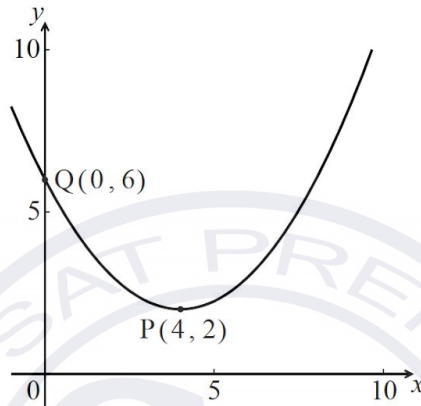


Subject – Math (Standard Level)
Topic - Functions and Equations
Year - Nov 2011 – Nov 2019

Question -1

[Maximum mark: 6]

Let f be a quadratic function. Part of the graph of f is shown below.



The vertex is at $P(4, 2)$ and the y -intercept is at $Q(0, 6)$.

(a) Write down the equation of the axis of symmetry. [1 mark]

The function f can be written in the form $f(x) = a(x - h)^2 + k$.

(b) Write down the value of h and of k . [2 marks]

(c) Find a . [3 marks]

Question -2

[Maximum mark: 8]

Let $f(x) = \frac{1}{2}x^2 + kx + 8$, where $k \in \mathbb{Z}$.

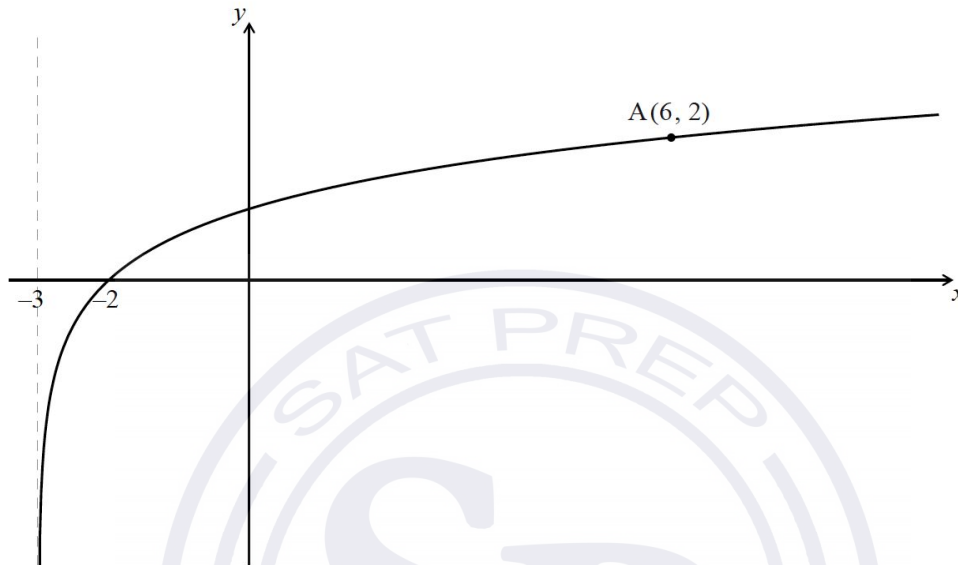
(a) Find the values of k such that $f(x) = 0$ has two equal roots. [4 marks]

(b) Each value of k is equally likely for $-5 \leq k \leq 5$. Find the probability that $f(x) = 0$ has no roots. [4 marks]

Question -3

[Maximum mark: 13]

Let $f(x) = \log_p(x+3)$ for $x > -3$. Part of the graph of f is shown below.



The graph passes through $A(6, 2)$, has an x -intercept at $(-2, 0)$ and has an asymptote at $x = -3$.

(a) Find p .

[4 marks]

The graph of f is reflected in the line $y = x$ to give the graph of g .

(b) (i) Write down the y -intercept of the graph of g .

(ii) Sketch the graph of g , noting clearly any asymptotes and the image of A .

[5 marks]

(c) Find $g(x)$.

[4 marks]

Question -4

[Maximum mark: 6]

Let $f(x) = 2x - 1$ and $g(x) = 3x^2 + 2$.

(a) Find $f^{-1}(x)$.

[3 marks]

(b) Find $(f \circ g)(1)$.

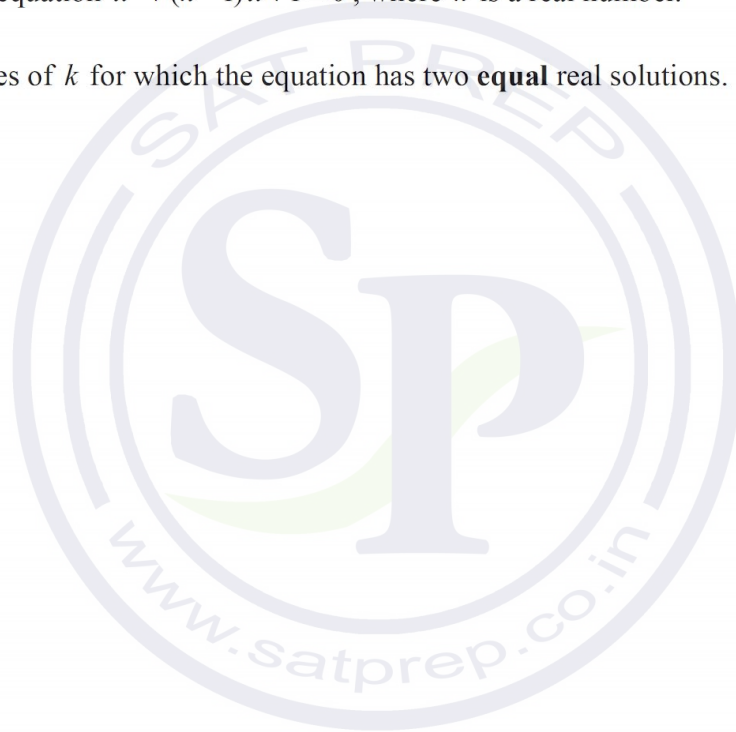
[3 marks]

Question -5

[Maximum mark: 7]

Consider the equation $x^2 + (k - 1)x + 1 = 0$, where k is a real number.

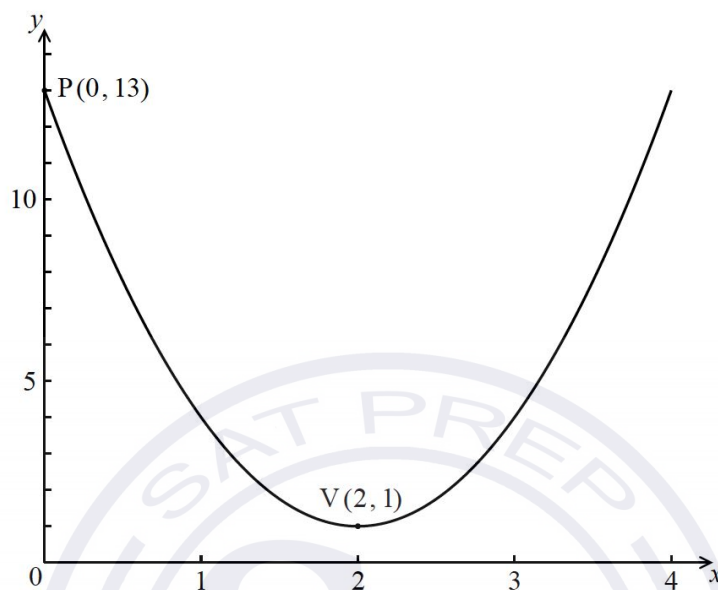
Find the values of k for which the equation has two **equal** real solutions.



