Subject – Math(Standard Level)
Topic - Algebra
Year - Nov 2011 – Nov 2019

Question -1

[Maximum mark: 7]

Given that $\left(1+\frac{2}{3}x\right)^n(3+nx)^2 = 9+84x+\ldots$, find the value of n.

Question -2

[Maximum mark: 7]

Find the value of

(a) $\log_2 40 - \log_2 5$; *[3 marks]*

(b) $8^{\log_2 5}$. *[4 marks]*

Question -3

[Maximum mark: 7]

Let $\log_3 p = 6$ and $\log_3 q = 7$.

(a) Find $\log_3 p^2$. *[2 marks]*

(b) Find $\log_3\left(\frac{p}{q}\right)$. *[2 marks]*

(c) Find $\log_3(9p)$. *[3 marks]*

Question -4

[Maximum mark: 16]

The first three terms of a infinite geometric sequence are $m-1$, 6, $m+4$, where $m \in \mathbb{Z}$.

(a) (i) Write down an expression for the common ratio, r.

(ii) Hence, show that m satisfies the equation $m^2 + 3m - 40 = 0$. *[4]*

(b) (i) Find the two possible values of m.

(ii) Find the possible values of r. *[6]*

(c) The sequence has a finite sum.

(i) State which value of r leads to this sum **and** justify your answer.

(ii) Calculate the sum of the sequence. *[6]*

Question -5

[Maximum mark: 7]

In an arithmetic sequence, the third term is 10 and the fifth term is 16.

(a) Find the common difference. *[2]*

(b) Find the first term. *[2]*

(c) Find the sum of the first 20 terms of the sequence. *[3]*

Question -6

[Maximum mark: 6]

(a) Write down the value of

(i) $\log_3 27$;

(ii) $\log_8 \frac{1}{8}$;

(iii) $\log_{16} 4$. *[3]*

(b) Hence, solve $\log_3 27 + \log_8 \frac{1}{8} - \log_{16} 4 = \log_4 x$. *[3]*

Question -7

[Maximum mark: 8]

The sums of the terms of a sequence follow the pattern

$S_1 = 1 + k$, $S_2 = 5 + 3k$, $S_3 = 12 + 7k$, $S_4 = 22 + 15k$, …, where $k \in \mathbb{Z}$.

(a) Given that $u_1 = 1 + k$, find u_2, u_3 and u_4. *[4]*

(b) Find a general expression for u_n. *[4]*

Question – 8

[Maximum mark: 6]

In an arithmetic sequence, the first term is 2 and the second term is 5.

(a) Find the common difference. *[2]*

(b) Find the eighth term. *[2]*

(c) Find the sum of the first eight terms of the sequence. *[2]*

Question -9

[Maximum mark: 6]

(a) Write the expression $3\ln 2 - \ln 4$ in the form $\ln k$, where $k \in \mathbb{Z}$. *[3]*

(b) Hence or otherwise, solve $3\ln 2 - \ln 4 = -\ln x$. *[3]*

Question -10

[Maximum mark: 6]

(a) Given that $2^m = 8$ and $2^n = 16$, write down the value of m and of n. [2]

(b) Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$. [4]

Question 11

[Maximum mark: 7]

In the expansion of $(3x+1)^n$, the coefficient of the term in x^2 is $135n$, where $n \in \mathbb{Z}^+$. Find n.

Question 12

[Maximum mark: 6]

An arithmetic sequence has the first term $\ln a$ and a common difference $\ln 3$.
The 13th term in the sequence is $8\ln 9$. Find the value of a.

Question 13

[Maximum mark: 6]

Let $x = \ln 3$ and $y = \ln 5$. Write the following expressions in terms of x and y.

(a) $\ln\left(\frac{5}{3}\right)$. [2]

(b) $\ln 45$. [4]

Question 14

[Maximum mark: 6]

Three consecutive terms of a geometric sequence are $x-3$, 6 and $x+2$.
Find the possible values of x.

