Subject – Math(Higher Level)
Topic - Statistics and Probability
Year - Nov 2011 – Nov 2017
Paper -2

Question -1

(a) $m = \frac{300}{60} = 5$ *(A1)*

$P(X = 0) = 0.00674$ **A1**

or e^{-5}

(b) $E(X) = 5 \times 2 = 10$ **A1**

(c) $P(X > 10) = 1 - P(X \leq 10)$ *(M1)*

$= 0.417$ **A1**

[5 marks]

Question-2

(a) $X \sim B(5, 0.1)$ *(M1)*

$P(X = 2) = 0.0729$ **A1**

(b) $P(X \geq 1) = 1 - P(X = 0)$ *(M1)*

$0.9 < 1 - \left(\frac{9}{10}\right)^n$ *(M1)*

$n > \frac{\ln 0.1}{\ln 0.9}$

$n = 22$ days **A1**

[5 marks]

Question 3

(a) $X \sim \text{N}(60.33, 1.95^2)$

$\text{P}(X < x) = 0.2 \Rightarrow x = 58.69$ m — ***(M1)A1***

[2 marks]

(b) $z = -0.8416\ldots$ — ***(A1)***

$-0.8416 = \dfrac{56.52 - 59.39}{\sigma}$ — ***(M1)***

$\sigma \approx 3.41$ — ***A1***

[3 marks]

(c) Jan $X \sim \text{N}(60.33, 1.95^2)$; Sia $X \sim \text{N}(59.50, 3.00^2)$

(i) Jan: $\text{P}(X > 65) \approx 0.00831$ — ***(M1)A1***

Sia: $\text{P}(Y > 65) \approx 0.0334$ — ***A1***

Sia is more likely to qualify — ***R1***

Note: Only award ***R1*** if ***(M1)*** has been awarded.

(ii) Jan: $\text{P}(X \geq 1) = 1 - \text{P}(X = 0)$ — ***(M1)***

$= 1 - (1 - 0.00831\ldots)^3 \approx 0.0247$ — ***(M1)A1***

Sia: $\text{P}(Y \geq 1) = 1 - \text{P}(Y = 0) = 1 - (1 - 0.0334\ldots)^3 \approx 0.0968$ — ***A1***

Note: Accept 0.0240 and 0.0969.

hence, $\text{P}(X \geq 1 \text{ and } Y \geq 1) = 0.0247 \times 0.0968 = 0.00239$ — ***(M1)A1***

[10 marks]

Total [15 marks]

Question -4

(a) $\begin{pmatrix} 10 \\ 6 \end{pmatrix} = 210$ — ***(M1)A1***

[2 marks]

(b) $2 \times \begin{pmatrix} 8 \\ 5 \end{pmatrix} = 112$ — ***(M1)A1A1***

Note: Accept 210 – 28 – 70 = 112

[3 marks]

(c) $\dfrac{112}{210} \left(= \dfrac{8}{15} = 0.533 \right)$ — ***(M1)A1***

[2 marks]

Total [7 marks]

Question -5

(a) 50 **A1**

[1 mark]

(b) Lower quartile is 4 so at least 26 obtained a 4 **R1**
Lower bound is 26 **A1**

Minimum is 2 but the rest could be 4 **R1**

So upper bound is 49 **A1**

Note:	Do not allow follow through for ***A*** marks.

Note:	If answers are incorrect award ***R0A0***, if argument is correct but no clear lower/upper bound is stated award ***R1A0***; award ***R0A1*** for correct answer without explanation or incorrect explanation.

[4 marks]

Total [5 marks]

Question – 6

$X \sim \mathrm{Po}(m)$

$\mathrm{P}(X=2)=\mathrm{P}(X<2)$ **(M1)**

$\frac{1}{2}m^2\mathrm{e}^{-m}=\mathrm{e}^{-m}(1+m)$ **(A1)(A1)**

$m=2.73 \quad \left(1+\sqrt{3}\right)$ **A1**

in four hours the expected value is 10.9 $\left(4+4\sqrt{3}\right)$ **A1**

te: Value of m does not need to be rounded.

[5 marks]

Question -7

(a) (i) $X \sim \text{Po}(11)$ *(M1)*

$\text{P}(X \le 11) = 0.579$ *(M1)A1*

(ii) $\text{P}(X > 8 \mid X < 12) =$ *(M1)*

$= \dfrac{\text{P}(8 < X < 12)}{\text{P}(X < 12)} \left(\text{or } \dfrac{\text{P}(X \le 11) - \text{P}(X \le 8)}{\text{P}(X \le 11)} \text{ or } \dfrac{0.3472\ldots}{0.5792\ldots} \right)$ *A1*

$= 0.600$ *A1* *N2*

[6 marks]

(b) (i) $Y \sim \text{Po}(m)$

$\text{P}(Y > 3) = 0.24$ *(M1)*

$\text{P}(Y \le 3) = 0.76$ *(M1)*

$e^{-m}\left(1 + m + \frac{1}{2}m^2 + \frac{1}{6}m^3\right) = 0.76$ *(A1)*

Note: At most two of the above lines can be implied.

Attempt to solve equation with GDC *(M1)*

$m = 2.49$ *A1*

(ii) $A \sim \text{Po}(4.98)$

$\text{P}(A > 5) = 1 - \text{P}(A \le 5) = 0.380\ldots$ *M1A1*

$W \sim \text{B}(4, 0.380\ldots)$ *(M1)*

$\text{P}(W \ge 2) = 1 - \text{P}(W \le 1) = 0.490$ *M1A1*

[10 marks]

(c) $\text{P}(A < 25) = 0.8,\ \text{P}(A < 18) = 0.4$

$\dfrac{25 - \mu}{\sigma} = 0.8416\ldots$ *(M1)(A1)*

$\dfrac{18 - \mu}{\sigma} = -0.2533\ldots$ (or -0.2534 from tables) *(M1)(A1)*

solving these equations *(M1)*

$\mu = 19.6$ *A1*

Note: Accept just 19.6, 19 or 20; award A0 to any other final answer.

[6 marks]

Total [22 marks]

Question -8

(a) $\mathrm{E}(X) = np$

$\Rightarrow 10 = 30p$

$\Rightarrow p = \dfrac{1}{3}$ *A1*

[1 mark]

(b) $\mathrm{P}(X = 10) = \binom{30}{10}\left(\dfrac{1}{3}\right)^{10}\left(\dfrac{2}{3}\right)^{20} = 0.153$ *(M1)A1*

[2 marks]

(c) $\mathrm{P}(X \geq 15) = 1 - \mathrm{P}(X \leq 14)$ *(M1)*

$= 1 - 0.9565\ldots = 0.0435$ *A1*

[2 marks]

Total [5 marks]

Question -9

(a) $\mathrm{P}(X = 5) = \mathrm{P}(X = 3) + \mathrm{P}(X = 4)$

$\dfrac{e^{-m}m^5}{5!} = \dfrac{e^{-m}m^3}{3!} + \dfrac{e^{-m}m^4}{4!}$ *M1(A1)*

$m^2 - 5m - 20 = 0$

$\Rightarrow m = \dfrac{5 + \sqrt{105}}{2} = (7.62)$ *A1*

[3 marks]

(b) $\mathrm{P}(X > 2) = 1 - \mathrm{P}(X \leq 2)$ *(M1)*

$= 1 - 0.018\ldots$

$= 0.982$ *A1*

[2 marks]

Total [5 marks]

Question 10

(a) $\int_0^a \frac{1}{1+x^4}\,dx = 1$ **M2**

$a = 1.40$ **A1**

[3 marks]

(b) $E(X) = \int_0^a \frac{x}{1+x^4}\,dx$ **M1**

$\left(= \frac{1}{2}\arctan(a^2)\right)$

$= 0.548$ **A1**

[2 marks]

Total [5 marks]

Question 11

(a) (i) $P(X > 225) = 0.158\ldots$ **(M1)(A1)**

expected number $= 450 \times 0.158\ldots = 71.4$ **A1**

(ii) $P(X < m) = 0.7$ **(M1)**

$\Rightarrow m = 213$ (grams) **A1**

[5 marks]

(b) $\frac{270-\mu}{\sigma} = 1.40\ldots$ **(M1)A1**

$\frac{250-\mu}{\sigma} = -1.03\ldots$ **A1**

Note: These could be seen in graphical form.

solving simultaneously **(M1)**

$\mu = 258,\ \sigma = 8.19$ **A1A1**

[6 marks]

(c)

$X \sim N(80, 4^2)$

$P(X > 82) = 0.3085\ldots$ **A1**

recognition of the use of binomial distribution. **(M1)**

$X \sim B(5, 0.3085\ldots)$

$P(X = 3) = 0.140$ **A1**

[3 marks]

Total [14 marks]

Question 12

$$\frac{\sum_{i=1}^{15} x_i}{15} = 11.5 \Rightarrow \sum_{i=1}^{15} x_i = 172.5 \qquad \textit{(A1)}$$

$$\text{new mean} = \frac{172.5 - 22.1}{14} \qquad \textit{(M1)}$$

$$= 10.7428\ldots = 10.7 \text{ (3sf)} \qquad \textbf{\textit{A1}}$$

$$\frac{\sum_{i=1}^{15} x_i^2}{15} - 11.5^2 = 9.3 \qquad \textit{(M1)}$$

$$\Rightarrow \sum_{i=1}^{15} x_i^2 = 2123.25$$

$$\text{new variance} = \frac{2123.25 - 22.1^2}{14} - (10.7428\ldots)^2 \qquad \textit{(M1)}$$

$$= 1.37 \text{ (3sf)} \qquad \textbf{\textit{A1}}$$

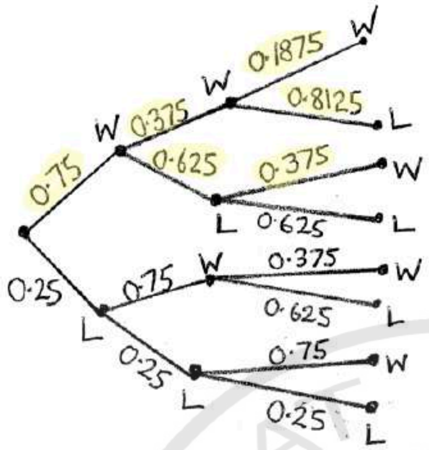
[6 marks]

Question 13

(a) $P(WWW) = 0.75 \times 0.375 \times 0.1875 = 0.0527$ (3sf) $\left(\frac{3}{4} \times \frac{3}{8} \times \frac{3}{16} = \frac{27}{512}\right)$ ***(M1)A1***

[2 marks]

(b)



(M1)(A1)

Note:	Award ***M1*** for any reasonable attempt to use a tree diagram showing that three games were played (do not award ***M1*** for tree diagrams that only show the first two games) and ***A1*** for the highlighted probabilities.

