Subject : Math (Standard Level)

Topic– Function and Equation

1. Let $f(x) = x^2 + 4$ and $g(x) = x - 1$.

    (a) Find $(f \circ g)(x)$.

    (2)

    The vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ translates the graph of $(f \circ g)$ to the graph of h.

    (b) Find the coordinates of the vertex of the graph of h.

    (3)

    (c) Show that $h(x) = x^2 - 8x + 19$.

    (2)

    (d) The line $y = 2x - 6$ is a tangent to the graph of h at the point P. Find the x-coordinate of P.

    (5)
    (Total 12 marks)

2. Let $g(x) = 3x - 2$, $h(x) = \dfrac{5x}{x-4}$, $x \neq 4$.

    (a) Find an expression for $(h \circ g)(x)$. Simplify your answer.

    (b) Solve the equation $(h \circ g)(x) = 0$.

    (Total 6 marks)

3. (a) Express $y = 2x^2 - 12x + 23$ in the form $y = 2(x - c)^2 + d$.

    The graph of $y = x^2$ is transformed into the graph of $y = 2x^2 - 12x + 23$ by the transformations

    > a vertical stretch with scale factor k **followed by**
    > a horizontal translation of p units **followed by**
    > a vertical translation of q units.
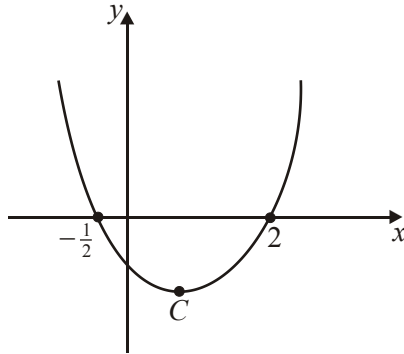
    (b) Write down the value of

    (i) k;

    (ii) p;

    (iii) q.

    (Total 6 marks)

4. The diagram represents the graph of the function

$$f : x \mapsto (x - p)(x - q).$$

(a) Write down the values of p and q.

(b) The function has a minimum value at the point C. Find the x-coordinate of C.
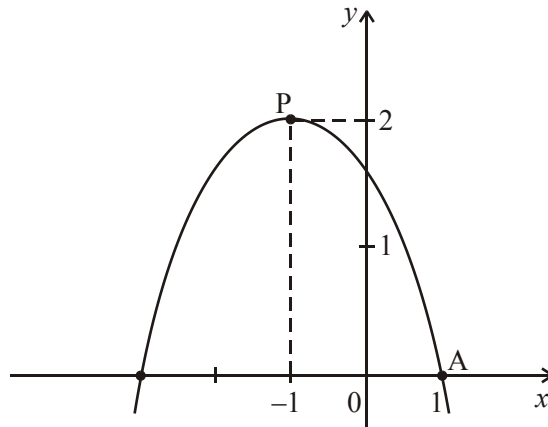
(Total 4 marks)

5. The mass m kg of a radio-active substance at time t hours is given by

$$m = 4e^{-0.2t}.$$

(a) Write down the initial mass.

(b) The mass is reduced to 1.5 kg. How long does this take?

(Total 6 marks)

6. The diagram shows part of the graph of $y = a(x - h)^2 + k$. The graph has its vertex at P, and passes through the point A with coordinates $(1, 0)$.

(a)   Write down the value of

(i) h;

(ii) k.

(b)   Calculate the value of a.

(Total 6 marks)

7. The diagrams show how the graph of $y = x^2$ is transformed to the graph of $y = f(x)$ in three

steps. For each diagram give the equation of the curve.

$y = x^2$

(a)

(b)