Question -6

[Maximum mark: 15]

The following diagram shows the graph of a quadratic function f , for $0 \leq x \leq 4$.



The graph passes through the point $P(0, 13)$, and its vertex is the point $V(2, 1)$.

(a) The function can be written in the form $f(x) = a(x-h)^2 + k$.

(i) Write down the value of h and of k .

(ii) Show that $a = 3$.

[4 marks]

(b) Find $f(x)$, giving your answer in the form $Ax^2 + Bx + C$.

[3 marks]

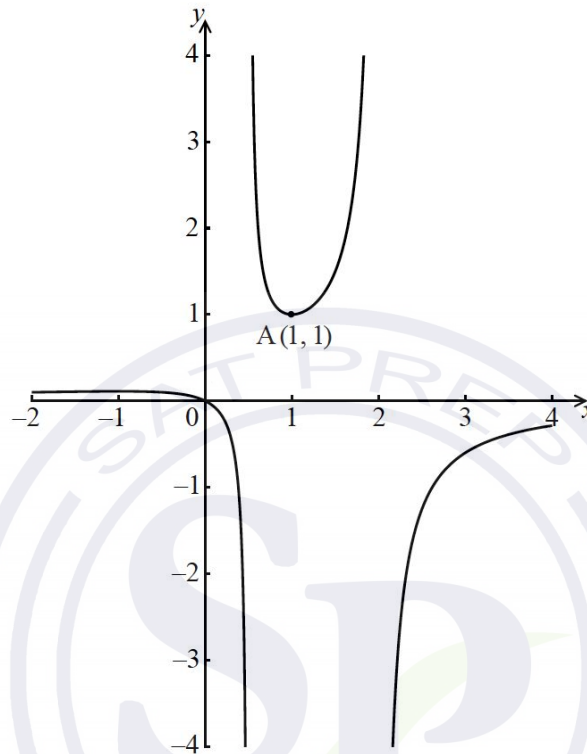
(c) Calculate the area enclosed by the graph of f , the x -axis, and the lines $x = 2$ and $x = 4$.

[8 marks]

Question -6

[Maximum mark: 16]

Let $f(x) = \frac{x}{-2x^2 + 5x - 2}$ for $-2 \leq x \leq 4$, $x \neq \frac{1}{2}$, $x \neq 2$. The graph of f is given below.



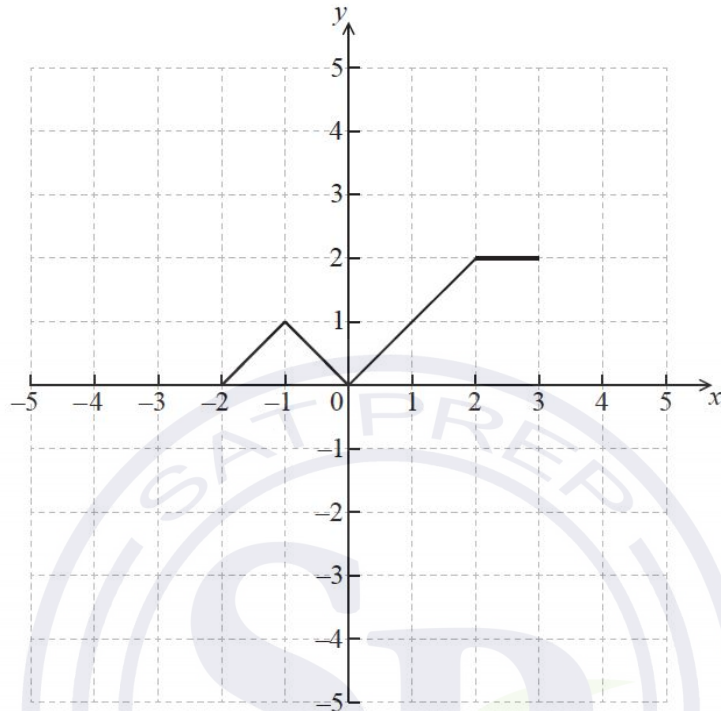
The graph of f has a local minimum at $A(1, 1)$ and a local maximum at B .

- (a) Use the quotient rule to show that $f'(x) = \frac{2x^2 - 2}{(-2x^2 + 5x - 2)^2}$. [6 marks]
- (b) Hence find the coordinates of B . [7 marks]
- (c) Given that the line $y = k$ does not meet the graph of f , find the possible values of k . [3 marks]

Question -7

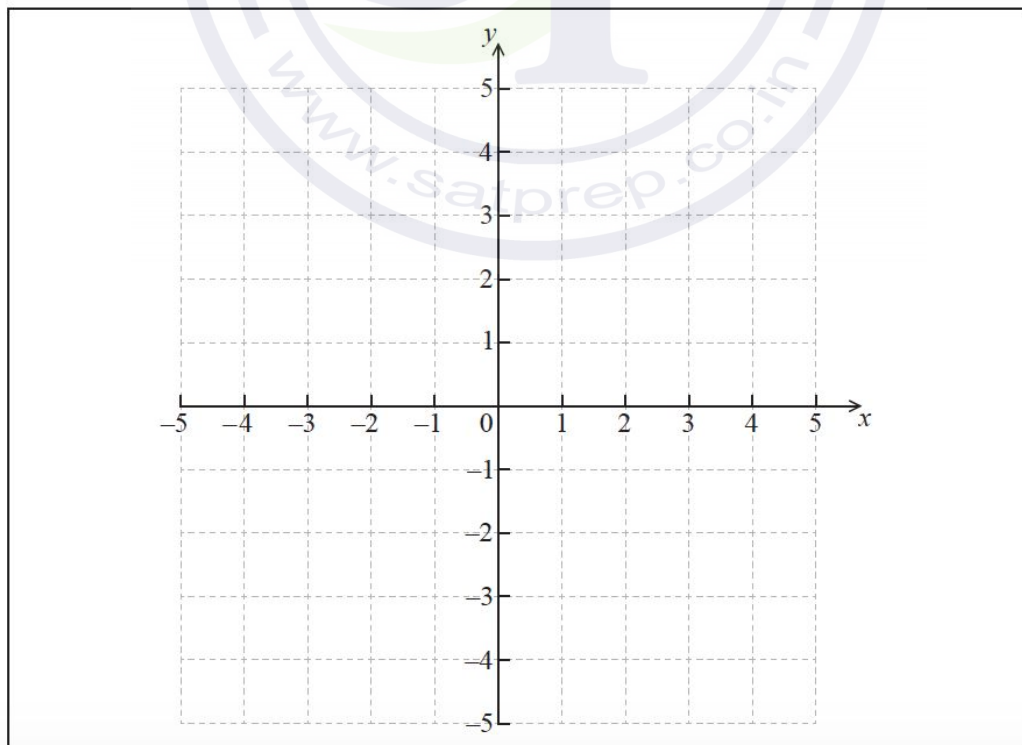
[Maximum mark: 6]

The diagram below shows the graph of a function $f(x)$, for $-2 \leq x \leq 3$.