$P(\text{wins 2 games} \mid \text{wins first game}) = \frac{P(WWL,\ WLW)}{P(\text{wins first game})}$ ***(M1)***

$= \frac{0.75 \times 0.375 \times 0.8125 + 0.75 \times 0.625 \times 0.375}{0.75}$ ***(A1)(A1)***

$= 0.539$ (3sf) $\left(\text{or } \frac{69}{128}\right)$ ***A1***

Note:	Candidates may use the tree diagram to obtain the answer without using the conditional probability formula, *ie*, $P(\text{wins 2 games} \mid \text{wins first game}) = 0.375 \times 0.8125 + 0.625 \times 0.375 = 0.539$.

[6 marks]

Total [8 marks]

Question 14

(a) $2.2 \times 6 \times 60 = 792$ *(M1)A1*

[2 marks]

(b) $V \sim \text{Po}(2.2 \times 60)$ *(M1)*

$\text{P}(V > 100) = 0.998$ *(M1)A1*

[3 marks]

(c) $(0.997801\ldots)^6 = 0.987$ *(M1)A1*

[2 marks]

(d) $A \sim \text{N}(\mu, \sigma^2)$

$\text{P}(A < 35) = 0.29$ and $\text{P}(A > 55) = 0.23 \Rightarrow \text{P}(A < 55) = 0.77$

$\text{P}\left(Z < \frac{35 - \mu}{\sigma}\right) = 0.29$ and $\text{P}\left(Z < \frac{55 - \mu}{\sigma}\right) = 0.77$ *(M1)*

use of inverse normal *(M1)*

$\frac{35 - \mu}{\sigma} = -0.55338\ldots$ and $\frac{55 - \mu}{\sigma} = 0.738846\ldots$ *(A1)*

solving simultaneously *(M1)*

$\mu = 43.564\ldots$ and $\sigma = 15.477\ldots$ *A1A1*

$\mu = 43.6$ and $\sigma = 15.5$ (3sf)

[6 marks]

(e) $0.29n = 100 \Rightarrow n = 344.82\ldots$ *(M1)(A1)*

$\text{P}(A < 50) = 0.66121\ldots$ *(A1)*

expected number of visitors under $50 = 228$ *(M1)A1*

[5 marks]

Total [18 marks]

Question 15

$\dfrac{5\times 6+6k+7\times 3+8\times 1+9\times 2+10\times 1}{13+k}=6.5$ (or equivalent) ***(M1)(A1)(A1)***

ote: Award ***(M1)(A1)*** for correct numerator, and ***(A1)*** for correct denominator.

$0.5k=2.5\Rightarrow k=5$ ***A1***

[4 marks]

Question 16

Let X represent the length of time a journey takes on a particular day.

(a) $P(X>15)=0.0912112819\ldots=0.0912$ ***(M1)A1***

(b) Use of correct Binomial distribution ***(M1)***

$N\sim B(5, 0.091...)$

$1-0.0912112819\ldots=0.9087887181\ldots$

$1-(0.9087887181\ldots)^5=0.380109935\ldots=0.380$ ***(M1)A1***

Note: Allow answers to be given as percentages.

[5 marks]

Question 17

(a) $X\sim \mathrm{Po}(0.25\mathrm{T})$ ***(A1)***

Attempt to solve $\mathrm{P}(X\le 3)=0.6$ ***(M1)***

$T=12.8453\ldots\ =\ 13$ (minutes) ***A1***

Note: Award ***A1M1A0*** if T found correctly but not stated to the nearest minute.

[3 marks]

(b) let X_1 be the number of cars that arrive during the first interval and X_2 be the number arriving during the second.

X_1 and X_2 are $\mathrm{Po}(2.5)$ ***(A1)***

P (all get on) $=\mathrm{P}(X_1\le 3)\times \mathrm{P}(X_2\le 3)+\mathrm{P}(X_1=4)\times \mathrm{P}(X_2\le 2)$

$+\mathrm{P}(X_1=5)\times \mathrm{P}(X_2\le 1)+\mathrm{P}(X_1=6)\times \mathrm{P}(X_2=0)$ ***(M1)***

$=0.573922\ldots+0.072654\ldots+0.019192\ldots+0.002285\ldots$ ***(M1)***

$=0.668\ (053\ldots)$ ***A1***

[4 marks]

Total [7 marks]

Question 18

(a) $X \sim N(13.5, 9.5)$

$13.5 - \sqrt{9.5} < X < 13.5 + \sqrt{9.5}$ *(M1)*

$10.4 < X < 16.6$ *A1*

Note: Accept 6.16.

[2 marks]

(b) $\mathrm{P}(X < 10) = 0.12807\ldots$ *(M1)(A1)*

estimate is 1281 (correct to the nearest whole number). *A1*

Note: Accept 1280.

[3 marks]

Total [5 marks]

Question 19

(a) $\int_0^{0.5} ax^2 \mathrm{d}x + \int_{0.5}^{1} 0.5a(1-x)\mathrm{d}x = 1$ *M1A1*

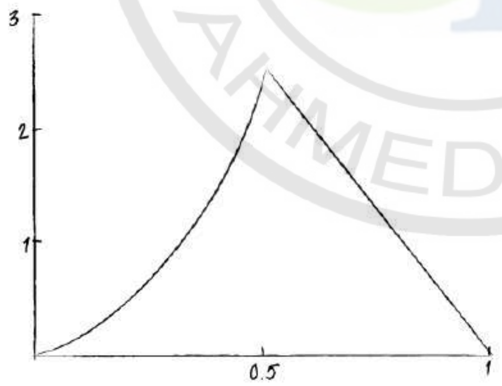
$\frac{5a}{48}$ (or equivalent) or $a \times 0.104\ldots = 1$ *A1*

Note: Award *M1* for considering two definite integrals. Award *A1* for equating to 1. Award *A1* for a correct equation. The *A1A1* can be awarded in any order.

$a = 9.6$ *AG*

[3 marks]

(b)



correct shape for $0 \le x \le 0.5$ and $f(0.5) \approx 2.4$ *A1*

correct shape for $0.5 \le x \le 1$ and $f(1) = 0$ *A1*

[2 marks]

(c) attempting to find $\mathrm{P}(X < 0.6)$ *(M1)*

direct GDC use or *eg* $\mathrm{P}(0 \le X \le 0.5) + \mathrm{P}(0.5 \le X \le 0.6)$ or $1 - \mathrm{P}(0.6 \le X \le 1)$

$\mathrm{P}(X < 0.6) = 0.616 \left(= \frac{77}{125}\right)$ *A1*

[2 marks]

Total [7 marks]

Question 20

(a) $X \sim \text{Po}(1.2)$

$\text{P}(X=3)\times\text{P}(X=0)$ **(M1)**

$=0.0867\ldots\times0.3011\ldots$

$=0.0261$ **A1**

[2 marks]

(b) Three requests over two days can occur as $(3,0),(0,3),(2,1)$ or $(1,2)$. **R1**

using conditional probability, for example

$\dfrac{\text{P}(3,0)}{\text{P}(3\text{ requests}, m=2.4)}=0.125$ or $\dfrac{\text{P}(2,1)}{\text{P}(3\text{ requests}, m=2.4)}=0.375$ **M1A1**

expected income is

$2\times0.125\times\text{US\$}120+2\times0.375\times\text{US\$}180$ **M1**

Note:	Award ***M1*** for attempting to find the expected income including both $(3,0)$ and $(2,1)$ cases.

$=\text{US\$}30+\text{US\$}135$

$=\text{US\$}165$ **A1**

[5 marks]

Total [7 marks]

Question 21

$\text{P}\left(Z<\dfrac{780-\mu}{\sigma}\right)=0.92$ and $\text{P}\left(Z<\dfrac{755-\mu}{\sigma}\right)=0.12$ ***(M1)***

use of inverse normal ***(M1)***

$\Rightarrow\dfrac{780-\mu}{\sigma}=1.405\ldots$ and $\dfrac{755-\mu}{\sigma}=-1.174\ldots$ ***(A1)***

solving simultaneously ***(M1)***

Note:	Award ***M1*** for attempting to solve an incorrect pair of equations *eg*, inverse normal not used.

$\mu=766.385$

$\sigma=9.6897$

$\mu=12$ hrs 46 mins $(=766$ mins$)$ ***A1***

$\sigma=10$ mins ***A1***

Total [6 marks]

Question 22

(a) $\mathrm{P}(F)=\left(\frac{1}{7}\times\frac{7}{9}\right)+\left(\frac{6}{7}\times\frac{4}{9}\right)$ ***(M1)(A1)***

Note: Award ***M1*** for the sum of two products.