(c)
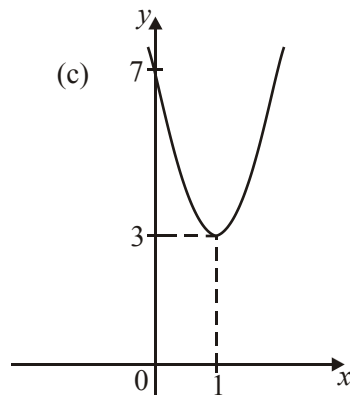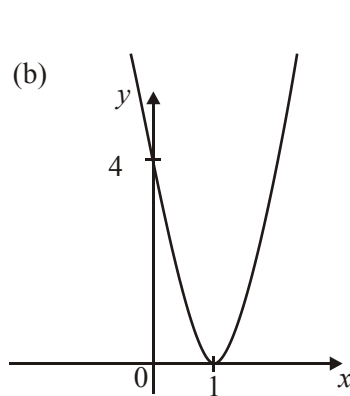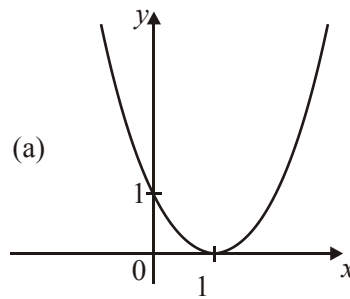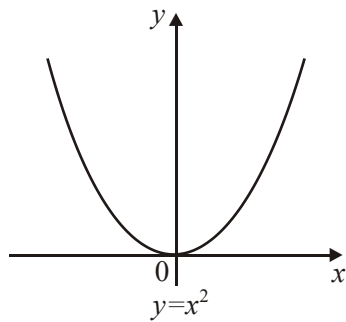
(Total 4 marks)

8. The function f is given by $f(x) = x^2 - 6x + 13$, for $x \geq 3$.
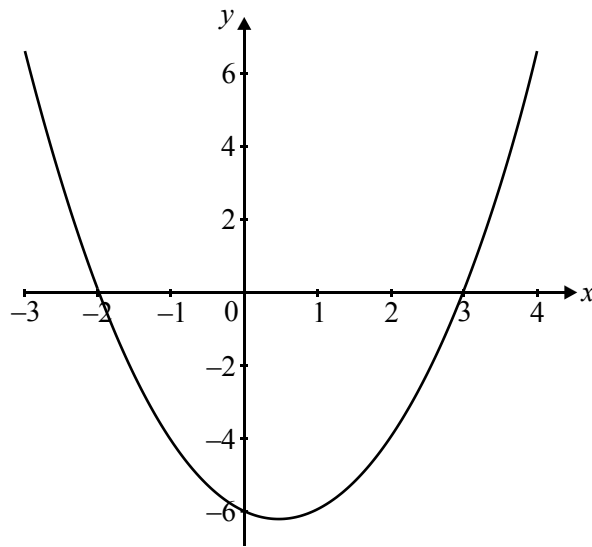
(a) Write $f(x)$ in the form $(x - a)^2 + b$.

(b) Find the inverse function f^{-1}.

(c) State the domain of f^{-1}.

(Total 6 marks)

9. The diagram shows part of the graph with equation $y = x^2 + px + q$. The graph cuts the x-axis at -2 and 3.

Find the value of

(a) p;

(b) q.

(Total 4 marks)

4

10. A group of ten leopards is introduced into a game park. After t years the number of leopards, N, is modelled by $N = 10 \, e^{0.4t}$.

(a) How many leopards are there after 2 years?

(b) How long will it take for the number of leopards to reach 100? Give your answers to an appropriate degree of accuracy.

Give your answers to an appropriate degree of accuracy.

(Total 4 marks)

11. The equation $x^2 - 2kx + 1 = 0$ has two distinct real roots. Find the set of all possible values of k.

(Total 6 marks)