(a) Sketch the graph of $f(-x)$ on the grid below.

[2 marks]



Question – 8

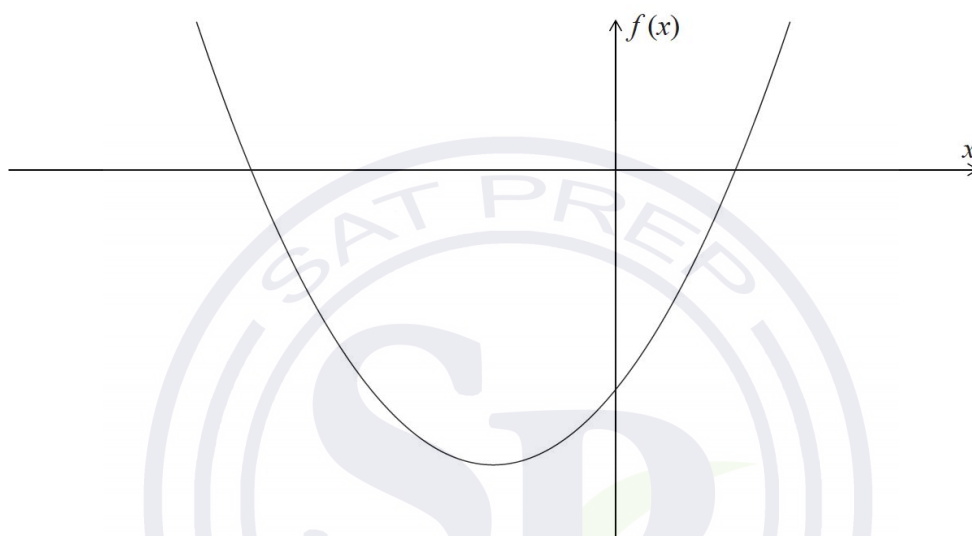
[Maximum mark: 6]

The equation $x^2 - 3x + k^2 = 4$ has two distinct real roots. Find the possible values of k .

Question -9

[Maximum mark: 6]

The diagram below shows part of the graph of $f(x) = (x-1)(x+3)$.



- (a) Write down the x -intercepts of the graph of f . [2 marks]
- (b) Find the coordinates of the vertex of the graph of f . [4 marks]

Question -10

[Maximum mark: 6]

Let $f(x) = \sqrt{x-5}$, for $x \geq 5$.

- (a) Find $f^{-1}(2)$. [3 marks]
- (b) Let g be a function such that g^{-1} exists for all real numbers. Given that $g(30) = 3$, find $(f \circ g^{-1})(3)$. [3 marks]

Question 11

[Maximum mark: 8]

The equation $x^2 + (k + 2)x + 2k = 0$ has two distinct real roots.

Find the possible values of k .

Question 12

[Maximum mark: 14]

Let $f(x) = 3x - 2$ and $g(x) = \frac{5}{3x}$, for $x \neq 0$.

(a) Find $f^{-1}(x)$. [2]

(b) Show that $(g \circ f^{-1})(x) = \frac{5}{x+2}$. [2]

Let $h(x) = \frac{5}{x+2}$, for $x \geq 0$. The graph of h has a horizontal asymptote at $y = 0$.

(c) (i) Find the y -intercept of the graph of h .
(ii) Hence, sketch the graph of h . [5]

(d) For the graph of h^{-1} ,
(i) write down the x -intercept;
(ii) write down the equation of the vertical asymptote. [2]

(e) Given that $h^{-1}(a) = 3$, find the value of a . [3]

Question 13

[Maximum mark: 6]

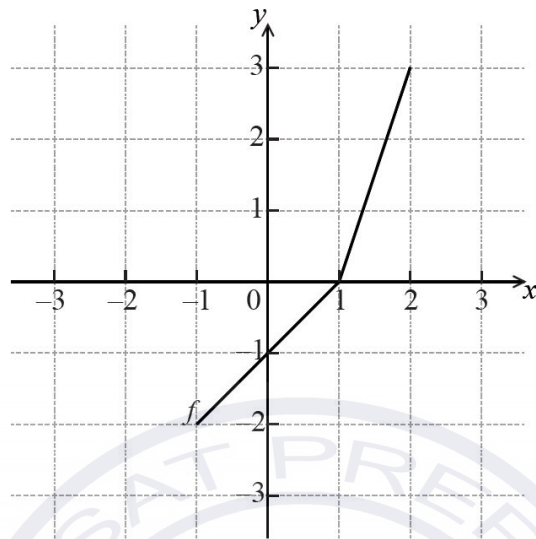
Let $f(x) = 4x - 2$ and $g(x) = -2x^2 + 8$.

(a) Find $f^{-1}(x)$. [3 marks]

(b) Find $(f \circ g)(1)$. [3 marks]

Question 14

The diagram below shows the graph of a function f , for $-1 \leq x \leq 2$.



(a) Write down the value of

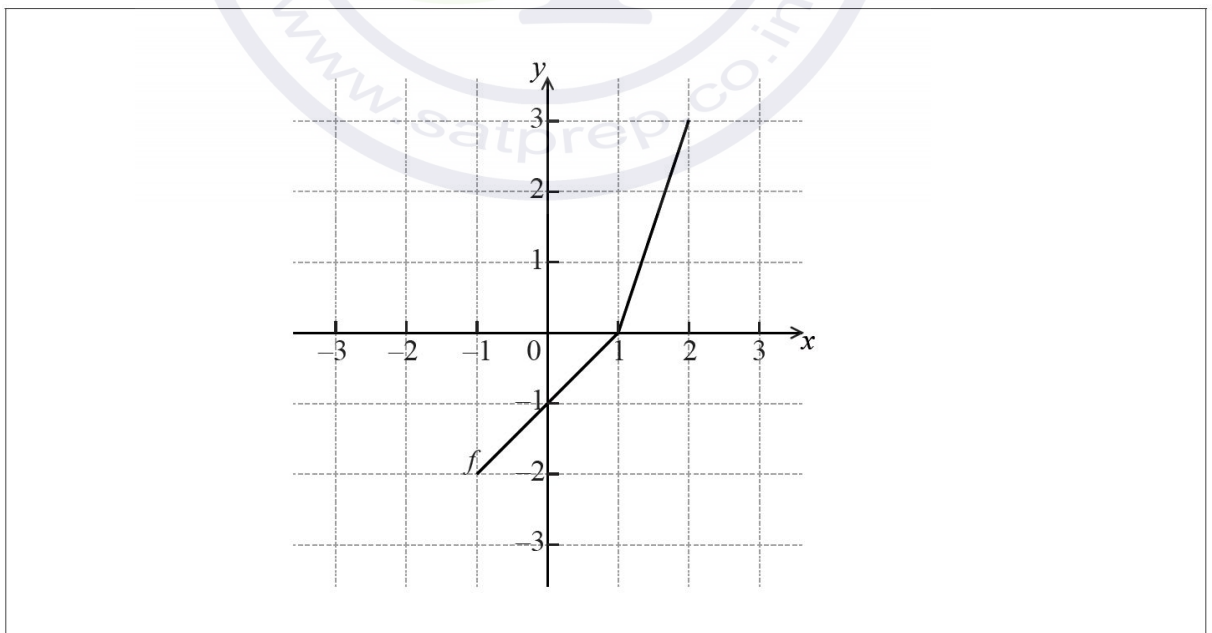
(i) $f(2)$;

