Subject : Math (Standard Level)

Topic: Functions and Equations

1. (a) attempt to form composition (in any order) (M1)

$(f \circ g)(x) = (x-1)^2 + 4 \quad (x^2 - 2x + 5)$ A1 N2

(b) **METHOD 1**

vertex of $f \circ g$ at (1, 4) (A1)

evidence of appropriate approach (M1)

e.g. adding $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ to the coordinates of the vertex of $f \circ g$

vertex of h at (4, 3) A1 N3

METHOD 2

attempt to find $h(x)$ (M1)

e.g. $((x-3)-1)^2 + 4 - 1$, $h(x) = (f \circ g)(x-3) - 1$

$h(x) = (x-4)^2 + 3$ (A1)

vertex of h at (4, 3) A1 N3

(c) evidence of appropriate approach (M1)

e.g. $(x-4)^2 + 3$, $(x-3)^2 - 2(x-3) + 5 - 1$

simplifying A1

e.g. $h(x) = x^2 - 8x + 16 + 3$, $x^2 - 6x + 9 - 2x + 6 + 4$

$h(x) = x^2 - 8x + 19$ AG N0

(d) **METHOD 1**

equating functions to find intersection point (M1)

e.g. $x^2 - 8x + 19 = 2x - 6$, $y = h(x)$

$x^2 - 10x + 25 = 0$ A1

evidence of appropriate approach to solve (M1)

e.g. factorizing, quadratic formula

appropriate working A1

e.g. $(x-5)^2 = 0$

$x = 5$ $(p = 5)$ A1 N3

METHOD 2

attempt to find $h'(x)$ (M1)
$h'(x) = 2x - 8$ A1

recognizing that the gradient of the tangent is the derivative (M1)
e.g. gradient at $p = 2$

$2x - 8 = 2$ $(2x = 10)$ A1

$x = 5$ A1 N3

[12]

2. (a) Evidence of attempting to form composition (M1)

Correct substitution $(h \circ g)(x) = \dfrac{5(3x-2)}{(3x-2)-4}$ A1

$= \dfrac{5(3x-2)}{(3x-6)} \quad \left(= \dfrac{15x-10}{3x-6}\right)\left(= \dfrac{5(3x-2)}{3(x-2)}\right)$ A1 N2

(b) Evidence of using numerator = 0 (M1)

eg $15x - 10 = 0$ $(3x - 2 = 0)$

$x = \dfrac{2}{3}$ $(= 0.667)$ A2 N3

[6]

3. (a) For attempting to complete the square or expanding $y = 2(x - c)^2 + d$, or for showing the vertex is at (3, 5) M1

$y = 2(x - 3)^2 + 5$ (accept $c = 3$, $d = 5$) A1A1 N2

(b) (i) $k = 2$ A1 N1

(ii) $p = 3$ A1 N1

(iii) $q = 5$ A1 N1

[6]

4. (a) $p = -\frac{1}{2}$, $q = 2$ (A1)(A1) (C2)

or vice versa

(b) By symmetry C is midway between p, q (M1)

Note: *This (M1) may be gained by implication.*

$\Rightarrow$ x-coordinate is $\frac{-\frac{1}{2}+2}{2} = \frac{3}{4}$ (A1) (C2)

[4]

5. (a) Initial mass $\Rightarrow t = 0$ (A1)

mass = 4 (A1) (C2)

(b) $1.5 = 4e^{-0.2t}$ (or $0.375 = e^{-0.2t}$) (M2)

$\ln 0.375 = -0.2t$ (M1)

$t = 4.90$ hours (A1) (C4)

[6]

6. (a) (i) $h = -1$ (A2) (C2)

(ii) $k = 2$ (A1) (C1)

(b) $a(1 + 1)^2 + 2 = 0$ (M1)(A1)

$a = -0.5$ (A1) (C3)

[6]

7. (a) $y = (x - 1)^2$ (A2) (C2)

(b) $y = 4(x - 1)^2$ (A1) (C1)

(c) $y = 4(x - 1)^2 + 3$ (A1) (C1)

Note: *Do not penalize if these are correctly expanded.*

[4]

8. (a) $a = 3$, $b = 4$ (A1)

$f(x) = (x - 3)^2 + 4$ A1 (C2)

(b) $y = (x - 3)^2 + 4$

METHOD 1

$x = (y - 3)^2 + 4$ (M1)
$x - 4 = (y - 3)^2$
$\sqrt{x-4} = y - 3$ (M1)
$y = \sqrt{x-4} + 3$ (A1) 3