12. The quadratic equation $4x^2 + 4kx + 9 = 0$, $k > 0$ has exactly one solution for x. Find the value of k.
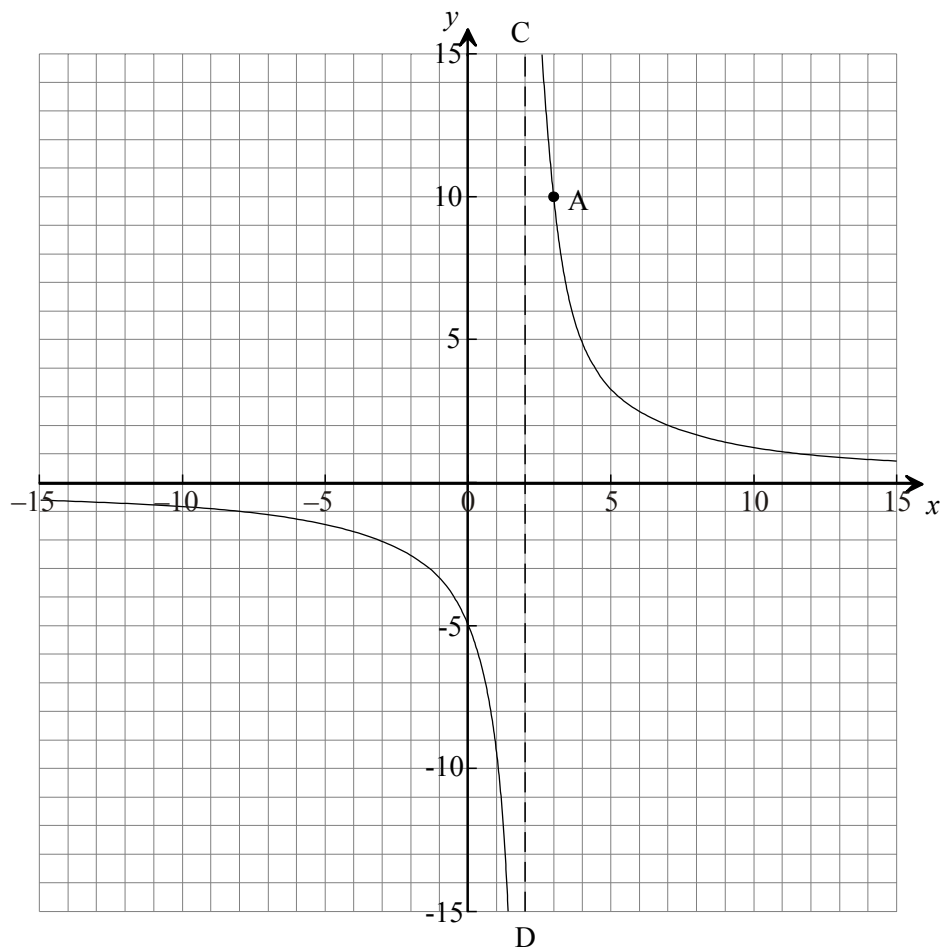
(Total 4 marks)

13. (a) Express $f(x) = x^2 - 6x + 14$ in the form $f(x) = (x - h)^2 + k$, where h and k are to be determined.

(b) Hence, or otherwise, write down the coordinates of the vertex of the parabola with equation $y - x^2 - 6x + 14$.
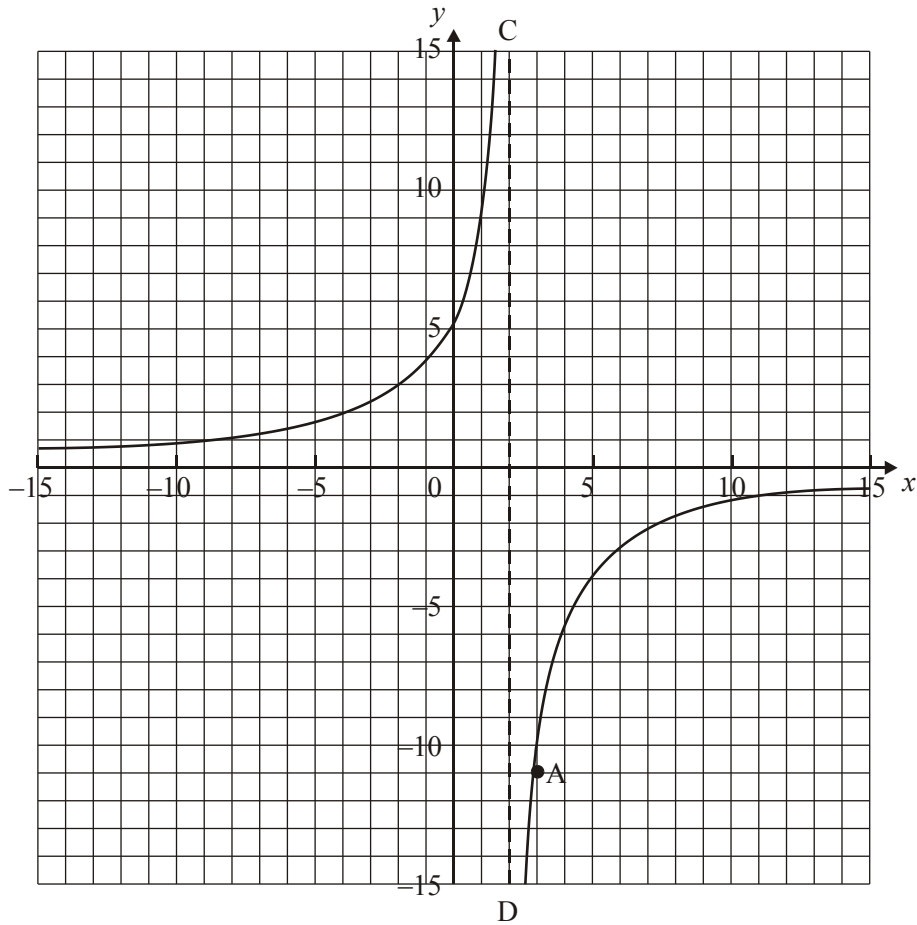
(Total 4 marks)

14. (a) The diagram shows part of the graph of the function $f(x) = \dfrac{q}{x-p}$. The curve passes

through the point A (3, 10). The line (CD) is an asymptote.