$=\frac{31}{63}\ (=0.4920\ldots)$ ***A1***

[3 marks]

(b) Use of $\mathrm{P}(S\mid F)=\frac{\mathrm{P}(S\cap F)}{\mathrm{P}(F)}$ to obtain $\mathrm{P}(S\mid F)=\frac{\frac{1}{7}\times\frac{7}{9}}{\frac{31}{63}}$. ***M1***

Note: Award ***M1*** only if the numerator results from the product of two probabilities.

$=\frac{7}{31}\ (=0.2258\ldots)$ ***A1***

[2 marks]

Total [5 marks]

Question 23

(a) (i) $X \sim \mathrm{Po}(0.6)$

$\mathrm{P}(X=0)=0.549 \ \left(=\mathrm{e}^{-0.6}\right)$ ***A1***

(ii) $\mathrm{P}(X \geq 3)=1-\mathrm{P}(X \leq 2)$ ***(M1)(A1)***

$=1-\left(\mathrm{e}^{-0.6}+\mathrm{e}^{-0.6} \times 0.6+\mathrm{e}^{-0.6} \times \frac{0.6^{2}}{2}\right)$

$=0.0231$ ***A1***

(iii) $Y \sim \mathrm{Po}(2.4)$ ***(M1)***

$\mathrm{P}(Y \leq 5)=0.964$ ***A1***

(iv) $Z \sim \mathrm{B}(12, 0.451\ldots)$ ***(M1)(A1)***

Note: Award ***M1*** for recognising binomial and ***A1*** for using correct parameters.

$\mathrm{P}(Z=4)=0.169$ ***A1***

[9 marks]

(b) (i) $k\int_{1}^{3} \ln x \, \mathrm{d}x=1$ ***(M1)***

$(k \times 1.2958\ldots=1)$

$k=0.771702$ ***A1***

(ii) $\mathrm{E}(X)=\int_{1}^{3} kx \ln x \, \mathrm{d}x$ ***(A1)***

attempting to evaluate their integral ***(M1)***

$=2.27$ ***A1***

(iii) $x=3$ ***A1***

(iv) $\int_{1}^{m} k \ln x \, \mathrm{d}x=0.5$ ***(M1)***

$k[x \ln x-x]_{1}^{m}=0.5$

attempting to solve for m ***(M1)***

$m=2.34$ ***A1***

[9 marks]

Total [18 marks]

Question 24

$X: \mathrm{N}\left(100, \sigma^{2}\right)$

$\mathrm{P}(X<124)=0.68$ ***(M1)(A1)***

$\frac{24}{\sigma}=0.4676\ldots$ ***(M1)***

$\sigma=51.315\ldots$ ***(A1)***

variance $=2630$ ***A1***

[5 marks]

Notes: Accept use of $\mathrm{P}(X<124.5)=0.68$ leading to variance $=2744$.

Question 25

(a) $\left(A\binom{6}{5}2^5 B+3\binom{6}{4}2^4 B^2\right)x^5$ ***M1A1A1***

$=\left(192AB+720B^2\right)x^5$ ***A1***

[4 marks]

(b) **METHOD 1**

$x=\frac{1}{6},\ A=\frac{3}{6}\left(=\frac{1}{2}\right),\ B=\frac{4}{6}\left(=\frac{2}{3}\right)$ ***A1A1A1***

probability is $\frac{4}{81}\ (=0.0494)$ ***A1***

METHOD 2

$\mathrm{P}(5 \text{ eaten})=\mathrm{P}(\text{M eats } 1)\,\mathrm{P}(\text{N eats } 4)+\mathrm{P}(\text{M eats } 0)\,\mathrm{P}(\text{N eats } 5)$ ***(M1)***

$=\frac{1}{2}\binom{6}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2+\frac{1}{2}\binom{6}{5}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)$ ***(A1)(A1)***

$=\frac{4}{81}\ (=0.0494)$ ***A1***

[4 marks]

Total [8 marks]

Question 26

(a) mean for week is 40.88 *(A1)*

$P(S > 40) = 1 - P(S \le 40) = 0.513$ *A1*

[2 marks]

(b) $\dfrac{\text{probability there were more than 10 on Monday AND more than 40 over the week}}{\text{probability there were more than 10 on Monday}}$

M1

possibilities for the numerator are:

there were more than 40 birds on the power line on Monday ***R1***
11 on Monday and more than 29 over the course of the next 6 days ***R1***
12 on Monday and more than 28 over the course of the next 6 days … until
40 on Monday and more than 0 over the course of the next 6 days ***R1***
hence if X is the number on the power line on Monday and Y, the number on the power line Tuesday – Sunday then the numerator is ***M1***

$$P(X > 40) + P(X = 11) \times P(Y > 29) + P(X = 12) \times P(Y > 28) + \ldots$$
$$+ P(X = 40) \times P(Y > 0)$$
$$= P(X > 40) + \sum_{r=11}^{40} P(X = r)P(Y > 40 - r)$$

hence solution is $\dfrac{P(X > 40) + \sum_{r=11}^{40} P(X = r)P(Y > 40 - r)}{P(X > 10)}$ ***AG***

[5 marks]

Total [7 marks]

Question 27

(a) $\int_2^3 (ax+b)\,dx\,(=1)$ **M1A1**

$\left[\frac{1}{2}ax^2+bx\right]_2^3 (=1)$ **A1**

$\frac{5}{2}a+b=1$ **M1**

$5a+2b=2$ **AG**

[4 marks]

(b) (i) $\int_2^3 \left(ax^2+bx\right)dx\,(=\mu)$ **M1A1**

$\left[\frac{1}{3}ax^3+\frac{1}{2}bx^2\right]_2^3 (=\mu)$ **A1**

$\frac{19}{3}a+\frac{5}{2}b=\mu$ **A1**

eliminating b **M1**

eg

$\frac{19}{3}a+\frac{5}{2}\left(1-\frac{5}{2}a\right)=\mu$ **A1**

$\frac{1}{12}a+\frac{5}{2}=\mu$

$a=12\mu-30$ **AG**

Note: Elimination of b could be at different stages.

(ii) $b=1-\frac{5}{2}(12\mu-30)$

$=76-30\mu$ **A1**

Note: This solution may be seen in part (i).

[7 marks]

(c) (i) $\int_2^{2.3} (ax+b)\,dx\,(=0.5)$ ***(M1)(A1)***

$\left[\frac{1}{2}ax^2+bx\right]_2^{2.3} (=0.5)$

$0.645a+0.3b(=0.5)$ ***(A1)***

$0.645(12\mu-30)+0.3(76-30\mu)=0.5$ **M1**

$\mu=2.34\left(=\frac{295}{126}\right)$ **A1**

(ii) $\mathrm{E}\left(X^2\right)=\int_2^3 x^2(ax+b)\,\mathrm{d}x$ ***(M1)***

$a=12\mu-30=-\dfrac{40}{21},\ b=76-30\mu=\dfrac{121}{21}$ ***(A1)***

$\mathrm{E}\left(X^2\right)=\int_2^3 x^2\left(-\dfrac{40}{21}x+\dfrac{121}{21}\right)\mathrm{d}x=5.539\ldots\left(=\dfrac{349}{63}\right)$ ***(A1)***

$\mathrm{Var}(X)=5.539\mathrm{K}-(2.341\mathrm{K})^2=0.05813\ldots$ ***(M1)***

$\sigma=0.241$ ***A1***

[10 marks]

Total [21 marks]

Question 28

(a) (i) $0.6^3\times0.4^3$ ***(M1)***

Note:	Award ***(M1)*** for use of the product of probabilities.

$=0.0138$ ***A1***

(ii) binomial distribution $X:\ \mathrm{B}(6,\,0.6)$ ***(M1)***

Note:	Award ***(M1)*** for recognizing the binomial distribution.

$\mathrm{P}(X=3)={}^6C_3\,(0.6)^3\,(0.4)^3$

$=0.276$ ***A1***

Note:	Award ***(M1)A1*** for ${}^6C_3\times0.0138=0.276$.

(b) $Y:\ \mathrm{B}(n,\,0.4)$

$\mathrm{P}(Y\geq1)>0.995$

$1-\mathrm{P}(Y=0)>0.995$

$\mathrm{P}(Y=0)<0.005$ ***(M1)***

Note:	Award ***(M1)*** for any of the last three lines. Accept equalities.

$0.6^n<0.005$ ***(M1)***

Note:	Award ***(M1)*** for attempting to solve $0.6^n<0.005$ using any method, *eg*, logs, graphically, use of solver. Accept an equality.