(ii) $f^{-1}(-1)$.

[3 marks]

(b) Sketch the graph of f^{-1} on the grid below.

[3 marks]



Question 15

[Maximum mark: 5]

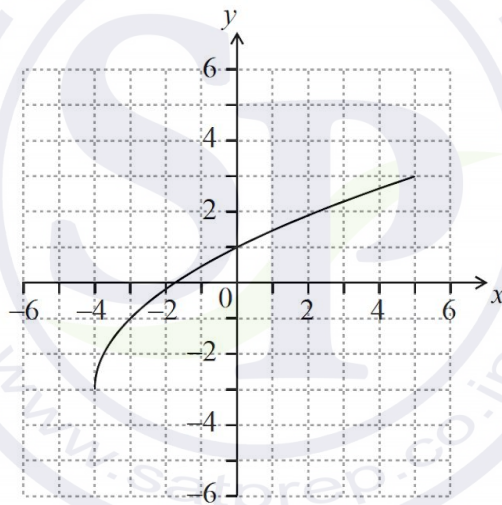
Let $f(x) = a(x-h)^2 + k$. The vertex of the graph of f is at $(2, 3)$ and the graph passes through $(1, 7)$.

- (a) Write down the value of h and of k . [2]
- (b) Find the value of a . [3]

Question 16

[Maximum mark: 6]

The following diagram shows the graph of $y = f(x)$, for $-4 \leq x \leq 5$.



- (a) Write down the value of
- (i) $f(-3)$;
- (ii) $f^{-1}(1)$. [2]
- (b) Find the domain of f^{-1} . [2]
- (c) On the grid above, sketch the graph of f^{-1} . [2]

Question 17

[Maximum mark: 15]

Let $f(x) = 3x^2 - 6x + p$. The equation $f(x) = 0$ has two equal roots.

(a) (i) Write down the **value** of the discriminant.

(ii) Hence, show that $p = 3$.

[3]

The graph of f has its vertex on the x -axis.

(b) Find the coordinates of the vertex of the graph of f .

[4]

(c) Write down the solution of $f(x) = 0$.

[1]

(d) The function can be written in the form $f(x) = a(x-h)^2 + k$. Write down the value of

(i) a ;

(ii) h ;

(iii) k .

[3]

(e) The graph of a function g is obtained from the graph of f by a reflection of f in the x -axis, followed by a translation by the vector $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$. Find g , giving your answer in the form $g(x) = Ax^2 + Bx + C$.

[4]

Question 18

[Maximum mark: 7]

Let $f(x) = x^2 + x - 6$.

(a) Write down the y -intercept of the graph of f .

[1]

(b) Solve $f(x) = 0$.

[3]

Question 19

[Maximum mark: 6]

Let $f(x) = p + \frac{9}{x-q}$, for $x \neq q$. The line $x = 3$ is a vertical asymptote to the graph of f .

- (a) Write down the value of q . [1]

The graph of f has a y -intercept at $(0, 4)$.

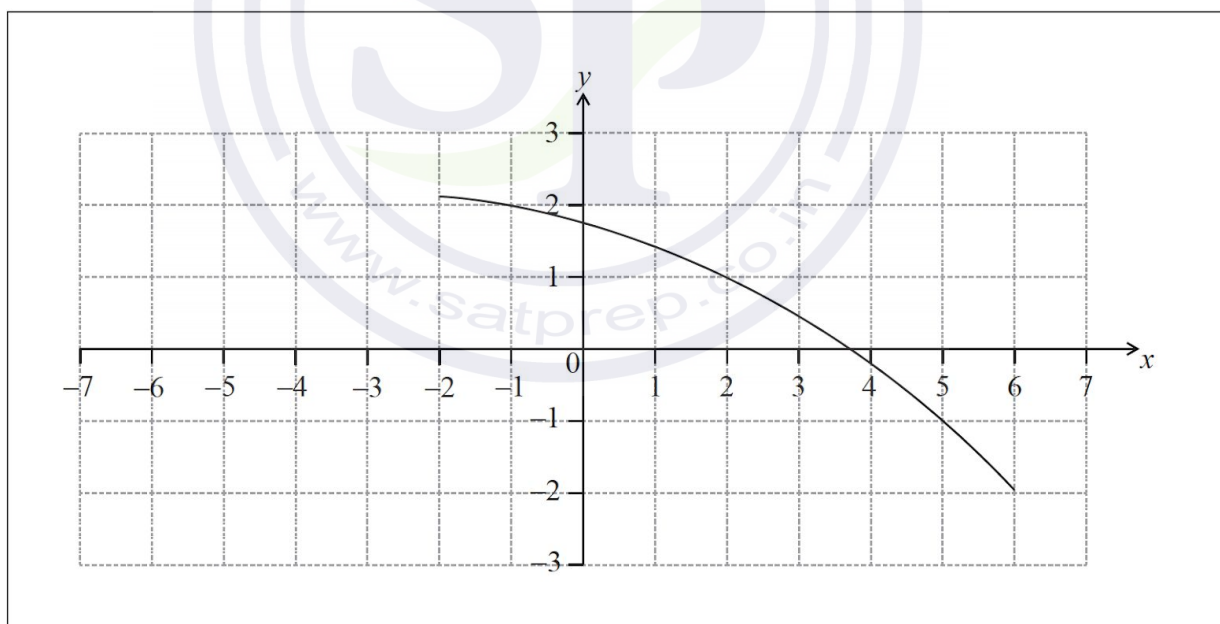
- (b) Find the value of p . [4]

- (c) Write down the equation of the horizontal asymptote of the graph of f . [1]

Question 20

[Maximum mark: 7]

The following diagram shows the graph of a function f .