Find the value of

(i) p;

(ii) q.

(b)   The graph of $f(x)$ is transformed as shown in the following diagram. The point A is transformed to A′ $(3, -10)$.

Give a full geometric description of the transformation.

(Total 6 marks)

15. Consider the function $f(x) = 2x^2 - 8x + 5$.

(a)   Express $f(x)$ in the form $a(x-p)^2 + q$, where $a, p, q \in \mathbb{Z}$.

(b)   Find the minimum value of $f(x)$.

(Total 6 marks)

16. The quadratic function f is defined by $f(x) = 3x^2 - 12x + 11$.

(a)   Write f in the form $f(x) = 3(x-h)^2 - k$.

(3)

(b)   The graph of f is translated 3 units in the positive x-direction and 5 units in the positive y-direction. Find the function g for the translated graph, giving your answer in the form $g(x) = 3(x-p)^2 + q$.
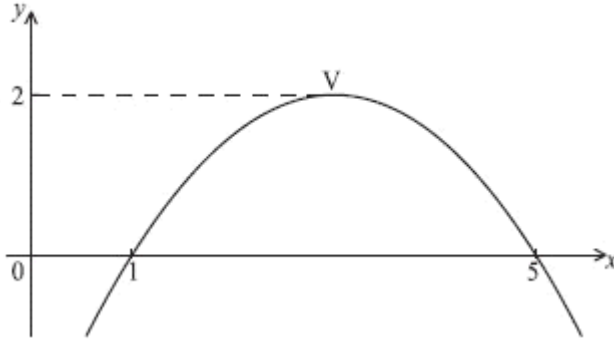
(3)
(Total 6 marks)

7

17. The equation $kx^2 + 3x + 1 = 0$ has exactly one solution. Find the value of k.

(Total 6 marks)

18. Part of the graph of the function $y = d\,(x - m)^2 + p$ is given in the diagram below.
The x-intercepts are $(1, 0)$ and $(5, 0)$. The vertex is $V(m, 2)$.

(a) Write down the value of

    (i) m;

    (ii) p.

(b) Find d.

(Total 6 marks)

19. Consider the function $f(x) = \dfrac{16}{x-10} + 8,\ x \neq 10$.

(a) Write down the **equation** of

    (i)   the vertical asymptote;

    (ii)   the horizontal asymptote.

(2)

(b) Find the

    (i) y-intercept;

    (ii) x-intercept.

(2)

(c) Sketch the graph of f, clearly showing the above information.

(4)

(d) Let $g(x) = \dfrac{16}{x},\ x \neq 0$.

The graph of g is transformed into the graph of f using two transformations.

The first is a translation with vector $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$. Give a full geometric description of the second transformation.

(2)
(Total 10 marks)

20. The quadratic function f is defined by $f(x) = 3x^2 - 12x + 11$.

   (a)   Write f in the form $f(x) = 3(x - h)^2 - k$.

   (b)   The graph of f is translated 3 units in the positive x-direction and 5 units in the positive y-direction. Find the function g for the translated graph, giving your answer in the form $g(x) = 3(x - p)^2 + q$.

(Total 6 marks)

21. Let $f(x) = 2x^2 + 4x - 6$.

   (a)   Express $f(x)$ in the form $f(x) = 2(x - h)^2 + k$.

(3)

   (b)   Write down the equation of the axis of symmetry of the graph of f.

(1)

   (c)   Express $f(x)$ in the form $f(x) = 2(x - p)(x - q)$.

(2)
(Total 6 marks)

22. A machine was purchased for $10000. Its value V after t years is given by $V = 100000e^{-0.3t}$. The machine must be replaced at the end of the year in which its value drops below $1500. Determine in how many years the machine will need to be replaced.

(Total 6 marks)

23. Let $f(x) = a(x - 4)^2 + 8$.

   (a)   Write down the coordinates of the vertex of the curve of f.

   (b)   Given that $f(7) = -10$, find the value of a.