Question 15

[Maximum mark: 13]

The first two terms of an infinite geometric sequence, in order, are

$$2\log_2 x,\ \log_2 x, \text{ where } x > 0.$$

(a) Find r. [2]

(b) Show that the sum of the infinite sequence is $4\log_2 x$. [2]

The first three terms of an arithmetic sequence, in order, are

$$\log_2 x,\ \log_2\left(\frac{x}{2}\right),\ \log_2\left(\frac{x}{4}\right), \text{ where } x > 0.$$

(c) Find d, giving your answer as an integer. [4]

Let S_{12} be the sum of the first 12 terms of the arithmetic sequence.

(d) Show that $S_{12} = 12\log_2 x - 66$. [2]

(e) Given that S_{12} is equal to half the sum of the infinite geometric sequence, find x, giving your answer in the form 2^p, where $p \in \mathbb{Q}$. [3]

Question 16

[Maximum mark: 8]

The first three terms of a geometric sequence are $\ln x^{16}$, $\ln x^8$, $\ln x^4$, for $x > 0$.

(a) Find the common ratio. [3]

(b) Solve $\sum_{k=1}^{\infty} 2^{5-k} \ln x = 64$. [5]

Question 17

[Maximum mark: 6]

In an arithmetic sequence, the first term is 3 and the second term is 7.

(a) Find the common difference. [2]

(b) Find the tenth term. [2]

(c) Find the sum of the first ten terms of the sequence. [2]

Question 18

[Maximum mark: 6]

The following figures consist of rows and columns of squares. The figures form a continuing pattern.

Figure 1 has two rows and one column. Figure 2 has three rows and two columns.

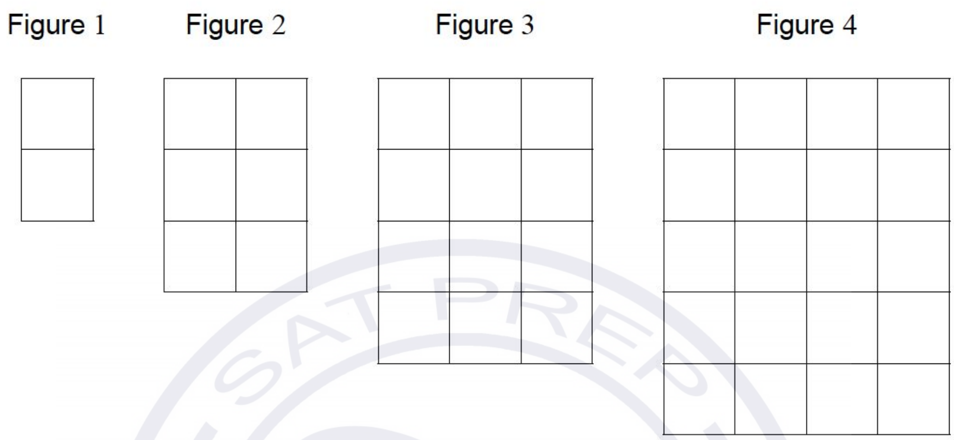


Figure 5 has p rows and q columns.

(a) Write down the value of

(i) p;

(ii) q. [2]

Each small square has an area of $1\ \text{cm}^2$. Let A_n be the total area of Figure n. The following table gives the first five values of A_n.

n	1	2	3	4	5
A_n (cm^2)	2	6	12	20	k

(b) Find the value of k. [2]

(c) Find an expression for A_n in terms of n. [2]

Question 19

[Maximum mark: 7]

Solve $\log_2(2\sin x) + \log_2(\cos x) = -1$, for $2\pi < x < \frac{5\pi}{2}$.

Question 20

[Maximum mark: 6]

In an arithmetic sequence, the first term is 8 and the second term is 5.