- (a) Find $f^{-1}(-1)$. [2]

- (b) Find $(f \circ f)(-1)$. [3]

- (c) On the same diagram, sketch the graph of $y = f(-x)$. [2]

Question 21

[Maximum mark: 6]

Let $f(x) = px^2 + (10 - p)x + \frac{5}{4}p - 5$.

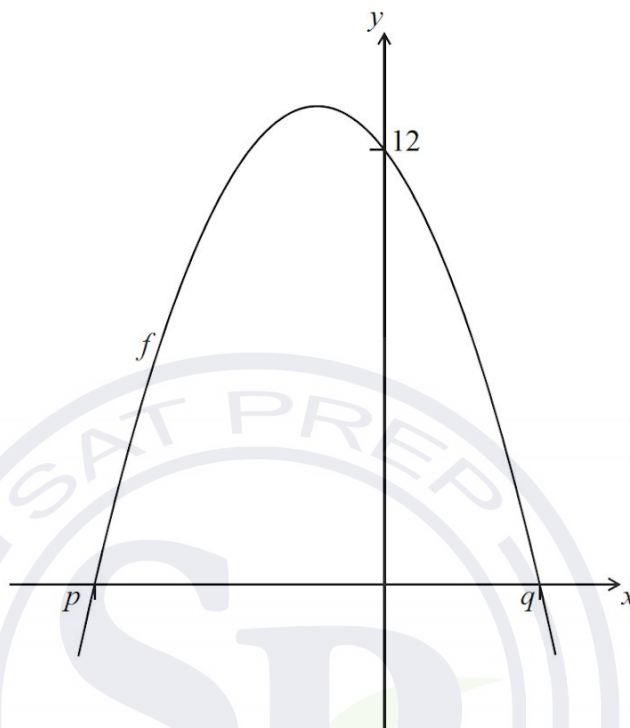
- (a) Show that the discriminant of $f(x)$ is $100 - 4p^2$. [3]
- (b) Find the values of p so that $f(x) = 0$ has two **equal** roots. [3]



Question 22

[Maximum mark: 15]

Let $f(x) = a(x + 3)(x - 1)$. The following diagram shows part of the graph of f .



The graph has x -intercepts at $(p, 0)$ and $(q, 0)$, and a y -intercept at $(0, 12)$.

- (a) (i) Write down the value of p and of q .
(ii) Find the value of a . [6]
- (b) Find the equation of the axis of symmetry of the graph of f . [3]
- (c) Find the largest value of f . [3]

The function f can also be written as $f(x) = a(x - h)^2 + k$.

- (d) Find the value of h and of k . [3]

Question 23

[Maximum mark: 6]

Let $f(x) = (x - 5)^3$, for $x \in \mathbb{R}$.

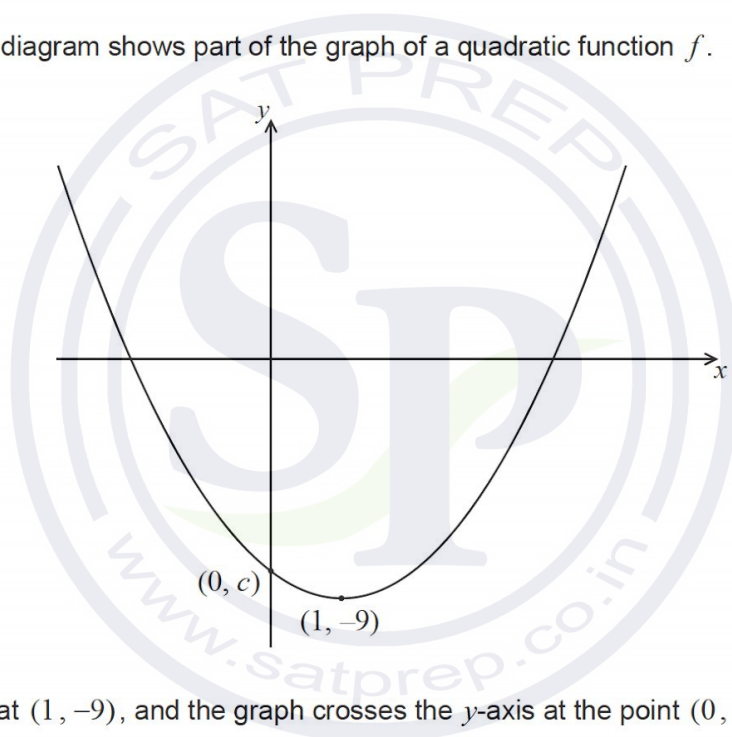
(a) Find $f^{-1}(x)$. [3]

(b) Let g be a function so that $(f \circ g)(x) = 8x^6$. Find $g(x)$. [3]

Question 24

[Maximum mark: 16]

The following diagram shows part of the graph of a quadratic function f .



The vertex is at $(1, -9)$, and the graph crosses the y -axis at the point $(0, c)$.

The function can be written in the form $f(x) = (x - h)^2 + k$.

(a) Write down the value of h and of k . [2]

(b) Find the value of c . [2]

Let $g(x) = -(x - 3)^2 + 1$. The graph of g is obtained by a reflection of the graph of f in the x -axis, followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

(c) Find the value of p and of q . [5]

(d) Find the x -coordinates of the points of intersection of the graphs of f and g . [7]

Question 25

[Maximum mark: 5]

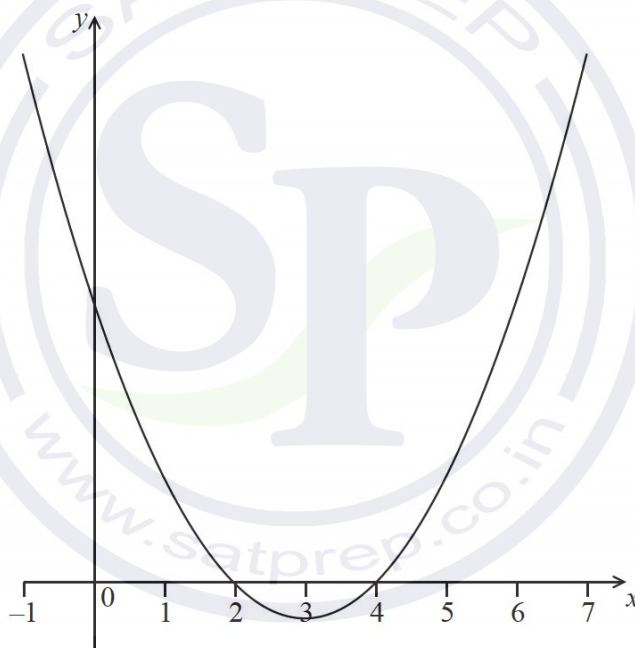
Let $f(x) = 8x + 3$ and $g(x) = 4x$, for $x \in \mathbb{R}$.

- (a) Write down $g(2)$. [1]
- (b) Find $(f \circ g)(x)$. [2]
- (c) Find $f^{-1}(x)$. [2]

Question 26

[Maximum mark: 6]

The following diagram shows part of the graph of a quadratic function f .