   (c)   Hence find the y-intercept of the curve of f.

(Total 6 marks)

24. Let $f(x) = x^2$ and $g(x) = 2(x - 1)^2$.

   (a)   The graph of g can be obtained from the graph of f using two transformations.
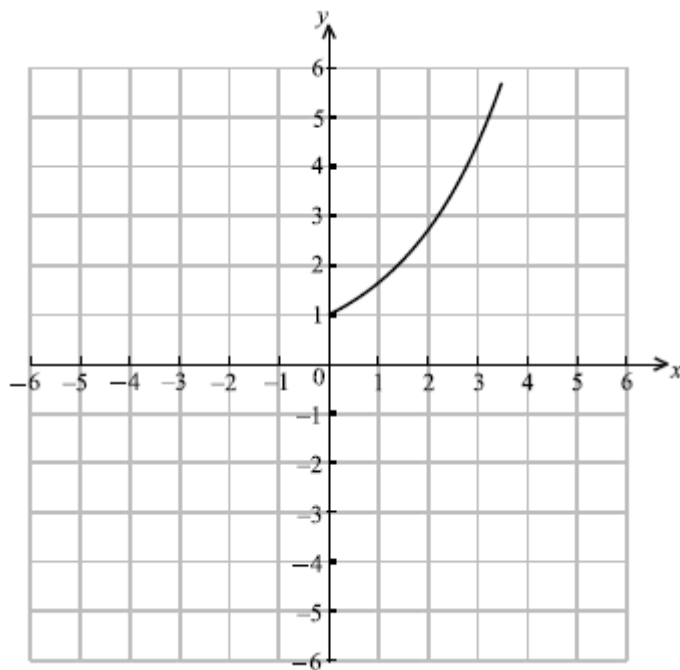   Give a full geometric description of each of the two transformations.

(2)

   (b)   The graph of g is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ to give the graph of h.

   The point $(-1, 1)$ on the graph of f is translated to the point P on the graph of h.
   Find the coordinates of P.

(4)
(Total 6 marks)

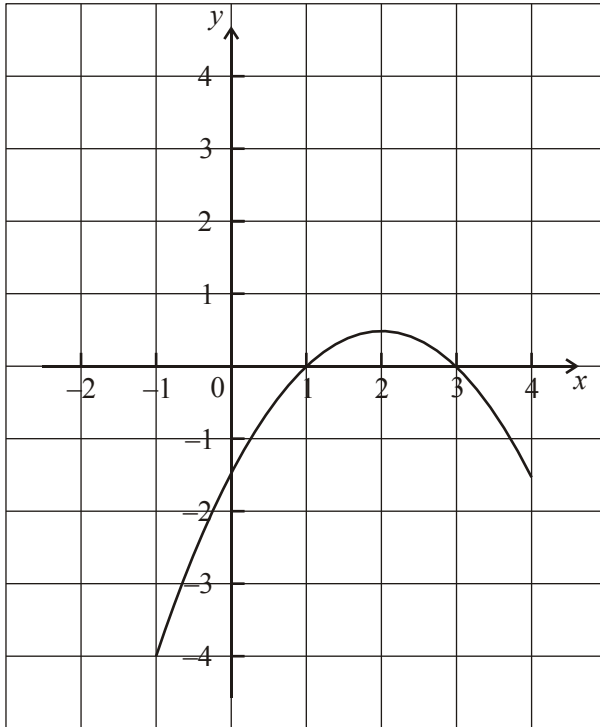**25.** Let f be the function given by $f(x) = e^{0.5x}$, $0 \leq x \leq 3.5$. The diagram shows the graph of f.

(a) On the same diagram, sketch the graph of f^{-1}.

(3)

(b) Write down the range of f^{-1}.

(1)

(c) Find $f^{-1}(x)$.

(3)
(Total 7 marks)

26. Part of the graph of a function *f* is shown in the diagram below.

(a) On the same diagram sketch the graph of $y = -f(x)$.

(2)

(b) Let $g(x) = f(x + 3)$.

  (i) Find $g(-3)$.

  (ii) Describe **fully** the transformation that maps the graph of *f* to the graph of *g*.

(4)
(Total 6 marks)

27. Let $f(x) = 7 - 2x$ and $g(x) = x + 3$.

(a) Find $(g \circ f)(x)$.

(2)

(b) Write down $g^{-1}(x)$.

(1)

(c) Find $(f \circ g^{-1})(5)$.