METHOD 2

$y - 4 = (x - 3)^2$ (M1)
$\sqrt{y-4} = x - 3$ (M1)
$\sqrt{y-4} + 3 = x$
$y = \sqrt{x-4} + 3$
$\Rightarrow f^{-1}(x) = \sqrt{x-4} + 3$ (A1) 3

(c) $x \geq 4$ (A1)(C1)

[6]

9. $y = (x + 2)(x - 3)$ (M1)
$= x^2 - x - 6$ (A1)
Therefore, $0 = 4 - 2p + q$ (A1)(A1)(C2)(C2)

OR

$y = x^2 - x - 6$ (C3)

OR

$0 = 4 - 2p + q$ (A1)
$0 = 9 + 3p + q$ (A1)
$p = -1, q = -6$ (A1)(A1)(C2)(C2)

[4]

10. (a) At $t = 2$, $N = 10e^{0.4(2)}$ (M1)
$N = 22.3$ (3 sf)
Number of leopards = 22 (A1)

(b) If $N = 100$, then solve $100 = 100e^{0.4t}$

$10 = e04^t$

$\ln 10 = 0.4t$

$t = \frac{\ln 10}{0.4} \sim 5.76$ years (3 sf) (A1)

[4]

11. Discriminant $\Delta = b^2 - 4ac$ $(= (-2k)^2 - 4)$ (A1)

$\Delta > 0$ (M2)

Note: *Award (M1)(M0) for* $\Delta \geq 0$.

$(2k)^2 - 4 > 0 \Rightarrow 4k^2 - 4 > 0$

EITHER

$4k^2 > 4$ $(k^2 > 1)$ (A1)

OR

$4(k - 1)(k + 1) > 0$ (A1)

OR

$(2k - 2)(2k + 2) > 0$ (A1)

THEN

$k < -1$ or $k > 1$ (A1)(A1) (C6)

Note: *Award (A1) for* $-1 < k < 1$.

[6]

12. $4x^2 + 4kx + 9 = 0$

Only one solution $\Rightarrow b^2 - 4ac = 0$ (M1)

$16k^2 - 4(4)(9) = 0$ (A1)

$k^2 = 9$

$k = \pm 3$ (A1)

But given $k > 0$, $k = 3$ (A1) (C4)

OR

One solution $\Rightarrow (4x^2 + 4kx + 9)$ is a perfect square (M1)

$4x^2 + 4kx + 9 = (2x \pm 3)^2$ by inspection (A2)

given $k > 0$, $k = 3$ (A1) (C4)

[4]

13. (a) $f(x) = x^2 - 6x + 14$

$f(x) = x^2 - 6x + 9 - 9 + 14$ (M1)

$f(x) = (x - 3)^2 + 5$ (M1)

(b) Vertex is (3, 5) (A1)(A1)

[4]

14. (a) (i) $p = 2$ (A2) (C2)

(ii) $10 = \dfrac{q}{3-2}$ (or equivalent) (M1)

$q = 10$ (A1) (C2)

(b) Reflection, in x-axis (A1)(A1) (C2)

[6]

15. (a) $2x^2 - 8x + 5 = 2(x^2 - 4x + 4) + 5 - 8$ (M1)

$= 2(x - 2)^2 - 3$ (A1)(A1)(A1)

$\Rightarrow$ $a = 2$, $p = 2$, $q = -3$ (C4)

(b) Minimum value of $2(x - 2)^2 = 0$ (or minimum value occurs when $x = 2$) (Ml)

$\Rightarrow$ Minimum value of $f(x) = -3$ (A1) (C2)

OR

Minimum value occurs at (2, –3) (M1)(A1) (C2)

[6]

16. (a) For a reasonable attempt to complete the square, (or expanding) (M1)

e.g. $3x^2 - 12x + 11 = 3(x^2 - 4x + 4) + 11 - 12$

$f(x) = 3(x - 2)^2 - 1$ (accept $h = 2$, $k = 1$) A1A1 N3

(b) **METHOD 1**

Vertex shifted to $(2 + 3, -1 + 5) = (5, 4)$ M1

so the new function is $3(x - 5)^2 + 4$ (accept $p = 5$, $q = 4$) A1A1 N2

METHOD 2

$g(x) = 3((x - 3) - h)^2 + k + 5 = 3((x - 3) - 2)^2 - 1 + 5$ M1

$= 3(x - 5)^2 + 4$ (accept $p = 5$, $q = 4$) A1A1 N2

[6]