(a) Find the common difference. [2]

(b) Find the tenth term. [2]

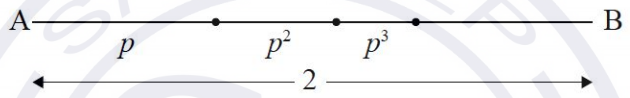
(c) Find the sum of the first ten terms. [2]

Question 21

[Maximum mark: 14]

(a) The following diagram shows [AB], with length 2 cm. The line is divided into an infinite number of line segments. The diagram shows the first three segments.

diagram not to scale

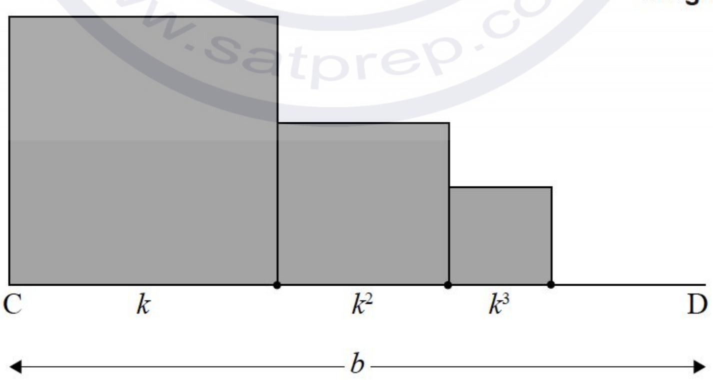


The length of the line segments are $p\text{ cm}, p^2\text{ cm}, p^3\text{ cm}, \ldots$, where $0 < p < 1$.

Show that $p = \frac{2}{3}$. [5]

(b) The following diagram shows [CD], with length $b\text{ cm}$, where $b > 1$. Squares with side lengths $k\text{ cm}, k^2\text{ cm}, k^3\text{ cm}, \ldots$, where $0 < k < 1$, are drawn along [CD]. This process is carried on indefinitely. The diagram shows the first three squares.

diagram not to scale



The **total** sum of the areas of all the squares is $\frac{9}{16}$. Find the value of b. [9]

Question 22

[Maximum mark: 7]

Consider $f(x) = \log_k(6x - 3x^2)$, for $0 < x < 2$, where $k > 0$.
The equation $f(x) = 2$ has exactly one solution. Find the value of k.

Question 23

[Maximum mark: 15]

The first two terms of an infinite geometric sequence are $u_1 = 18$ and $u_2 = 12\sin^2\theta$, where $0 < \theta < 2\pi$, and $\theta \neq \pi$.

(a) (i) Find an expression for r in terms of θ.

(ii) Find the possible values of r. [5]

(b) Show that the sum of the infinite sequence is $\frac{54}{2+\cos(2\theta)}$. [4]

(c) Find the values of θ which give the greatest value of the sum. [6]

Question 24

[Maximum mark: 7]

Let $f(x) = px^2 + qx - 4p$, where $p \neq 0$. Find the number of roots for the equation $f(x) = 0$. Justify your answer.

Question 25

[Maximum mark: 8]

An arithmetic sequence has $u_1 = \log_c(p)$ and $u_2 = \log_c(pq)$, where $c > 1$ and $p, q > 0$.

(a) Show that $d = \log_c(q)$. [2]

(b) Let $p = c^2$ and $q = c^3$. Find the value of $\sum_{n=1}^{20} u_n$. [6]

Question 26

[Maximum mark: 15]

The first two terms of an infinite geometric sequence are $u_1 = 18$ and $u_2 = 12\sin^2\theta$, where $0 < \theta < 2\pi$, and $\theta \neq \pi$.

(a) (i) Find an expression for r in terms of θ.

(ii) Find the possible values of r. [5]

(b) Show that the sum of the infinite sequence is $\dfrac{54}{2+\cos(2\theta)}$. [4]

(c) Find the values of θ which give the greatest value of the sum. [6]

Question 27

[Maximum mark: 7]

Let $f(x) = ax^2 - 4x - c$. A horizontal line, L, intersects the graph of f at $x = -1$ and $x = 3$.

(a) (i) The equation of the axis of symmetry is $x = p$. Find p.