(2)
(Total 5 marks)

28. Let $f(x) = 3 \ln x$ and $g(x) = \ln 5x^3$.

(a) Express $g(x)$ in the form $f(x) + \ln a$, where $a \in \mathbb{Z}^+$.

(4)

(b) The graph of *g* is a transformation of the graph of *f*. Give a full geometric description of this transformation.

(3)
(Total 7 marks)

29. Let $f(x) = 3x^2$. The graph of f is translated 1 unit to the right and 2 units down. The graph of g is the image of the graph of f after this translation.

(a) Write down the coordinates of the vertex of the graph of g.

(2)

(b) Express g in the form $g(x) = 3(x - p)^2 + q$.

(2)

The graph of h is the reflection of the graph of g in the x-axis.
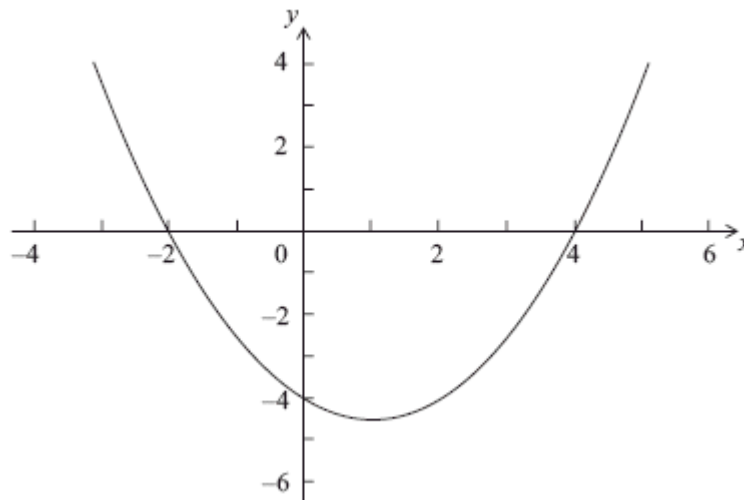
(c) Write down the coordinates of the vertex of the graph of h.

(2)
(Total 6 marks)

30. Let $f(x) = 3x$, $g(x) = 2x - 5$ and $h(x) = (f \circ g)(x)$.

(a) Find $h(x)$.

(2)

(b) Find $h^{-1}(x)$.

(3)
(Total 5 marks)

31. Let $f(x) = p(x - q)(x - r)$. Part of the graph of f is shown below.
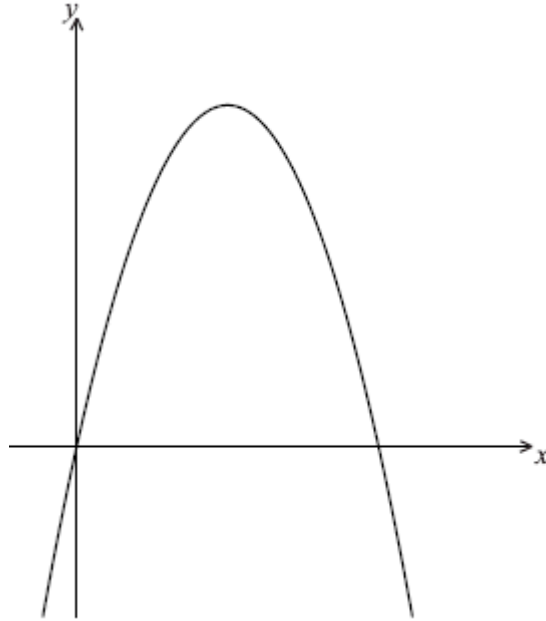

The graph passes through the points $(-2, 0)$, $(0, -4)$ and $(4, 0)$.

(a) Write down the value of q and of r.

(2)

(b) Write down the **equation** of the axis of symmetry.

(1)

(c) Find the value of p.

(3)
(Total 6 marks)

12

32. Let $f(x) = 8x - 2x^2$. Part of the graph of f is shown below.

(a)   Find the x-intercepts of the graph.

(4)

(b)   (i)   Write down the equation of the axis of symmetry.

(ii)   Find the y-coordinate of the vertex.

(3)
(Total 7 marks)

33. Let $f(x) = \log_3 \sqrt{x}$, for $x > 0$.

(a)   Show that $f^{-1}(x) = 3^{2x}$.

(2)

(b)   Write down the range of f^{-1}.

(1)

Let $g(x) = \log_3 x$, for $x > 0$.

(c)   Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer.

(4)
(Total 7 marks)