17. One solution ⇒ discriminant = 0 (M2)

$3^2 - 4k = 0$ (A2)

$9 = 4k$

$k = \frac{9}{4}\left(= 2\frac{1}{4}, 2.25\right)$ (A2) (C6)

Note: *If candidates correctly solve an incorrect equation, award M2 A0 A2(ft), if they have the first line or equivalent, otherwise award no marks.*

[6]

18. (a) (i) $m = 3$ A2 N2

(ii) $p = 2$ A2 N2

(b) Appropriate substitution M1

eg $0 = d(1-3)^2 + 2$, $0 = d(5-3)^2 + 2$, $2 = d(3-1)(3-5)$

$d = -\frac{1}{2}$ A1 N1

[6]

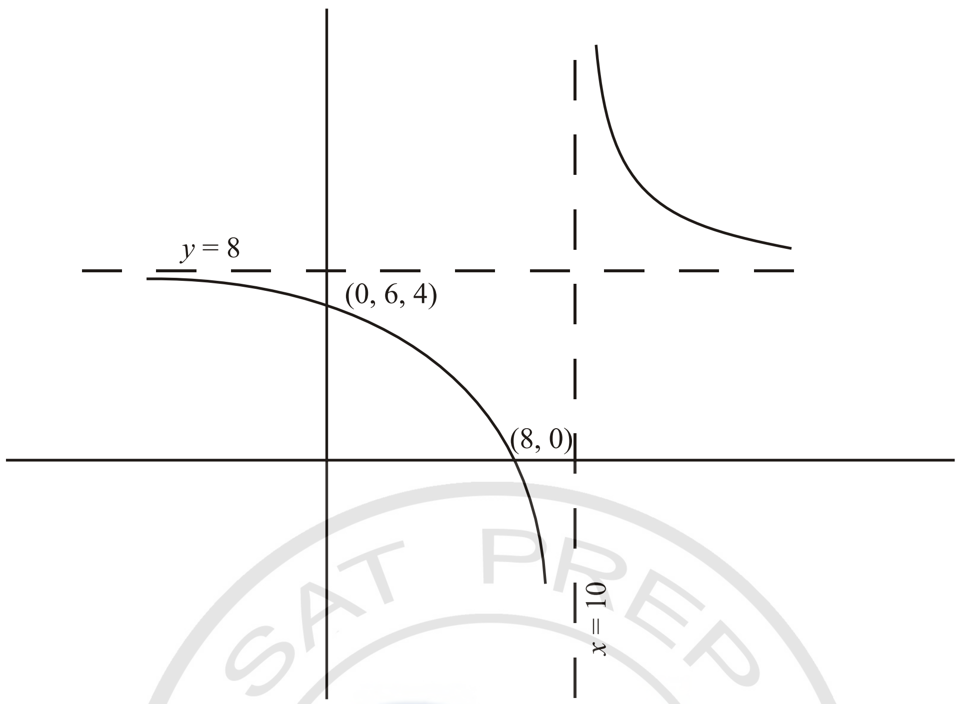
19. (a) (i) $x = 10$ (A1) (N1)

(ii) $y = 8$ (A1) (N1)

(b) (i) 6.4 (or (0, 6.4)) (A1) (N1)

(ii) 8 (or (8, 0)) (A1) (N1)

(c)



(A1)(A1)(A1)(A1) (N4)

Note: *Award (A1) for both asymptotes correctly drawn, (A1) for both intercepts correctly marked, (A1)(A1) for each branch drawn in approximately correct positions. Asymptotes and intercepts need not be labelled.*

(d) There is a vertical translation of 8 units.

(accept translation of $\begin{pmatrix} 0 \\ 8 \end{pmatrix}$) (A2) (N2)

[10]

20. (a) For a reasonable attempt to complete the square, (or expanding)

$3x^2 - 12x + 11 = 3(x^2 - 4x + 4) + 11 - 12$

$= 3(x - 2)^2 - 1$ (Accept $h = 2$, $k = 1$) A1A1 2

(b) **METHOD 1**

Vertex shifted to $(2 + 3, -1 + 5) = (5, 4)$ M1

so the new function is $3(x - 5)^2 + 4$ (Accept $p = 5$, $q = 4$) A1A1 2

METHOD 2

$g(x) = 3((x - 3) - h)^2 + k + 5 = 3((x - 3) - 2)^2 - 1 + 5$ M1

$= 3(x - 5)^2 + 4$ (Accept $p = 5$, $q = 4$) A1A1 2

[6]