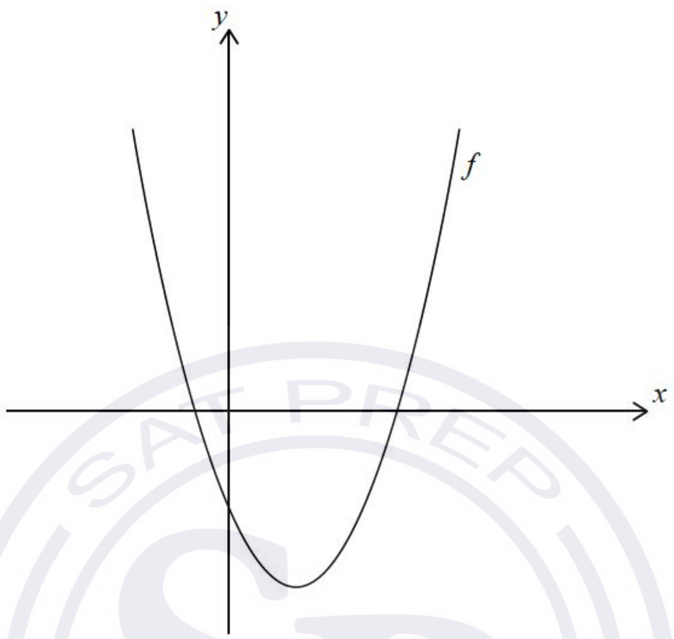
(ii) Hence, show that $a = 2$. [4]

(b) The equation of L is $y = 5$. Find the value of c. [3]

Question 28

[Maximum mark: 16]

Let $f(x) = x^2 - 4x - 5$. The following diagram shows part of the graph of f.



(a) Find the x-intercepts of the graph of f. [5]

(b) Find the equation of the axis of symmetry of the graph of f. [2]

(c) The function can be written in the form $f(x) = (x - h)^2 + k$.

(i) Write down the value of h.

(ii) Find the value of k. [4]

The graph of a second function, g, is obtained by a reflection of the graph of f in the y-axis, followed by a translation of $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$.

(d) Find the coordinates of the vertex of the graph of g. [5]

Question 29

[Maximum mark: 6]

Let $b = \log_2 a$, where $a > 0$. Write down each of the following expressions in terms of b.

(a) $\log_2 a^3$ [2]

(b) $\log_2 8a$ [2]

(c) $\log_8 a$ [2]

Question 30

[Maximum mark: 6]

In an arithmetic sequence, $u_1 = -5$ and $d = 3$.

(a) Find u_8. [2]

(b) Find the value of n for which $u_n = 67$. [4]

Question 31

[Maximum mark: 7]

Solve $\log_4(2-x) = \log_{16}(13-4x)$.

Question 32

[Maximum mark: 17]

Consider $f(x) = \sqrt{x}\sin\left(\frac{\pi}{4}x\right)$ and $g(x) = \sqrt{x}$ for $x \geq 0$. The first time the graphs of f and g intersect is at $x = 0$.

