ii) Find the total number of hours she spends training during the first 15 weeks.

(4)

(c) In which week will the time Billie spends training first exceed 50 hours?

(4)

(Total 11 marks)

6. Solve the equation $\log_9 81 + \log_9 \frac{1}{9} + \log_9 3 = \log_9 x$.

(Total 4 marks)

7. Let $f(x) = k \log_2 x$.

(a) Given that $f^{-1}(1) = 8$, find the value of k .

(3)

(b) Find $f^{-1}\left(\frac{2}{3}\right)$.

(4)

(Total 7 marks)

8. (a) Find $\log_2 32$. (1)

(b) Given that $\log_2 \left(\frac{32^x}{8^y} \right)$ can be written as $px + qy$, find the value of p and of q . (4)
(Total 5 marks)

9. Solve the equation $\log_{27} x = 1 - \log_{27} (x - 0.4)$. (Total 6 marks)

10. Consider the expansion of $\left(3x^2 - \frac{1}{x} \right)^9$.
(a) How many terms are there in this expansion?
(b) Find the constant term in this expansion. (Total 6 marks)

11. When the expression $(2 + ax)^{10}$ is expanded, the coefficient of the term in x^3 is 414 720. Find the value of a . (Total 6 marks)

12. Find the term containing x^{10} in the expansion of $(5 + 2x^2)^7$. (Total 6 marks)

13. The first term of an infinite geometric sequence is 18, while the third term is 8. There are two possible sequences. Find the sum of each sequence. (Total 6 marks)

14. Consider the infinite geometric sequence 25, 5, 1, 0.2,

- (a) Find the common ratio.
- (b) Find
 - (i) the 10th term;
 - (ii) an expression for the n^{th} term.
- (c) Find the sum of the infinite sequence.

(Total 6 marks)

15. Find the term in x^3 in the expansion of $\left(\frac{2}{3}x - 3\right)^8$.

(Total 5 marks)

16. (a) Given that $\log_3 x - \log_3 (x - 5) = \log_3 A$, express A in terms of x .
- (b) Hence or otherwise, solve the equation $\log_3 x - \log_3 (x - 5) = 1$.

(Total 6 marks)

17. Consider the arithmetic sequence 2, 5, 8, 11,

- (a) Find u_{101} .
- (b) Find the value of n so that $u_n = 152$.

(3)

(3)

(Total 6 marks)

18. Let S_n be the sum of the first n terms of an arithmetic sequence, whose first three terms are u_1 , u_2 and u_3 . It is known that $S_1 = 7$, and $S_2 = 18$.

- (a) Write down u_1 .
- (b) Calculate the common difference of the sequence.
- (c) Calculate u_4 .

(Total 6 marks)

19. Find the **exact** solution of the equation $9^{2x} = 27^{(1-x)}$.

(Total 6 marks)

20. Let $f(x) = \log_3 \sqrt{x}$, for $x > 0$.

(a) Show that $f^{-1}(x) = 3^{2x}$.

(2)

(b) Write down the range of f^{-1} .

(1)

Let $g(x) = \log_3 x$, for $x > 0$.

(c) Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer.

(4)

(Total 7 marks)

21. In an arithmetic sequence $u_{21} = -37$ and $u_4 = -3$.

(a) Find

(i) the common difference;

(ii) the first term.

(4)

(b) Find S_{10} .

(3)

(Total 7 marks)

(a) Show that $f^{-1}(x) = 3^{2x}$.

(2)

(b) Write down the range of f^{-1} .

(1)

Let $g(x) = \log_3 x$, for $x > 0$.

(c) Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer.

(4)

(Total 7 marks)

22. Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

(a) Show that $f(x) = \log_3 2x$.

(2)

(b) Find the value of $f(0.5)$ and of $f(4.5)$.

(3)

The function f can also be written in the form $f(x) = \frac{\ln ax}{\ln b}$.

(c) (i) Write down the value of a and of b .

(ii) Hence on graph paper, **sketch** the graph of f , for $-5 \leq x \leq 5$, $-5 \leq y \leq 5$, using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote.

(6)

(d) Write down the value of $f^{-1}(0)$.

(1)

The point A lies on the graph of f . At A, $x = 4.5$.

(e) On your diagram, sketch the graph of f^{-1} , noting clearly the image of point A.

(4)

(Total 16 marks)

23. A theatre has 20 rows of seats. There are 15 seats in the first row, 17 seats in the second row, and each successive row of seats has two more seats in it than the previous row.

(a) Calculate the number of seats in the 20th row.

(b) Calculate the **total** number of seats.

(Total 6 marks)

24. (a) Consider the geometric sequence $-3, 6, -12, 24, \dots$

(i) Write down the common ratio.

(ii) Find the 15th term.

Consider the sequence $x - 3, x + 1, 2x + 8, \dots$

(3)

(b) When $x = 5$, the sequence is geometric.

(i) Write down the first three terms.

(ii) Find the common ratio.

(2)

(c) Find the other value of x for which the sequence is geometric.

(4)

(d) For this value of x , find

(i) the common ratio;

(ii) the sum of the infinite sequence.

(3)

(Total 12 marks)