21. (a) evidence of obtaining the vertex (M1)

e.g. a graph, $x = -\frac{b}{2a}$, completing the square

$f(x) = 2(x + 1)^2 - 8$ A2 N3

(b) $x = -1$ (equation must be seen) A1 N1

(c) $f(x) = 2(x - 1)(x + 3)$ A1A1 N2

[6]

22. $10\,000e^{-0.3t} = 1500$ (A1)

For taking logarithms (M1)

$-0.3t \ln e = \ln 0.15$ (A1)

$t = \frac{\ln 0.15}{-0.3}$ (A1)

$= 6.32$ (A1)

7 (years) (A1) (C6)

***Note**: Candidates may use a graphical method. Award (A1) for setting up the correct equation, (M1)(A1) for a sketch, (A1) for showing the point of intersection, (A1) for 6.32, and (A1) for 7.*

[6]

23. (a) Vertex is $(4, 8)$ A1A1 N2

(b) Substituting $-10 = a(7 - 4)^2 + 8$ M1

$a = -2$ A1 N1

(c) For y-intercept, $x = 0$ (A1)

$y = -24$ A1 N2

[6]

24. (a) in any order

translated 1 unit to the right A1 N1

stretched vertically by factor 2 A1 N1

(b) **METHOD 1**

Finding coordinates of image on g (A1)(A1)

e.g. $-1 + 1 = 0$, $1 \times 2 = 2$, $(-1, 1) \rightarrow (-1 + 1, 2 \times 1)$, $(0, 2)$

P is (3, 0) A1A1 N4

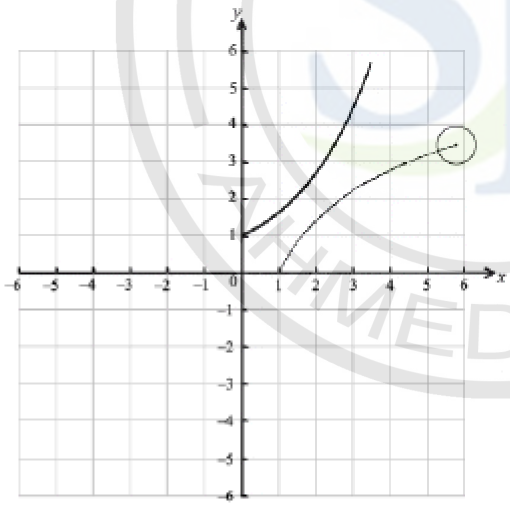
METHOD 2

$h(x) = 2(x - 4)^2 - 2$ (A1)(A1)

P is (3, 0) A1A1 N4

[6]

25. (a)



A1A1A1 N3

Note: *Award A1 for approximately correct (reflected) shape, A1 for right end point in circle, A1 for through (1, 0).*

(b) $0 \leq y \leq 3.5$ A1 N1

(c) interchanging x and y (seen anywhere) M1

e.g. $x = e^{0.5y}$

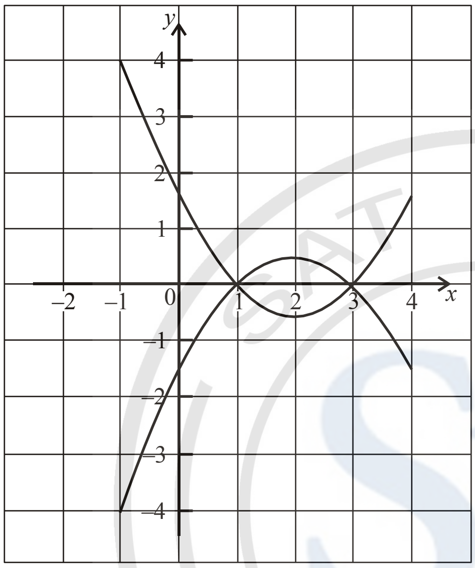
evidence of changing to log form A1

e.g. $\ln x = 0.5y$, $\ln x = \ln e^{0.5y}$ (any base), $\ln x = 0.5\, y \ln e$ (any base)

$f^{-1}(x) = 2 \ln x$ A1 N1

[7]

26. (a)



M1A1 N2

Note: *Award M1 for evidence of reflection in x-axis, A1 for correct vertex* ***and*** *all intercepts approximately correct.*

(b) (i) $g(-3) = f(0)$ (A1)

$f(0) = -1.5$ A1 N2

(ii) translation (accept shift, slide, *etc.*) of $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ A1A1 N2

[6]