$n>10.4$

$\therefore n=11$ ***A1***

[3 marks]

Total [7 marks]

Question 29

(a) $\frac{\mu^2 e^{-\mu}}{2!} + \frac{\mu^3 e^{-\mu}}{3!} = \frac{\mu^5 e^{-\mu}}{5!}$ ***(M1)***

$\frac{\mu^2}{2} + \frac{\mu^3}{6} - \frac{\mu^5}{120} = 0$

$\mu = 5.55$ ***A1***

[2 marks]

(b) $\sigma = \sqrt{5.55...} = 2.35598...$ ***(M1)***

$\mathrm{P}(3.19 \leq X \leq 7.9)$

$\mathrm{P}(4 \leq X \leq 7)$

$= 0.607$ ***A1***

[2 marks]

Total [4 marks]

Question 30

(a) $a\int_0^{\frac{\pi}{2}} x\cos x\,dx = 1$ *(M1)*

integrating by parts:

$u = x \qquad v' = \cos x$

$u' = 1 \qquad v = \sin x$ *M1*

$\int x\cos x\,dx = x\sin x + \cos x$ *A1*

$[x\sin x + \cos x]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 1$ *A1*

$a = \frac{1}{\frac{\pi}{2} - 1}$ *A1*

$= \frac{2}{\pi - 2}$ *AG*

[5 marks]

(b) $P\left(X < \frac{\pi}{4}\right) = \frac{2}{\pi - 2}\int_0^{\frac{\pi}{4}} x\cos x\,dx = 0.460$ *(M1)A1*

Note:	Accept $\frac{2}{\pi-2}\left(=\frac{\pi\sqrt{2}}{8}+\frac{\sqrt{2}}{2}-1\right)$ or equivalent

[2 marks]

(c) (i) mode = 0.860 *A1*

(x-value of a maximum on the graph over the given domain)

(ii) $\frac{2}{\pi - 2}\int_0^m x\cos x\,dx = 0.5$ *(M1)*

$\int_0^m x\cos x\,dx = \frac{\pi - 2}{4}$

$m\sin m + \cos m - 1 = \frac{\pi - 2}{4}$ *(M1)*

median = 0.826 *A1*

Note: Do not accept answers containing additional solutions.

[4 marks]

(d) $\mathrm{P}\left(X < \frac{\pi}{8} \mid X < \frac{\pi}{4}\right) = \dfrac{\mathrm{P}\left(X < \frac{\pi}{8}\right)}{\mathrm{P}\left(X < \frac{\pi}{4}\right)}$ ***M1***

$= \dfrac{0.129912}{0.459826}$

$= 0.283$ ***A1***

[2 marks]

Total [13 marks]

Question 31

(a) $\mathrm{P}(X > x) = 0.99 \quad (= \mathrm{P}(X < x) = 0.01)$ ***(M1)***

$\Rightarrow x = 54.6(\mathrm{cm})$ ***A1***

[2 marks]

(b) $\mathrm{P}(60.15 \le X \le 60.25)$ ***(M1)(A1)***

$= 0.0166$ ***A1***

[3 marks]

Total [5 marks]

Question 32

use of $\mu = \frac{\sum_{i=1}^{k} f_i x_i}{n}$ to obtain $\frac{2+x+y+10+17}{5} = 8$ ***(M1)***

$x + y = 11$ ***A1***

EITHER

use of $\sigma^2 = \frac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n}$ to obtain $\frac{(-6)^2 + (x-8)^2 + (y-8)^2 + 2^2 + 9^2}{5} = 27.6$ ***(M1)***

$(x-8)^2 + (y-8)^2 = 17$ ***A1***

OR

use of $\sigma^2 = \frac{\sum_{i=1}^{k} f_i {x_i}^2}{n} - \mu^2$ to obtain $\frac{2^2 + x^2 + y^2 + 10^2 + 17^2}{5} - 8^2 = 27.6$ ***(M1)***

$x^2 + y^2 = 65$ ***A1***

THEN

attempting to solve the two equations ***(M1)***
$x = 4$ and $y = 7$ (only as $x < y$) ***A1*** ***N4***

Note: Award ***A0*** for $x = 7$ and $y = 4$.

Note: Award ***(M1)A1(M0)A0(M1)A1*** for $x + y = 11 \Rightarrow x = 4$ and $y = 7$.

Total [6 marks]

Question 33

(a) (i) $\mathrm{P}(X = 0) = 0.549 \left(= \mathrm{e}^{-0.6}\right)$ ***A1***

(ii) $\mathrm{P}(X \geq 3) = 1 - \mathrm{P}(X \leq 2)$ ***(M1)***
$\mathrm{P}(X \geq 3) = 0.0231$ ***A1***

[3 marks]

(b) **EITHER**

using $Y \sim \mathrm{Po}(3)$ ***(M1)***

OR

using $(0.549)^5$ ***(M1)***

THEN

$\mathrm{P}(Y = 0) = 0.0498 \left(= \mathrm{e}^{-3}\right)$ ***A1***

[2 marks]

(c) $P(X=0)$ (most likely number of complaints received is zero) *A1*

EITHER

calculating $P(X=0)=0.549$ and $P(X=1)=0.329$ ***M1A1***

OR

sketching an appropriate (discrete) graph of $P(X=x)$ against x ***M1A1***

OR

finding $P(X=0)=e^{-0.6}$ and stating that $P(X=0)>0.5$ ***M1A1***

OR

using $P(X=x)=P(X=x-1)\times\dfrac{\mu}{x}$ where $\mu<1$ ***M1A1***

[3 marks]

(d) $P(X=0)=0.8(\Rightarrow e^{-\lambda}=0.8)$ ***(A1)***

$\lambda=0.223\left(=\ln\dfrac{5}{4},=-\ln\dfrac{4}{5}\right)$ ***A1***

[2 marks]

Total [10 marks]

Question 34

(a) $\mathrm{P}(\text{Ava wins on her first turn}) = \frac{1}{3}$ *A1*

[1 mark]

(b) $\mathrm{P}(\text{Barry wins on his first turn}) = \left(\frac{2}{3}\right)^2$ *(M1)*

$= \frac{4}{9} (= 0.444)$ *A1*

[2 marks]

(c) $\mathrm{P}(\text{Ava wins in one of her first three turns})$

$= \frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\frac{1}{3}$ *M1A1A1*

Note:	Award ***M1*** for adding probabilities, award ***A1*** for a correct second term and award ***A1*** for a correct third term. Accept a correctly labelled tree diagram, awarding marks as above.

$= \frac{103}{243} (= 0.424)$ *A1*

[4 marks]

(d) $\mathrm{P}(\text{Ava eventually wins}) = \frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\frac{1}{3} + \ldots$ *(A1)*

using $S_\infty = \frac{a}{1-r}$ with $a = \frac{1}{3}$ and $r = \frac{2}{9}$ *(M1)(A1)*

Note:	Award ***(M1)*** for using $S_\infty = \frac{a}{1-r}$ and award ***(A1)*** for $a = \frac{1}{3}$ and $r = \frac{2}{9}$.

$= \frac{3}{7} (= 0.429)$ *A1*

[4 marks]

Total [11 marks]

Question 35

(a) $X \sim N(210, 22^2)$

$\mathrm{P}(X < 180) = 0.0863$ *(M1)A1*

[2 marks]

(b) $\mathrm{P}(X < T) = 0.9 \Rightarrow T = 238$ (mins) *(M1)A1*

[2 marks]

Total [4 marks]

Question 36

(a) $W \sim B(1000, 0.1)$ (accept $C_k^{1000}(0.1)^k(0.9)^{1000-k}$) ***A1A1***

Note: First ***A1*** is for recognizing the binomial, second ***A1*** for both parameters if stated explicitly in this part of the question.

[2 marks]

(b) $\mu(=1000 \times 0.1) = 100$ ***A1***

[1 mark]

(c) $\mathrm{P}(W > 89) = \mathrm{P}(W \geq 90)\ \left(= 1 - \mathrm{P}(W \leq 89)\right)$ ***(M1)***

$= 0.867$ ***A1***

Notes: Award ***M0A0*** for 0.889

[2 marks]

Total [5 marks]

Question 37

(a) $2\frac{e^{-m}m^4}{4!} = \frac{e^{-m}m^5}{5!}$ ***M1A1***

$\frac{2}{4!} = \frac{m}{5!}$ or other simplification ***M1***

Note: accept a labelled graph showing clearly the solution to the equation. Do not accept simple verification that $m = 10$ is a solution.

$\Rightarrow m = 10$ ***AG***

[3 marks]

(b) $\mathrm{P}(X = 6 \mid X \leq 11) = \frac{\mathrm{P}(X = 6)}{\mathrm{P}(X \leq 11)}$ ***(M1) (A1)***

$= \frac{0.063055...}{0.696776...}$ ***(A1)***

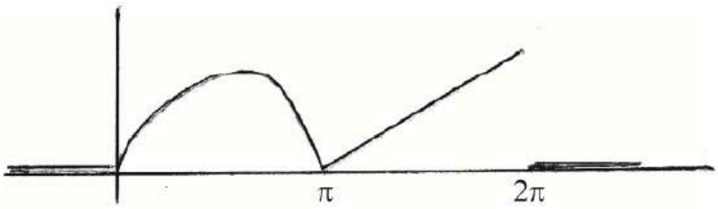
$= 0.0905$ ***A1***

[4 marks]

Total [7 marks]

Question 38

(a)



Award ***A1*** for sine curve from 0 to π, award ***A1*** for straight line from π to 2π ***A1A1***

[2 marks]

(b) $\int_0^{\pi} \frac{\sin x}{4} \mathrm{d}x = \frac{1}{2}$ ***(M1)A1***

[2 marks]

(c) **METHOD 1**

require $\frac{1}{2} + \int_{\pi}^{2\pi} a(x-\pi)\,\mathrm{d}x = 1$ ***(M1)***

$\Rightarrow \frac{1}{2} + a\left[\frac{(x-\pi)^2}{2}\right]_{\pi}^{2\pi} = 1$ (or $\frac{1}{2} + a\left[\frac{x^2}{2} - \pi x\right]_{\pi}^{2\pi} = 1$) ***A1***

$\Rightarrow a\frac{\pi^2}{2} = \frac{1}{2}$ ***A1***

$\Rightarrow a = \frac{1}{\pi^2}$ ***AG***

Note: Must obtain the exact value. Do not accept answers obtained with calculator.