The vertex is at $(3, -1)$ and the x -intercepts at 2 and 4.

The function f can be written in the form $f(x) = (x - h)^2 + k$.

- (a) Write down the value of h and of k . [2]

The function can also be written in the form $f(x) = (x - a)(x - b)$.

- (b) Write down the value of a and of b . [2]
- (c) Find the y -intercept. [2]

Question 27

[Maximum mark: 7]

Let $f(x) = 6x\sqrt{1-x^2}$, for $-1 \leq x \leq 1$, and $g(x) = \cos(x)$, for $0 \leq x \leq \pi$.
Let $h(x) = (f \circ g)(x)$.

- (a) Write $h(x)$ in the form $a \sin(bx)$, where $a, b \in \mathbb{Z}$. [5]
- (b) Hence find the range of h . [2]

Question 28

[Maximum mark: 6]

Let $f(x) = x^2 - 4x + 5$.

- (a) Find the equation of the axis of symmetry of the graph of f . [2]

The function can also be expressed in the form $f(x) = (x - h)^2 + k$.

- (b) (i) Write down the value of h .
(ii) Find the value of k . [4]

Question 29

[Maximum mark: 7]

Let $f(x) = m - \frac{1}{x}$, for $x \neq 0$. The line $y = x - m$ intersects the graph of f in two distinct points. Find the possible values of m .

Question 30

[Maximum mark: 5]

Let $f(x) = 5x$ and $g(x) = x^2 + 1$, for $x \in \mathbb{R}$.

- (a) Find $f^{-1}(x)$. [2]
- (b) Find $(f \circ g)(7)$. [3]

Question 31

[Maximum mark: 14]

A quadratic function f can be written in the form $f(x) = a(x - p)(x - 3)$. The graph of f has axis of symmetry $x = 2.5$ and y -intercept at $(0, -6)$.

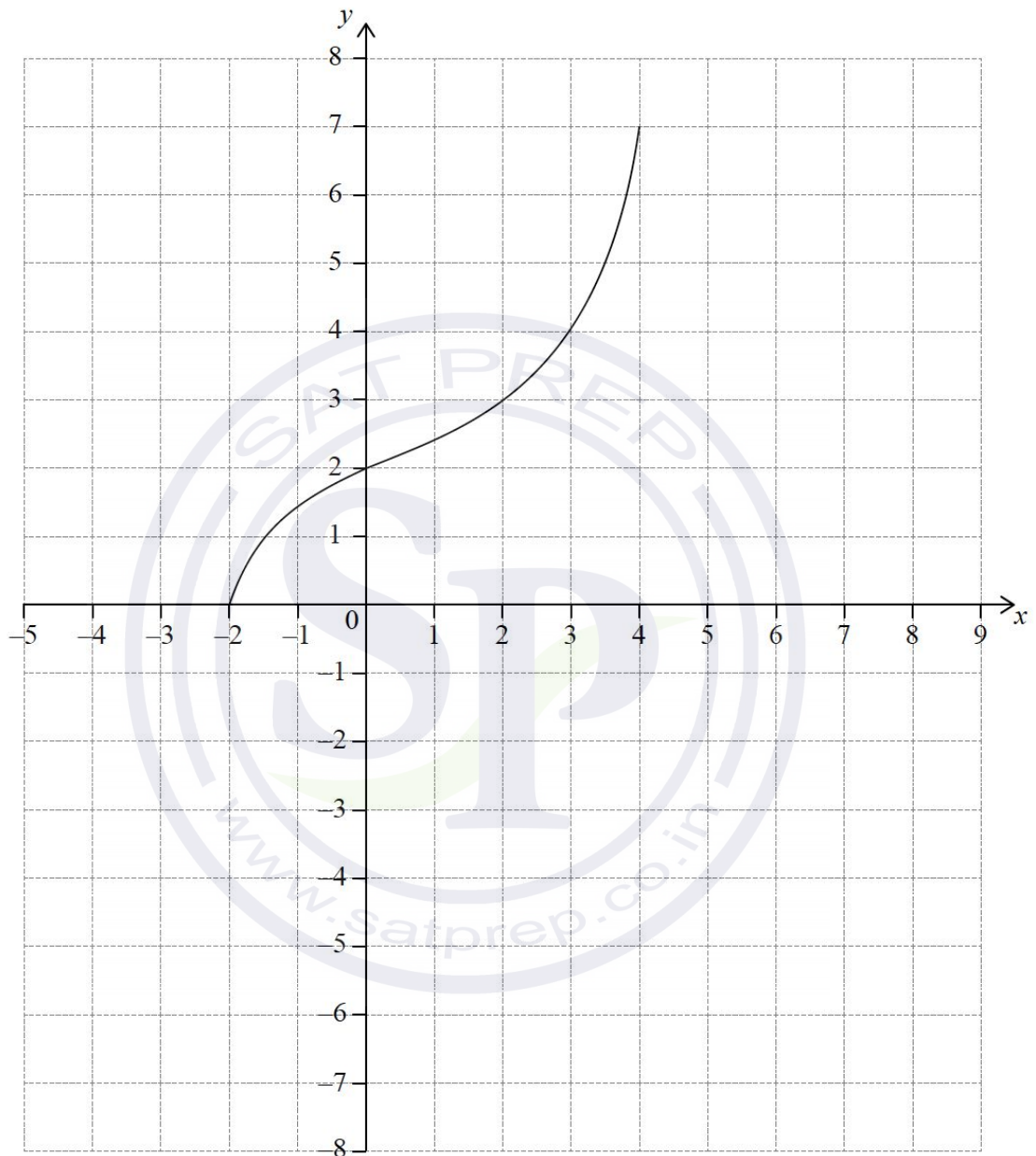
- (a) Find the value of p . [3]
- (b) Find the value of a . [3]
- (c) The line $y = kx - 5$ is a tangent to the curve of f . Find the values of k . [8]



Question 32

[Maximum mark: 6]

The following diagram shows the graph of a function f , with domain $-2 \leq x \leq 4$.



The points $(-2, 0)$ and $(4, 7)$ lie on the graph of f .

- (a) Write down the range of f . [1]
- (b) Write down
- (i) $f(2)$;
- (ii) $f^{-1}(2)$. [2]
- (c) On the grid opposite, sketch the graph of f^{-1} . [3]

Question 33

[Maximum mark: 6]

Let $f(x) = 1 + e^{-x}$ and $g(x) = 2x + b$, for $x \in \mathbb{R}$, where b is a constant.

- (a) Find $(g \circ f)(x)$. [2]
- (b) Given that $\lim_{x \rightarrow +\infty} (g \circ f)(x) = -3$, find the value of b . [4]

Question 34

[Maximum mark: 7]

Consider $f(x) = \log_k(6x - 3x^2)$, for $0 < x < 2$, where $k > 0$.
The equation $f(x) = 2$ has exactly one solution. Find the value of k .