27. (a) attempt to form composite (M1)

e.g. $g(7-2x), 7-2x+3$

$(g \circ f)(x) = 10 - 2x$ A1 N2 2

(b) $g^{-1}(x) = x - 3$ A1 N1 1

(c) **METHOD 1**

valid approach (M1)

e.g. $g^{-1}(5)$, 2, $f(5)$

$f(2) = 3$ A1 N2 2

METHOD 2

attempt to form composite of f and g^{-1} (M1)

e.g. $(f \circ g^{-1})(x) = 7 - 2(x - 3)$, $13 - 2x$

$(f \circ g^{-1})(5) = 3$ A1 N2 2

[5]

28. (a) attempt to apply rules of logarithms (M1)

e.g. $\ln a^b = b\ln a$, $\ln ab = \ln a + \ln b$

correct application of $\ln a^b = b\ln a$ (seen anywhere) A1

e.g. $3\ln x = \ln x^3$

correct application of $\ln ab = \ln a + \ln b$ (seen anywhere) A1

e.g. $\ln 5x^3 = \ln 5 + \ln x^3$

so $\ln 5x^3 = \ln 5 + 3\ln x$

$g(x) = f(x) + \ln 5$ (accept $g(x) = 3\ln x + \ln 5$) A1 N1 4

(b) transformation with correct name, direction, and value A3

e.g. translation by $\begin{pmatrix} 0 \\ \ln 5 \end{pmatrix}$, shift up by ln 5, vertical translation of ln 5 3

[7]

29. (a) $(1, -2)$ A1A1 N2 2

(b) $g(x) = 3(x - 1)^2 - 2$ (accept $p = 1$, $q = -2$) A1A1 N2 2

(c) $(1, 2)$ A1A1 N2 2

[6]

30. (a) attempt to form composite (M1)

e.g. $f(2x-5)$

$h(x) = 6x - 15$ A1 N2 2

(b) interchanging x and y (M1)

evidence of correct manipulation (A1)

e.g. $y+15-6x,\ \frac{x}{6} = y - \frac{5}{2}$

$h^{-1}(x) = \frac{x+15}{6}$ A1 N3 3

[5]

31. (a) $q = -2$, $r = 4$ or $q = 4$, $r = -2$ A1A1 N2

(b) $x = 1$ (must be an equation) A1 N1

(c) substituting $(0, -4)$ into the equation (M1)

e.g. $-4 = p(0-(-2))(0-4)$, $-4 = p(-4)(2)$

correct working towards solution (A1)

e.g. $-4 = -8p$

$p = \frac{4}{8}\left(=\frac{1}{2}\right)$ A1 N2

[6]

32. (a) evidence of setting function to zero (M1)

e.g. $f(x) = 0$, $8x = 2x^2$

evidence of correct working A1

e.g. $0 = 2x(4-x)$, $\frac{-8 \pm \sqrt{64}}{-4}$

x-intercepts are at 4 and 0 (accept (4, 0) and (0, 0), or $x = 4$, $x = 0$) A1A1 N1N1

(b) (i) $x = 2$ (must be equation) A1 N1

(ii) substituting $x = 2$ into $f(x)$ (M1)

$y = 8$ A1 N2

[7]

33. (a) interchanging x and y (seen anywhere) (M1)

e.g. $x = \log\sqrt{y}$ (accept any base)

evidence of correct manipulation A1

e.g. $3^x = \sqrt{y}, 3^y = x^{\frac{1}{2}}, x = \frac{1}{2}\log_3 y, 2y = \log_3 x$

$f^{-1}(x) = 3^{2x}$ AG N0

(b) $y > 0, f^{-1}(x) > 0$ A1 N1

(c) **METHOD 1**

finding $g(2) = \log_3 2$ (seen anywhere) A1

attempt to substitute (M1)

e.g. $(f^{-1} \circ g)(2) = 3^{\log_3 2}$

evidence of using log or index rule (A1)

e.g. $(f^{-1} \circ g)(2) = 3^{\log_3 4}, 3^{\log_3 2^2}$

$(f^{-1} \circ g)(2) = 4$ A1 N1

METHOD 2

attempt to form composite (in any order) (M1)

e.g. $(f^{-1} \circ g)(x) = 3^{2\log_3 x}$

evidence of using log or index rule (A1)

e.g. $(f^{-1} \circ g)(x) = 3^{\log_3 x^2}, 3^{\log_3 x^2}$

$(f^{-1} \circ g)(x) = x^2$ A1

$(f^{-1} \circ g)(2) = 4$ A1 N1

[7]