METHOD 2

$0.5 +$ area of triangle $= 1$ ***R1***

area of triangle $= \frac{1}{2}\pi \times a\pi = 0.5$ ***M1A1***

Note: Award ***M1*** for correct use of area formula $= 0.5$, ***A1*** for $a\pi$.

$a = \frac{1}{\pi^2}$ ***AG***

[3 marks]

(d) median is π ***A1***

[1 mark]

(e) $\mu = \int_0^{\pi} x \cdot \frac{\sin x}{4}\,dx + \int_{\pi}^{2\pi} x \cdot \frac{x-\pi}{\pi^2}\,dx$ ***(M1)(A1)***

$= 3.40339\ldots = 3.40$ (or $\frac{\pi}{4} + \frac{5\pi}{6} = \frac{13}{12}\pi$) ***A1***

[3 marks]

(f) For $\mu = 3.40339\ldots$

EITHER

$\sigma^2 = \int_0^{\pi} x^2 \cdot \frac{\sin x}{4}\,dx + \int_{\pi}^{2\pi} x^2 \cdot \frac{x-\pi}{\pi^2}\,dx - \mu^2$ ***(M1)(A1)***

OR

$\sigma^2 = \int_0^{\pi} (x-\mu)^2 \cdot \frac{\sin x}{4}\,dx + \int_{\pi}^{2\pi} (x-\mu)^2 \cdot \frac{x-\pi}{\pi^2}\,dx$ ***(M1)(A1)***

THEN

$= 3.866277\ldots = 3.87$ ***A1***

[3 marks]

(g) $\int_{\frac{\pi}{2}}^{\pi} \frac{\sin x}{4}\,dx + \int_{\pi}^{\frac{3\pi}{2}} \frac{x-\pi}{\pi^2}\,dx = 0.375$ (or $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$) ***(M1)A1***

[2 marks]

(h) $P\left(\pi \le X \le 2\pi \,\middle|\, \frac{\pi}{2} \le X \le \frac{3\pi}{2}\right) = \frac{P\left(\pi \le X \le \frac{3\pi}{2}\right)}{P\left(\frac{\pi}{2} \le X \le \frac{3\pi}{2}\right)}$ ***(M1)(A1)***

$= \frac{\int_{\pi}^{\frac{3\pi}{2}} \frac{(x-\pi)}{\pi^2}\,dx}{0.375} = \frac{0.125}{0.375}$ (or $= \frac{\frac{1}{8}}{\frac{3}{8}}$ from diagram areas) ***(M1)***

$= \frac{1}{3}$ (0.333) ***A1***

[4 marks]

Total [20 marks]

Question 39

(a) (i) $X \sim Po(5)$

$\mathrm{P}(X \geq 8) = 0.133$ ***(M1)A1***

(ii) $7 \times 0.133\ldots$ ***M1***

≈ 0.934 days ***A1***

Note: Accept "1 day".

[4 marks]

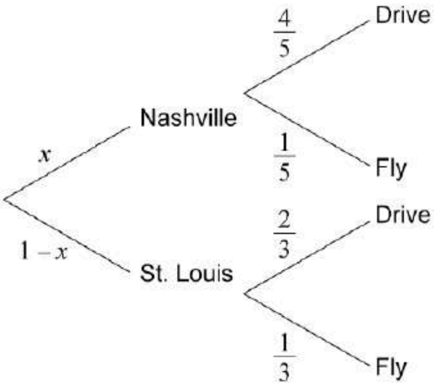
(b) $7 \times 5 = 35 \; (Y \sim Po(35))$ ***(A1)***

$\mathrm{P}(Y \leq 29) = 0.177$ ***(M1)A1***

[3 marks]

Total [7 marks]

Question 40



(a) attempt to set up the problem using a tree diagram and/or an equation, with the unknown x ***M1***

$\frac{4}{5}x+\frac{2}{3}(1-x)=\frac{13}{18}$ ***A1***

$\frac{4x}{5}-\frac{2x}{3}=\frac{13}{18}-\frac{2}{3}$

$\frac{2x}{15}=\frac{1}{18}$

$x=\frac{5}{12}$ ***A1***

[3 marks]

(b) attempt to set up the problem using conditional probability ***M1***

EITHER

$\dfrac{\frac{5}{12}\times\frac{1}{5}}{1-\frac{13}{18}}$ ***A1***

OR

$\dfrac{\frac{5}{12}\times\frac{1}{5}}{\frac{1}{12}+\frac{7}{36}}$ ***A1***

THEN

$=\frac{3}{10}$ ***A1***

Question 41

(a) (i) $P(110 < X < 130) = = 0.49969... = 0.500 = 50.0\%$ **(M1)A1**

Note: Accept 50

Note: Award ***M1A0*** for 0.50 (0.500)

(ii) $P(X > 130) = (1 - 0.707\ldots) = 0.293\ldots$ **M1**

expected number of turnips $= 29.3$ **A1**

Note: Accept 29.

(iii) no of turnips weighing more than 130 is $Y \sim B(100, 0.293)$ **M1**

$P(Y \geq 30) = 0.478$ **A1**

[6 marks]

(b) (i) $X \sim N\left(144, \sigma^2\right)$

$P(X \leq 130) = \frac{1}{15} = 0.0667$ **(M1)**

$P\left(Z \leq \frac{130 - 144}{\sigma}\right) = 0.0667$

$\frac{14}{\sigma} = 1.501$ **(A1)**

$\sigma = 9.33 g$ **A1**

(ii) $P(X > 150 \mid X > 130) = \frac{P(X > 150)}{P(X > 130)}$ **M1**

$= \frac{0.26008\ldots}{1 - 0.06667} = 0.279$ **A1**

expected number of turnips $= 55.7$ **A1**

[6 marks]

Total [12 marks]

Question 42

(a) $0.818 = 0.65 + 0.48 - \mathrm{P}(A \cap B)$ **(M1)**

$\mathrm{P}(A \cap B) = 0.312$ **A1**

[2 marks]

(b) $\mathrm{P}(A)\,\mathrm{P}(B) = 0.312 (= 0.48 \times 0.65)$ **A1**

since $\mathrm{P}(A)\,\mathrm{P}(B) = \mathrm{P}(A \cap B)$ then A and B are independent **R1**

:e: Only award the ***R1*** if numerical values are seen. Award ***A1R1*** for a correct conditional probability approach.

[2 marks]

Total [4 marks]

Question 43

(a) $\frac{0\cdot 4 + 1\cdot k + 2\cdot 3 + 3\cdot 2 + 4\cdot 3 + 8\cdot 1}{13 + k} = 1.95 \ \left(\frac{k+32}{k+13} = 1.95\right)$ ***(M1)***

attempting to solve for k ***(M1)***

$k = 7$ ***A1***

[3 marks]

(b) (i) $\frac{7 + 32 + 22}{7 + 13 + 1} = 2.90\left(= \frac{61}{21}\right)$ ***(M1)A1***

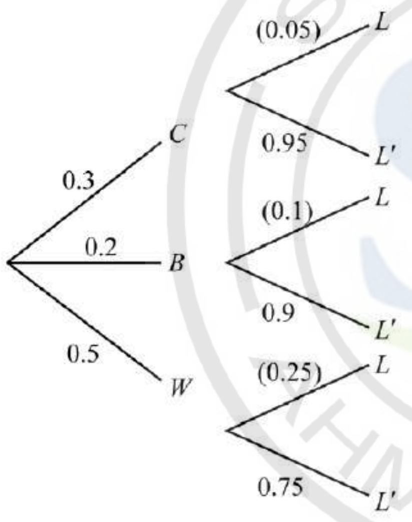
(ii) standard deviation $= 4.66$ ***A1***

: Award ***A0*** for 4.77.

[3 marks]

Total [6 marks]

Question 44



M1A1A1

e: Award ***M1*** for a two-level tree diagram, ***A1*** for correct first level probabilities, and ***A1*** for correct second level probabilities.

OR

$$\mathrm{P}(B \mid L') = \frac{\mathrm{P}(L' \mid B)\,\mathrm{P}(B)}{\mathrm{P}(L' \mid B)\,\mathrm{P}(B) + \mathrm{P}(L' \mid C)\,\mathrm{P}(C) + \mathrm{P}(L' \mid W)\,\mathrm{P}(W)}\left(= \frac{\mathrm{P}(B \cap L')}{\mathrm{P}(L')}\right)$$ ***(M1)(A1)(A1)***

THEN

$$\mathrm{P}(B \mid L') = \frac{0.9 \times 0.2}{0.9 \times 0.2 + 0.95 \times 0.3 + 0.75 \times 0.5}\left(= \frac{0.18}{0.84}\right)$$ ***M1A1***

$$= 0.214\left(= \frac{3}{14}\right)$$ ***A1***

[6 marks]

Question 45

(a) $A\int_1^5 \sin(\ln x)\,dx = 1$ *(M1)*

$A = 0.323$ (3 dp only) ***A1***

[2 marks]

(b) either a graphical approach or $f'(x) = \dfrac{\cos(\ln x)}{x} = 0$ *(M1)*

$x = 4.81 \left(= e^{\frac{\pi}{2}}\right)$ ***A1***

Note: Do not award ***A1FT*** for a candidate working in degrees.

[2 marks]

(c) $P(X \le 3 \mid X \ge 2) = \dfrac{P(2 \le X \le 3)}{P(X \ge 2)} \left(= \dfrac{\int_2^3 \sin(\ln(x))\,dx}{\int_2^5 \sin(\ln(x))\,dx} \right)$ *(M1)*

$= 0.288$ ***A1***

Note: Do not award ***A1FT*** for a candidate working in degrees.