Question 35

[Maximum mark: 6]

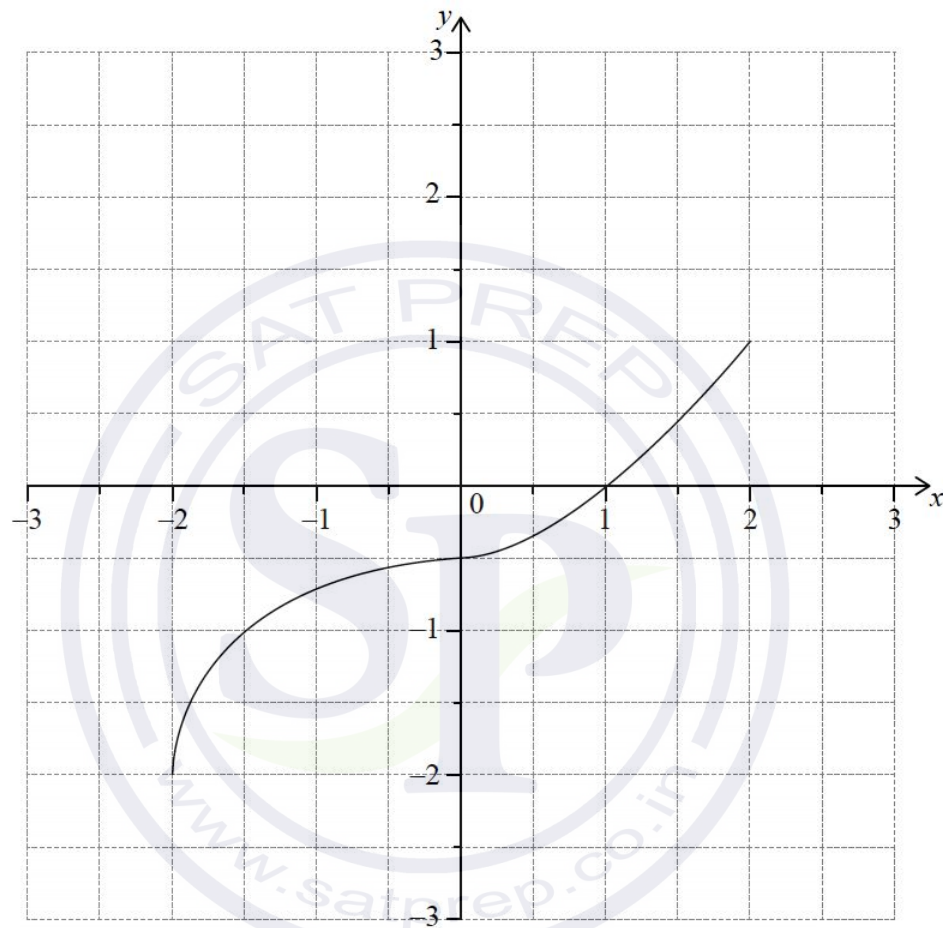
Let $f(x) = \sqrt{x+2}$ for $x \geq -2$ and $g(x) = 3x - 7$ for $x \in \mathbb{R}$.

- (a) Write down $f(14)$. [1]
- (b) Find $(g \circ f)(14)$. [2]
- (c) Find $g^{-1}(x)$. [3]

Question 36

[Maximum mark: 7]

Consider a function $f(x)$, for $-2 \leq x \leq 2$. The following diagram shows the graph of f .



(a) Write down the value of

(i) $f(0)$;

(ii) $f^{-1}(1)$.

[2]

(b) Write down the range of f^{-1} .

[1]

(c) On the grid above, sketch the graph of f^{-1} .

[4]

Question 37

[Maximum mark: 7]

Let $f(x) = ax^2 - 4x - c$. A horizontal line, L , intersects the graph of f at $x = -1$ and $x = 3$.

(a) (i) The equation of the axis of symmetry is $x = p$. Find p .

(ii) Hence, show that $a = 2$.

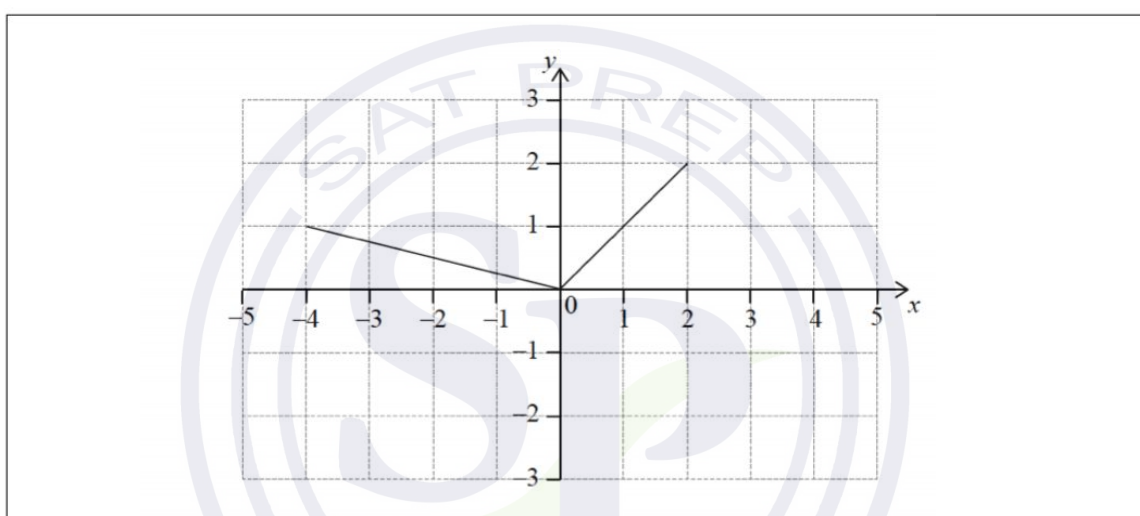
[4]

(b) The equation of L is $y = 5$. Find the value of c .

[3]

Question 38

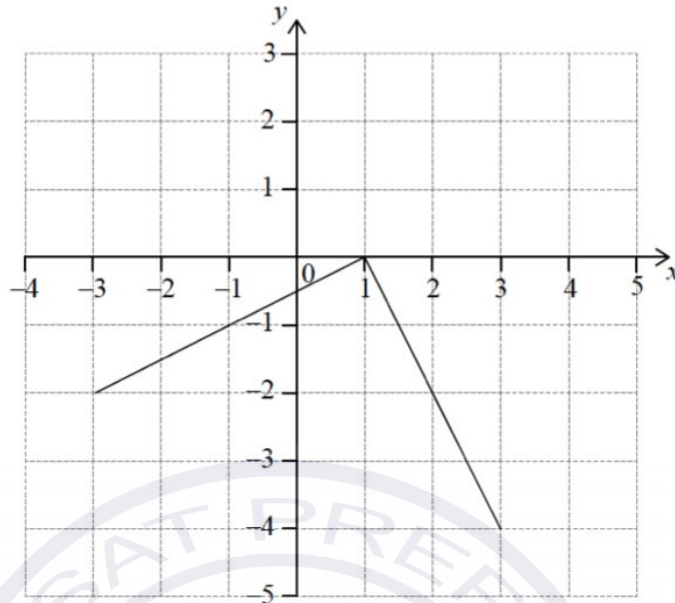
The following diagram shows the graph of a function f , for $-4 \leq x \leq 2$.