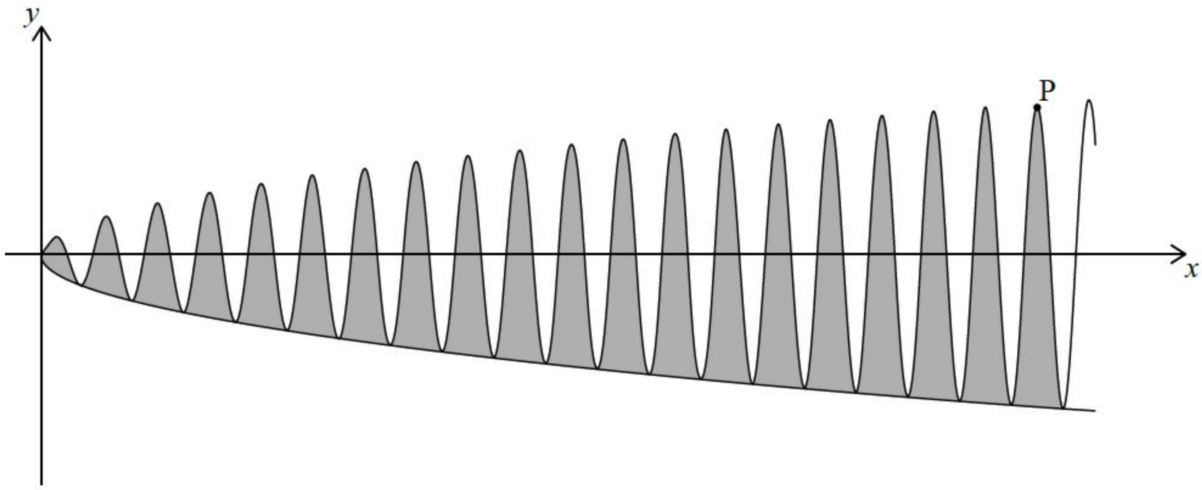
(a) Find the **two** smallest non-zero values of x for which $f(x) = g(x)$. [5]

The set of all non-zero values that satisfy $f(x) = g(x)$ can be described as an arithmetic sequence, $u_n = a + bn$ where $n \geq 1$.

(b) Find the value of a and of b. [4]

(c) At point P, the graphs of f and g intersect for the 21st time. Find the coordinates of P. [4]

The following diagram shows part of the graph of g **reflected** in the x-axis. It also shows part of the graph of f and the point P.



(d) Find an expression for the area of the shaded region. Do not calculate the value of the expression. [4]

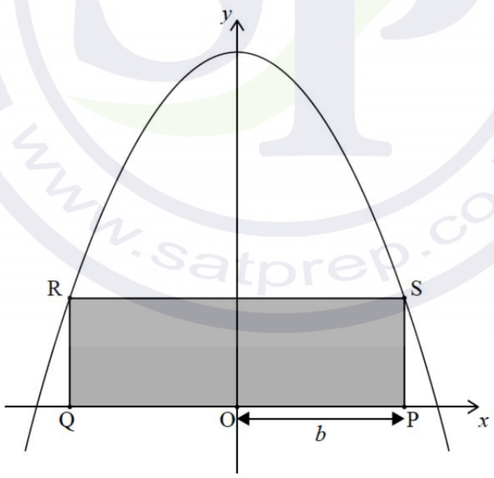
Question33

[Maximum mark: 16]

Let $f(x) = 9 - x^2$, $x \in \mathbb{R}$.

(a) Find the x-intercepts of the graph of f. [2]

The following diagram shows part of the graph of f.



Rectangle PQRS is drawn with P and Q on the x-axis and R and S on the graph of f.

Let $OP = b$.

(b) Show that the area of PQRS is $18b - 2b^3$. [2]

(c) Hence find the value of b such that the area of PQRS is a maximum. [5]

Consider another function $g(x) = (x-3)^2 + k$, $x \in \mathbb{R}$.

(d) Show that when the graphs of f and g intersect, $2x^2 - 6x + k = 0$. [2]

(e) Given that the graphs of f and g intersect only once, find the value of k. [5]

Question 34

[Maximum mark: 6]

Consider the function f, with derivative $f'(x) = 2x^2 + 5kx + 3k^2 + 2$ where x, $k \in \mathbb{R}$.

(a) Show that the discriminant of $f'(x)$ is $k^2 - 16$. [2]

(b) Given that f is an increasing function, find all possible values of k. [4]

Question 35

[Maximum mark: 6]

Consider $\binom{11}{a} = \frac{11!}{a!\,9!}$.

(a) Find the value of a. [2]

(b) Hence or otherwise find the coefficient of the term in x^9 in the expansion of $(x+3)^{11}$. [4]

Question 36

[Maximum mark: 6]

In an arithmetic sequence, $u_2 = 5$ and $u_3 = 11$.

(a) Find the common difference. [2]

(b) Find the first term. [2]

(c) Find the sum of the first 20 terms. [2]