[2 marks]

Question 46

(a) (i) let W be the weight of a worker and $W \sim N(\mu, \sigma^2)$

$P\left(Z < \dfrac{62-\mu}{\sigma}\right) = 0.3$ and $P\left(Z < \dfrac{98-\mu}{\sigma}\right) = 0.75$ *(M1)*

$\dfrac{62-\mu}{\sigma} = \Phi^{-1}(0.3)\ (= -0.524\ldots)$ and

$\dfrac{98-\mu}{\sigma} = \Phi^{-1}(0.75)\ (= 0.674\ldots)$

or linear equivalents ***A1A1***

(ii) attempting to solve simultaneously *(M1)*

$\mu = 77.7,\ \sigma = 30.0$ ***A1A1***

[6 marks]

(b) $P(W > 100) = 0.229$ ***A1***

[1 mark]

(c) let X represent the number of workers over $100\,\text{kg}$ in a lift of ten passengers

$X \sim B(10, 0.229\ldots)$ *(M1)*

$P(X \ge 4) = 0.178$ ***A1***

[2 marks]

(d) $P(X < 4 | X \geq 1) = \dfrac{P(1 \leq X \leq 3)}{P(X \geq 1)}$ **M1(A1)**

Note: Award the ***M1*** for a clear indication of conditional probability.

$= 0.808$ **A1**

[3 marks]

(e) $L \sim \text{Po}(50)$ ***(M1)***

$P(L > 60) = 1 - P(L \leq 60)$ ***(M1)***

$= 0.0722$ **A1**

[3 marks]

(f) 400 workers require at least 40 elevators ***(A1)***

$P(L \geq 40) = 1 - P(L \leq 39)$ ***(M1)***

$= 0.935$ **A1**

[3 marks]

Question 47

let the heights of the students be X

$P(X < 1.62) = 0.4$, $P(X > 1.79) = 0.25$ **M1**

Note: Award ***M1*** for either of the probabilities above.

$P\left(Z < \dfrac{1.62 - \mu}{\sigma}\right) = 0.4$, $P\left(Z < \dfrac{1.79 - \mu}{\sigma}\right) = 0.75$ **M1**

Note: Award ***M1*** for either of the expressions above.

$\dfrac{1.62 - \mu}{\sigma} = -0.2533\ldots$, $\dfrac{1.79 - \mu}{\sigma} = 0.6744\ldots$ **M1A1**

Note: ***A1*** for both values correct.

$\mu = 1.67(\text{m})$, $\sigma = 0.183(\text{m})$ **A1A1**

Note: Accept answers that round to $1.7(\text{m})$ and $0.18(\text{m})$.

Note: Accept answers in centimetres.

[6 marks]

Question 48

P(3 in the first hour) $= \dfrac{\lambda^3 e^{-\lambda}}{3!}$ **A1**

number to arrive in the four hours follows $Po(4\lambda)$ **M1**

P(5 arrive in total) $= \dfrac{(4\lambda)^5 e^{-4\lambda}}{5!}$ **A1**

attempt to find P(2 arrive in the next three hours) **M1**

$= \dfrac{(3\lambda)^2 e^{-3\lambda}}{2!}$ **A1**

use of conditional probability formula **M1**

P(3 in the first hour given 5 in total) $= \dfrac{\dfrac{\lambda^3 e^{-\lambda}}{3!} \times \dfrac{(3\lambda)^2 e^{-3\lambda}}{2!}}{\dfrac{(4\lambda)^5 e^{-4\lambda}}{5!}}$ **A1**

$\dfrac{\left(\dfrac{9}{2!3!}\right)}{\left(\dfrac{4^5}{5!}\right)} = \dfrac{45}{512} = 0.0879$ **A1**

[8 marks]

Question 49

(a) $E(X) = 1 \times \dfrac{1}{6} + 2 \times \dfrac{2}{6} + 3 \times \dfrac{3}{6} = \dfrac{14}{6}\left(= \dfrac{7}{3} = 2.33\right)$ ***(M1)A1***

[2 marks]

(b) (i) $3 \times P(113) + 3 \times P(122)$ ***(M1)***

$3 \times \dfrac{1}{6} \times \dfrac{1}{6} \times \dfrac{1}{2} + 3 \times \dfrac{1}{6} \times \dfrac{1}{3} \times \dfrac{1}{3} = \dfrac{7}{72} (= 0.0972)$ ***A1***

Note: Award ***M1*** for attempt to find at least four of the cases.

(ii) recognising 111 as a possibility (implied by $\dfrac{1}{216}$) ***(M1)***

recognising 112 and 113 as possibilities (implied by $\dfrac{2}{216}$ and $\dfrac{3}{216}$) ***(M1)***

seeing the three arrangements of 112 and 113 ***(M1)***

$P(111) + 3 \times P(112) + 3 \times P(113)$

$= \dfrac{1}{216} + \dfrac{6}{216} + \dfrac{9}{216} = \dfrac{16}{216}\left(= \dfrac{2}{27} = 0.0741\right)$ ***A1***

[6 marks]

(c) let the number of twos be X, $X \sim B\left(10, \frac{1}{3}\right)$ **(M1)**

$P(X < 4) = P(X \leq 3) = 0.559$ **(M1)A1**

[3 marks]

(d) let n be the number of balls drawn

$P(X \geq 1) = 1 - P(X = 0)$ **M1**

$= 1 - \left(\frac{2}{3}\right)^n > 0.95$ **M1**

$\left(\frac{2}{3}\right)^n < 0.05$

$n = 8$ **A1**

[3 marks]

(e) $8p_1 = 4.8 \Rightarrow p_1 = \frac{3}{5}$ **(M1)A1**

$8p_2(1 - p_2) = 1.5$ **(M1)**

$p_2^2 - p_2 - 0.1875 = 0$ **(M1)**

$p_2 = \frac{1}{4}\left(\text{or } \frac{3}{4}\right)$ **A1**

reject $\frac{3}{4}$ as it gives a total greater than one

$P(1 \text{ or } 2) = \frac{17}{20}$ or $P(3) = \frac{3}{20}$ **(A1)**

recognising LCM as 20 so min total number is 20 **(M1)**

the least possible number of 3's is 3 **A1**

[8 marks]

Total [22 marks]

Question 50

(a) $P(0 \leq X \leq 2) = 0.242$ **(M1)A1**

[2 marks]

(b) **METHOD 1**

$P(|X| > 1) = P(X < -1) + P(X > 1)$ **(M1)**

$= 0.02275\ldots + 0.84134\ldots$ **(A1)**

$= 0.864$ **A1**

METHOD 2

$P(|X| > 1) = 1 - P(-1 < X < 1)$ **(M1)**

$= 1 - 0.13590\ldots$ **(A1)**

$= 0.864$ **A1**

[3 marks]

(c) $c = 3.30$ **(M1)A1**

[2 marks]

Total [7 marks]

Question 51

(a) $X \sim \text{Po}(0.5)$ ***(A1)***

$\text{P}(X \geq 1) = 0.393 \; \left(= 1 - \text{e}^{-0.5}\right)$ ***(M1)A1***

[3 marks]

(b) $\text{P}(X = 0) = 0.607\ldots$ ***(A1)***

$\text{E}(P) = (0.607\ldots \times 5) - (0.393\ldots \times 3)$ ***(M1)***

the expected profit is $\$1.85$ per glass sheet ***A1***

[3 marks]

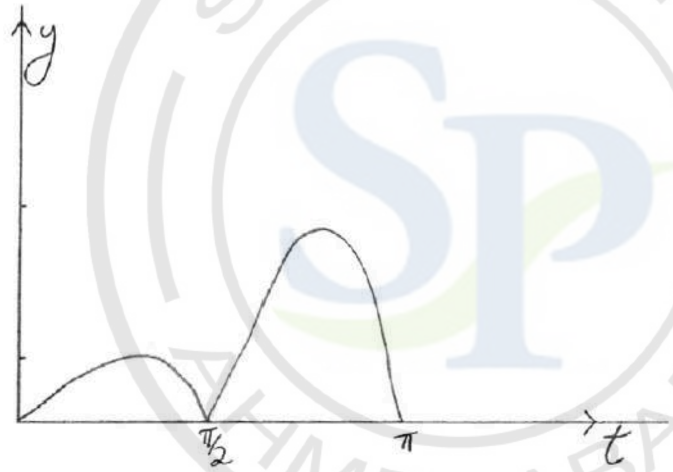
(c) $Y \sim \text{Po}(2)$ ***(M1)***

$\text{P}(Y = 0) = 0.135 \; \left(= \text{e}^{-2}\right)$ ***A1***

[2 marks]

Total [8 marks]

Question 52



(a) two enclosed regions ($0 \leq t \leq \frac{\pi}{2}$ and $\frac{\pi}{2} \leq t \leq \pi$) bounded by the curve and the t-axis ***A1***

correct non-symmetrical shape for $0 \leq t \leq \frac{\pi}{2}$ and

$\frac{\pi}{2} <$ mode of $T < \pi$ clearly apparent ***A1***

[2 marks]

(b) mode $= 2.46$ ***A1***

[1 mark]

(c) $\text{E}(T) = \frac{1}{\pi}\int_0^{\pi} t^2 \left|\sin 2t\right| \text{d}t$ ***(M1)***

$= 2.04$ ***A1***

[2 marks]

(d) **EITHER**

$$\text{Var}(T) = \int_0^{\pi} (t - 2.03788\ldots)^2 \left(\frac{t\left|\sin 2t\right|}{\pi} \right) \mathrm{d}t$$ ***(M1)(A1)***