(a) On the same axes, sketch the graph of $f(-x)$.

[2]

- (b) Another function, g , can be written in the form $g(x) = a \times f(x + b)$. The following diagram shows the graph of g .



Write down the value of a and of b .

[4]

Question 39

[Maximum mark: 5]

Two functions, f and g , are defined in the following table.

x	-2	1	3	6
$f(x)$	6	3	1	-2
$g(x)$	-7	-2	5	9

- (a) Write down the value of $f(1)$.

[1]

- (b) Find the value of $(g \circ f)(1)$.

[2]

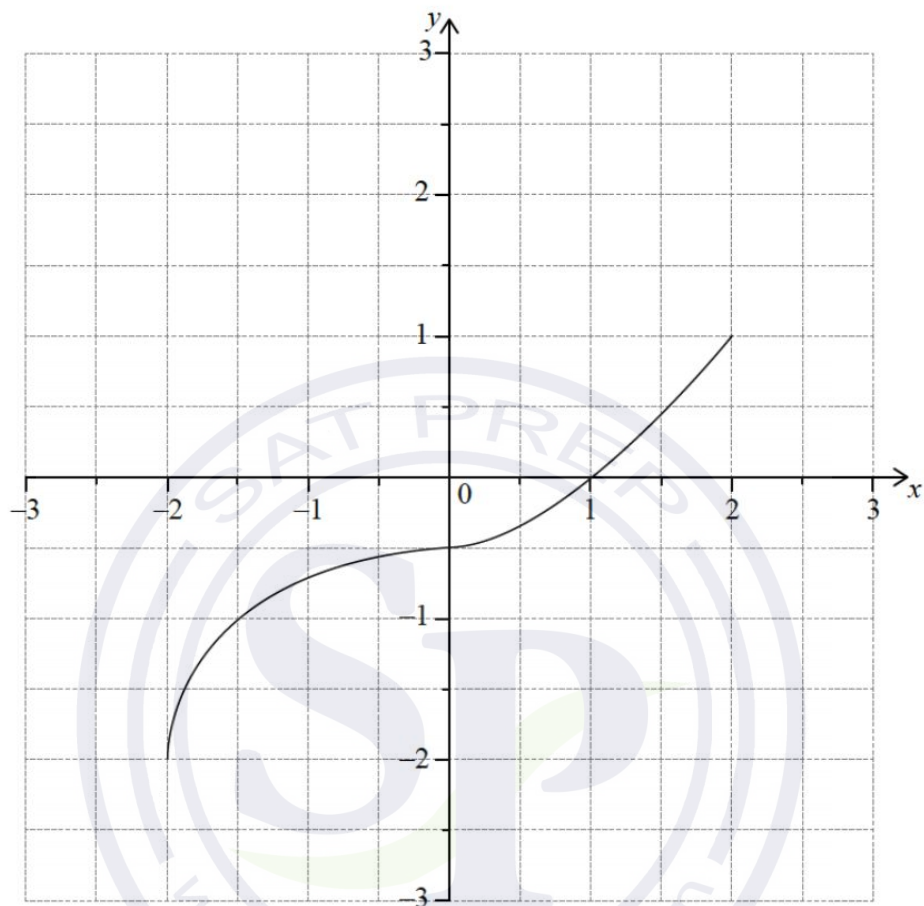
- (c) Find the value of $g^{-1}(-2)$.

[2]

Question 40

[Maximum mark: 7]

Consider a function $f(x)$, for $-2 \leq x \leq 2$. The following diagram shows the graph of f .



- (a) Write down the value of
- (i) $f(0)$;
 - (ii) $f^{-1}(1)$. [2]
- (b) Write down the range of f^{-1} . [1]
- (c) On the grid above, sketch the graph of f^{-1} . [4]

Question 41

[Maximum mark: 6]

Let $f(x) = \sqrt{x+2}$ for $x \geq -2$ and $g(x) = 3x - 7$ for $x \in \mathbb{R}$.

- (a) Write down $f(14)$. [1]
- (b) Find $(g \circ f)(14)$. [2]
- (c) Find $g^{-1}(x)$. [3]

Question 42

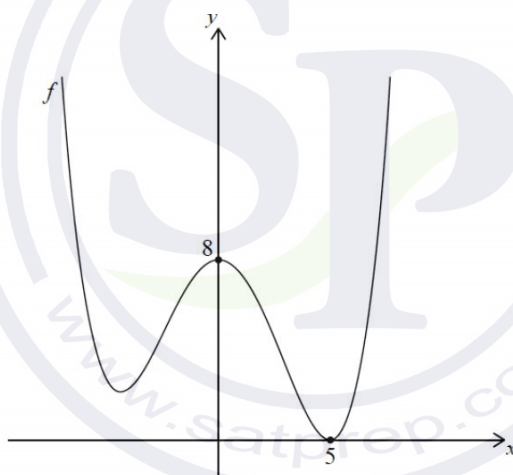
[Maximum mark: 6]

Consider the function $f(x) = (1 - k)x^2 + x + k$, $x \in \mathbb{R}$. Find the value of k for which $f(x)$ has two equal real roots.

Question 43

[Maximum mark: 7]

The following diagram shows part of the graph of f with x -intercept $(5, 0)$ and y -intercept $(0, 8)$.



- (a) Find the y -intercept of the graph of
 - (i) $f(x) + 3$;
 - (ii) $f(4x)$. [3]
- (b) Find the x -intercept of the graph of $f(2x)$. [2]
- (c) Describe the transformation $f(x + 1)$. [2]

Question 44

[Maximum mark: 6]

Consider the function $f(x) = \frac{3x+1}{x-2}$, $x \neq 2$.

- (a) For the graph of f ,
- (i) write down the equation of the vertical asymptote;
 - (ii) find the equation of the horizontal asymptote. [3]

Let $g(x) = x^2 + 4$, $x \in \mathbb{R}$.

- (b) Find $(f \circ g)(1)$. [3]

Question 45

[Maximum mark: 6]

Let $f(x) = \frac{2x-1}{x+3}$, $x \neq -3$.

- (a) Write down the equation of the vertical asymptote of the graph of f . [1]
- (b) Find $f^{-1}(x)$. [3]
- (c) Find the equation of the horizontal asymptote of the graph of f^{-1} . [2]

Question 46

[Maximum mark: 7]

Let $g(x) = x^2 + bx + 11$. The point $(-1, 8)$ lies on the graph of g .

- (a) Find the value of b . [3]
- (b) The graph of $f(x) = x^2$ is transformed to obtain the graph of g .
Describe this transformation. [4]