OR

$$\text{Var}(T) = \int_0^{\pi} t^2 \left(\frac{t\left|\sin 2t\right|}{\pi} \right) \mathrm{d}t - (2.03788\ldots)^2$$ ***(M1)(A1)***

THEN

$\text{Var}(T) = 0.516$ ***A1***

[3 marks]

(e) $$\frac{1}{\pi}\int_{2.03788\ldots}^{2.456590\ldots} t\left|\sin 2t\right| \mathrm{d}t = 0.285$$ ***(M1)A1***

[2 marks]

(f) (i) attempting integration by parts ***(M1)***

($u = t$, $\mathrm{d}u = \mathrm{d}t$, $\mathrm{d}v = \sin 2t\ \mathrm{d}t$ and $v = -\frac{1}{2}\cos 2t$)

$$\frac{1}{\pi}\left[t\left(-\frac{1}{2}\cos 2t \right) \right]_0^T - \frac{1}{\pi}\int_0^T \left(-\frac{1}{2}\cos 2t \right) \mathrm{d}t$$ ***A1***

Note: Award ***A1*** if the limits are not included.

$$= \frac{\sin 2T}{4\pi} - \frac{T\cos 2T}{2\pi}$$ ***A1***

(ii) $$\frac{\sin \pi}{4\pi} - \frac{\frac{\pi}{2}\cos \pi}{2\pi} = \frac{1}{4}$$ ***A1***

as $\mathrm{P}\left(0 \le T \le \frac{\pi}{2} \right) = \frac{1}{4}$ (or equivalent), then the lower quartile of T is $\frac{\pi}{2}$ ***R1AG***

[5 marks]

Total [15 marks]

Question 53

(a) $\mathrm{E}\left(X^2\right) = \sum x^2 \cdot \mathrm{P}(X = x) = 10.37$ $(= 10.4$ 3 sf) ***(M1)A1***

[2 marks]

(b) **METHOD 1**

$\mathrm{sd}(X) = 1.44069\ldots$ ***(M1)(A1)***

$\mathrm{Var}(X) = 2.08$ $(= 2.0756)$ ***A1***

METHOD 2

$\mathrm{E}(X) = 2.88$ $(= 0.06 + 0.27 + 0.5 + 0.98 + 0.63 + 0.44)$ ***(A1)***

use of $\mathrm{Var}(X) = \mathrm{E}\left(X^2\right) - \left(\mathrm{E}(X)\right)^2$ ***(M1)***

Note: Award ***(M1)*** only if $\left(\mathrm{E}(X)\right)^2$ is used correctly.

$(\mathrm{Var}(X) = 10.37 - 8.29)$

$\mathrm{Var}(X) = 2.08$ $(= 2.0756)$ ***A1***

Note: Accept 2.11.

METHOD 3

$\mathrm{E}(X) = 2.88$ $(= 0.06 + 0.27 + 0.5 + 0.98 + 0.63 + 0.44)$ ***(A1)***

use of $\mathrm{Var}(X) = \mathrm{E}\left(\left(X - \mathrm{E}(X)\right)^2\right)$ ***(M1)***

$(0.679728 + \ldots + 0.549152)$

$\mathrm{Var}(X) = 2.08$ $(= 2.0756)$ ***A1***

[3 marks]

Total [5 marks]

Question 54

(a) **METHOD 1**

$\mathrm{P}(X = x+1) = \dfrac{\mu^{x+1}}{(x+1)!}\mathrm{e}^{-\mu}$ ***A1***

$= \dfrac{\mu}{x+1} \times \dfrac{\mu^{x}}{x!}\mathrm{e}^{-\mu}$ ***M1A1***

$= \dfrac{\mu}{x+1} \times \mathrm{P}(X = x)$ ***AG***

METHOD 2

$\dfrac{\mu}{x+1} \times \mathrm{P}(X = x) = \dfrac{\mu}{x+1} \times \dfrac{\mu^{x}}{x!}\mathrm{e}^{-\mu}$ ***A1***

$= \dfrac{\mu^{x+1}}{(x+1)!}\mathrm{e}^{-\mu}$ ***M1A1***

$= \mathrm{P}(X = x+1)$ ***AG***

METHOD 3

$\dfrac{\mathrm{P}(X = x+1)}{\mathrm{P}(X = x)} = \dfrac{\dfrac{\mu^{x+1}}{(x+1)!}\mathrm{e}^{-\mu}}{\dfrac{\mu^{x}}{x!}\mathrm{e}^{-\mu}}$ ***(M1)***

$= \dfrac{\mu^{x+1}}{\mu^{x}} \times \dfrac{x!}{(x+1)!}$ ***A1***

$= \dfrac{\mu}{x+1}$ ***A1***

and so $\mathrm{P}(X = x+1) = \dfrac{\mu}{x+1} \times \mathrm{P}(X = x)$ ***AG***

[3 marks]

(b) $\mathrm{P}(X = 3) = \dfrac{\mu}{3} \cdot \mathrm{P}(X = 2) \left(0.112777 = \dfrac{\mu}{3} \cdot 0.241667\right)$ ***A1***

attempting to solve for μ ***(M1)***

$\mu = 1.40$ ***A1***

[3 marks]

Total [6 marks]

Question 55

(a) $P(X < 42.52) = 0.6940$ ***(M1)***

either $P\left(Z < \frac{30.31-\mu}{\sigma}\right) = 0.1180$ or $P\left(Z < \frac{42.52-\mu}{\sigma}\right) = 0.6940$ ***(M1)***

$\frac{30.31-\mu}{\sigma} = \underbrace{\Phi^{-1}(0.1180)}_{-1.1850\ldots}$ ***(A1)***

$\frac{42.52-\mu}{\sigma} = \underbrace{\Phi^{-1}(0.6940)}_{0.5072\ldots}$ ***(A1)***

attempting to solve simultaneously ***(M1)***

$\mu = 38.9$ and $\sigma = 7.22$ ***A1***

[6 marks]

(b) $P(\mu - 1.2\sigma < X < \mu + 1.2\sigma)$ (or equivalent *eg.* $2P(\mu < X < \mu + 1.2\sigma)$) ***(M1)***

$= 0.770$ ***A1***

Note: Award ***(M1)A1*** for $P(-1.2 < Z < 1.2) = 0.770$.

[2 marks]

Total [8 marks]

Question 56

(a) $P(X = 3) = (0.1)^3$ ***A1***

$= 0.001$ ***AG***

$P(X = 4) = P(VV\bar{V}V) + P(V\bar{V}VV) + P(\bar{V}VVV)$ ***(M1)***

$= 3 \times (0.1)^3 \times 0.9$ (or equivalent) ***A1***

$= 0.0027$ ***AG***

[3 marks]

(b) **METHOD 1**

attempting to form equations in a and b ***M1***

$\frac{9+3a+b}{2000} = \frac{1}{1000}$ $(3a + b = -7)$ ***A1***

$\frac{16+4a+b}{2000} \times \frac{9}{10} = \frac{27}{10000}$ $(4a + b = -10)$ ***A1***

attempting to solve simultaneously ***(M1)***

$a = -3, b = 2$ ***A1***

METHOD 2

$P(X = n) = \binom{n-1}{2} \times 0.1^3 \times 0.9^{n-3}$ ***M1***

$= \frac{(n-1)(n-2)}{2000} \times 0.9^{n-3}$ ***(M1)A1***

$= \frac{n^2 - 3n + 2}{2000} \times 0.9^{n-3}$ ***A1***

$a = -3, b = 2$ ***A1***

Note: Condone the absence of 0.9^{n-3} in the determination of the values of a and b.

[5 marks]

(c) **METHOD 1**

EITHER

$$\mathrm{P}(X = n) = \frac{n^2 - 3n + 2}{2000} \times 0.9^{n-3}$$ ***(M1)***

OR

$$\mathrm{P}(X = n) = \binom{n-1}{2} \times 0.1^3 \times 0.9^{n-3}$$ ***(M1)***

THEN

$$= \frac{(n-1)(n-2)}{2000} \times 0.9^{n-3}$$ ***A1***

$$\mathrm{P}(X = n-1) = \frac{(n-2)(n-3)}{2000} \times 0.9^{n-4}$$ ***A1***

$$\frac{\mathrm{P}(X = n)}{\mathrm{P}(X = n-1)} = \frac{(n-1)(n-2)}{(n-2)(n-3)} \times 0.9$$ ***A1***

$$= \frac{0.9(n-1)}{n-3}$$ ***AG***

METHOD 2

$$\frac{\mathrm{P}(X = n)}{\mathrm{P}(X = n-1)} = \frac{\frac{n^2 - 3n + 2}{2000} \times 0.9^{n-3}}{\frac{(n-1)^2 - 3(n-1) + 2}{2000} \times 0.9^{n-4}}$$ ***(M1)***

$$= \frac{0.9\left(n^2 - 3n + 2\right)}{\left(n^2 - 5n + 6\right)}$$ ***A1A1***

Note: Award ***A1*** for a correct numerator and ***A1*** for a correct denominator.

$$= \frac{0.9(n-1)(n-2)}{(n-2)(n-3)}$$ ***A1***

$$= \frac{0.9(n-1)}{n-3}$$ ***AG***

[4 marks]

(d) (i) attempting to solve $\frac{0.9(n-1)}{n-3}=1$ for n **M1**

$n=21$ **A1**

$\frac{0.9(n-1)}{n-3}<1 \Rightarrow n>21$ **R1**

$\frac{0.9(n-1)}{n-3}>1 \Rightarrow n<21$ **R1**

X has two modes **AG**

Note: Award ***R1R1*** for a clearly labelled graphical representation of the two inequalities (using $\frac{P(X=n)}{P(X=n-1)}$).

(ii) the modes are 20 and 21 **A1**

[5 marks]

(e) **METHOD 1**

$Y \sim B(x, 0.1)$ ***(A1)***

attempting to solve $P(Y \geq 3) > 0.5$ (or equivalent *eg* $1-P(Y \leq 2) > 0.5$) for x ***(M1)***

Note: Award ***(M1)*** for attempting to solve an equality (obtaining $x = 26.4$).

$x = 27$ **A1**

METHOD 2

$\sum_{n=0}^{x} P(X=n) > 0.5$ ***(A1)***

attempting to solve for x ***(M1)***

$x = 27$ **A1**

[3 marks]

Total [20 marks]

Question 57

(a) use of $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ **M1**

$0.5 = k + 3k - k^2$ **A1**

$k^2 - 4k + 0.5 = 0$

$k = 0.129$ **A1**

Note: Do not award the final ***A1*** if two solutions are given.

[3 marks]

(b) use of $P(A' \cap B) = P(B) - P(A \cap B)$ or alternative ***(M1)***

$P(A' \cap B) = 3k - k^2$ ***(A1)***

$= 0.371$ **A1**

[3 marks]

Total [6 marks]

Question 58

(a) $\lambda = 4 \times 0.5$ *(M1)*

$\lambda = 2$ *(A1)*

$P(X \leq 2) = 0.677$ ***A1***

[3 marks]

(b) $Y \sim B(10, 0.677)$ *(M1)(A1)*

$P(Y = 7) = 0.263$ ***A1***

Note: Award ***M1*** for clear recognition of binomial distribution.

[3 marks]

Total [6 marks]

Question 59

(a) $T \sim N(196, 24^2)$

$P(T < 180) = 0.252$ *(M1)A1*

[2 marks]

(b) $P(T < T_1) = 0.05$ *(M1)*

$T_1 = 157$ ***A1***

[2 marks]

(c) $F \sim N(210, \sigma^2)$

$P(F < 235) = 0.79$ *(M1)*

$\frac{235 - 210}{\sigma} = 0.806421$ or equivalent *(M1)(A1)*

$\sigma = 31.0$ ***A1***

[4 marks]

Total [8 marks]

Question 60

(a) $P(5 \text{ or more}) = \frac{29}{75} (= 0.387)$ *(M1)A1*

[2 marks]

(b) mean score $= \frac{2 \times 3 + 3 \times 15 + 4 \times 28 + 5 \times 17 + 6 \times 9 + 7 \times 3}{75}$ *(M1)*

$= \frac{323}{75} (= 4.31)$ ***A1***

[2 marks]

Total [4 marks]

Question 61

(a) $\mathrm{P}(X < 250) = 0.0228$ ***(M1)A1***

[2 marks]

(b) $\frac{250 - \mu}{1.5} = -2.878\ldots$ ***(M1)(A1)***

$\Rightarrow \mu = 254.32$ ***A1***

Notes: Only award ***A1*** here if the correct 2dp answer is seen.
Award ***M0*** for use of 1.5^2.

[3 marks]

(c) $\frac{250 - 253}{\sigma} = -2.878\ldots$ ***(A1)***

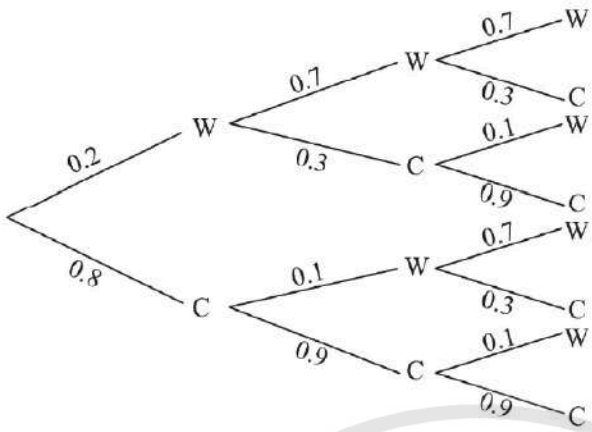
$\Rightarrow \sigma = 1.04$ ***A1***

[2 marks]

Total [7 marks]

Question 62

(a)



M1A2

Note: Award ***M1*** for 3 stage tree-diagram, ***A2*** for 0.8,0.9,0.3 probabilities correctly placed.

[3 marks]

(b) $0.2 \times 0.7 \times 0.3 + 0.2 \times 0.3 \times 0.9 + 0.8 \times 0.1 \times 0.3 + 0.8 \times 0.9 \times 0.9 = 0.768$

(M1)A1

[2 marks]

(c) $\text{P(1st July is calm} \mid \text{3rd July is windy)} = \dfrac{\text{P(1st July is calm and 3rd July is windy)}}{\text{P(3rd July is windy)}}$

(M1)

$= \dfrac{0.8 \times 0.1 \times 0.7 + 0.8 \times 0.9 \times 0.1}{1 - 0.768}$

OR $\dfrac{0.8 \times 0.1 \times 0.7 + 0.8 \times 0.9 \times 0.1}{0.2 \times 0.7 \times 0.7 + 0.2 \times 0.3 \times 0.1 + 0.8 \times 0.1 \times 0.7 + 0.8 \times 0.9 \times 0.1}$

OR $\dfrac{0.128}{0.232}$ ***(A1)(A1)***

Note: Award ***A1*** for correct numerator, ***A1*** for correct denominator.

$= 0.552$ ***A1***

[4 marks]

Total [9 marks]

Question 64

(a) $\int_0^4 \left(\frac{x^2}{a}+b\right)dx = 1 \Rightarrow \left[\frac{x^3}{3a}+bx\right]_0^4 = 1 \Rightarrow \frac{64}{3a}+4b=1$ **M1A1**

$\int_2^4 \left(\frac{x^2}{a}+b\right)dx = 0.75 \Rightarrow \frac{56}{3a}+2b=0.75$ **M1A1**

Note: $\int_0^2 \left(\frac{x^2}{a}+b\right)dx = 0.25 \Rightarrow \frac{8}{3a}+2b=0.25$ could be seen/used in place of either of the above equations.

evidence of an attempt to solve simultaneously
(or check given a, b values are consistent) **M1**

$a=32,\ b=\frac{1}{12}$ **AG**

[5 marks]

(b) $E(X)=\int_0^4 x\left(\frac{x^2}{32}+\frac{1}{12}\right)dx$ ***(M1)***

$E(X)=\frac{8}{3}(=2.67)$ **A1**

[2 marks]

(c) $E\left(X^2\right)=\int_0^4 x^2\left(\frac{x^2}{32}+\frac{1}{12}\right)dx$ ***(M1)***

$\text{Var(X)}=E\left(X^2\right)-\left[E(X)\right]^2=\frac{16}{15}(=1.07)$ **A1**

[2 marks]

(d) $\int_0^m \left(\frac{x^2}{32}+\frac{1}{12}\right)dx = 0.5$ ***(M1)***

$\frac{m^3}{96}+\frac{m}{12}=0.5\left(\Rightarrow m^3+8m-48=0\right)$ ***(A1)***

$m=2.91$ **A1**

[3 marks]

(e) $Y \sim B(8,\ 0.75)$ ***(M1)***

$E(Y)=8\times 0.75=6$ **A1**

[2 marks]

(f) $P(Y\geq 3)=0.996$ **A1**

[1 mark]

Total [15 marks]

Question 64

(a) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow 0.75 = \frac{0.6}{P(B)}$ **(M1)**

$\Rightarrow P(B)\left(= \frac{0.6}{0.75}\right) = 0.8$ **A1**

[2 marks]

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.95 = P(A) + 0.8 - 0.6$ **(M1)**

$\Rightarrow P(A) = 0.75$ **A1**

[2 marks]

(c) **METHOD 1**

$P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{0.2}{0.8} = 0.25$ **A1**

$P(A'|B) = P(A')$ **R1**

hence A' and B are independent **AG**

Question 65

let X be the random variable "amount of caffeine content in coffee"

$P(X > 120) = 0.2$, $P(X > 110) = 0.6$ **(M1)**

$(\Rightarrow P(X < 120) = 0.8$, $P(X < 110) = 0.4)$

ote: Award ***M1*** for at least one correct probability statement.

$\frac{120 - \mu}{\sigma} = 0.84162\ldots$, $\frac{110 - \mu}{\sigma} = -0.253347\ldots$ **(M1)(A1)(A1)**

ote: Award ***M1*** for attempt to find at least one appropriate z-value.

$120 - \mu = 0.84162\sigma$, $110 - \mu = -0.253347\sigma$

attempt to solve simultaneous equations **(M1)**

$\mu = 112$, $\sigma = 9.13$ **A1**

[6 marks]

Question 66

(a) let X be the number of bananas eaten in one day

$X \sim \text{Po}(0.2)$

$\text{P}(X \geq 1) = 1 - \text{P}(X = 0)$ **(M1)**

$= 0.181\left(= 1 - e^{-0.2}\right)$ **A1**

[2 marks]

(b) **EITHER**

let Y be the number of bananas eaten in one week

$Y \sim \text{Po}(1.4)$ ***(A1)***

$\text{P}(Y = 0) = 0.246596\ldots\left(= e^{-1.4}\right)$ ***(A1)***

OR

let Z be the number of days in one week at least one banana is eaten

$Z \sim \text{B}(7, 0.181\ldots)$ ***(A1)***

$\text{P}(Z = 0) = 0.246596\ldots$ ***(A1)***

THEN

$52 \times 0.246596\ldots$ ***(M1)***

$= 12.8\left(= 52e^{-1.4}\right)$ ***A1***

[4 marks]

Total [6 marks]