Subject – Math (Standard Level)
Topic - Functions and Equations
Year - Nov 2011 – Nov 2019

Question -1

(a) $x = 4$ (must be an equation) ***A1*** ***N1***

[1 mark]

(b) $h = 4$, $k = 2$ ***A1A1*** ***N2***

[2 marks]

(c) attempt to substitute coordinates of any point on the graph into f ***(M1)***

e.g. $f(0) = 6$, $6 = a(0-4)^2 + 2$, $f(4) = 2$

correct equation (do **not** accept an equation that results from $f(4) = 2$) ***(A1)***

e.g. $6 = a(-4)^2 + 2$, $6 = 16a + 2$

$$a = \frac{4}{16}\left(= \frac{1}{4}\right)$$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

Question-2

(a) **METHOD 1**

evidence of discriminant *(M1)*
e.g. $b^2 - 4ac$, discriminant $= 0$

correct substitution into discriminant ***A1***
e.g. $k^2 - 4 \times \frac{1}{2} \times 8,\ k^2 - 16 = 0$

$k = \pm 4$ ***A1A1*** ***N3***

METHOD 2

Recognising that equal roots means perfect square *(R1)*
e.g. attempt to complete the square, $\frac{1}{2}\left(x^2 + 2kx + 16\right)$

correct working
e.g $\frac{1}{2}(x+k)^2,\ \frac{1}{2}k^2 = 8$ ***A1***

$k = \pm 4$ ***A1A1*** ***N3***

[4 marks]

(b) evidence of appropriate approach *(M1)*
e.g. $b^2 - 4ac < 0$

correct working for k ***A1***
e.g. $-4 < k < 4,\ k^2 < 16$, list all correct values of k

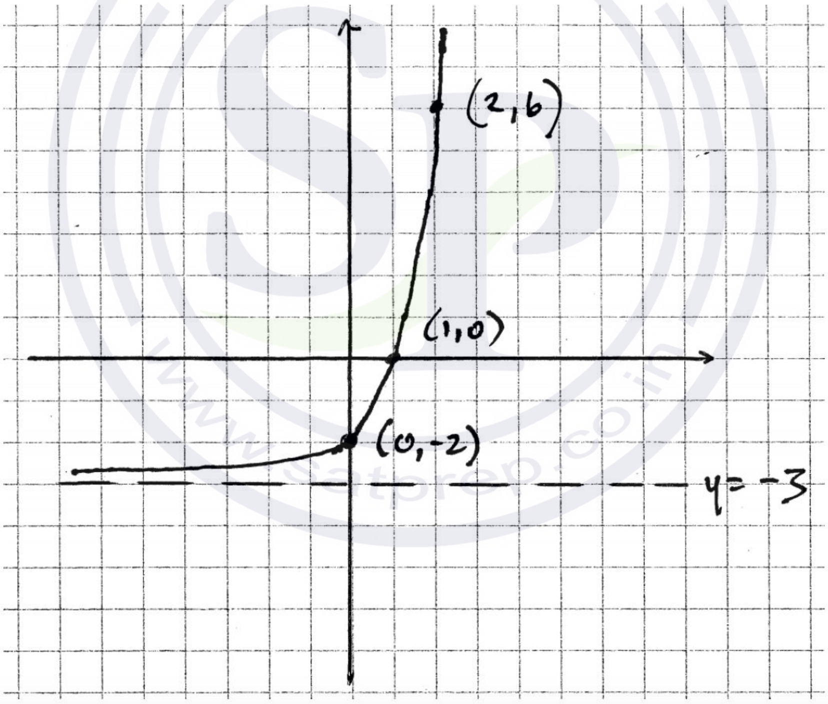
$p = \frac{7}{11}$ ***A2*** ***N3***

[4 marks]

Total [8 marks]

Question -3

(a)	evidence of substituting the point A *e.g.* $2 = \log_p (6+3)$	*(M1)*	
	manipulating logs *e.g.* $p^2 = 9$	***A1***	
	$p = 3$	***A2***	***N2***
			[4 marks]
(b)	(i) $y = -2$ (accept $(0, -2)$)	***A1***	***N1***
	(ii)		



(c) **METHOD 1**

recognizing that $g = f^{-1}$ ***(R1)***

evidence of valid approach ***(M1)***
e.g. switching x and y (seen anywhere), solving for x

correct manipulation ***(A1)***
e.g. $3^x = y + 3$

$g(x) = 3^x - 3$ ***A1*** ***N3***

[4 marks]

METHOD 2

recognizing that $g(x) = a^x + b$ ***(R1)***

identifying vertical translation ***(A1)***
e.g. graph shifted down 3 units, $f(x) - 3$

evidence of valid approach ***(M1)***
e.g. substituting point to identify the base

$g(x) = 3^x - 3$ ***A1*** ***N3***

[4 marks]

Total [13 marks]

Question -4

(a) interchanging x and y (seen anywhere) ***(M1)***
e.g. $x = 2y - 1$

correct manipulation ***(A1)***
e.g. $x + 1 = 2y$

$f^{-1}(x) = \frac{x+1}{2}$ ***A1*** ***N2***

[3 marks]

(b) **METHOD 1**

attempt to find $g(1)$ or $f(1)$ ***(M1)***
$g(1) = 5$ ***(A1)***
$f(5) = 9$ ***A1*** ***N2***

[3 marks]

METHOD 2

attempt to form composite (in any order) ***(M1)***
e.g. $2(3x^2 + 2) - 1$, $3(2x - 1)^2 + 2$

$(f \circ g)(1) = 2(3 \times 1^2 + 2) - 1 \quad (= 6 \times 1^2 + 3)$ ***(A1)***
$(f \circ g)(1) = 9$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

Question – 5

evidence of valid approach *e.g.* b^2-4ac, quadratic formula	***(M1)***	
correct substitution into b^2-4ac (may be seen in formula) *e.g.* $(k-1)^2-4\times1\times1$; $(k-1)^2-4$; k^2-2k-3	***(A1)***	
setting **their** discriminant equal to zero $e.g.\ \Delta=0, (k-1)^2-4=0$	***M1***	
attempt to solve the quadratic *e.g.* $(k-1)^2=4$, factorizing	***(M1)***	
correct working *e.g.* $(k-1)=\pm2$, $(k-3)(k+1)$	***A1***	
$k=-1$, $k=3$ (do not accept inequalities)	***A1A1***	***N2***

[7 marks]

Question -6

(a) (i) $h=2$, $k=1$ *A1A1* *N2*

(ii) attempt to substitute coordinates of any point (except the vertex) on the graph into f *M1*

e.g. $13=a(0-2)^2+1$

working towards solution *A1*

e.g. $13=4a+1$

$a=3$ *AG* *N0*

[4 marks]

(b) attempting to expand **their** binomial *(M1)*

e.g. $f(x)=3(x^2-2\times 2x+4)+1$, $(x-2)^2=x^2-4x+4$

correct working *(A1)*

e.g. $f(x)=3x^2-12x+12+1$

$f(x)=3x^2-12x+13$ (accept $A=3, B=-12, C=13$) *A1* *N2*

[3 marks]

(c) **METHOD 1**

integral expression *(A1)*

e.g. $\int_2^4(3x^2-12x+13)$, $\int f\,\mathrm{d}x$

Area $=\left[x^3-6x^2+13x\right]_2^4$ *A1A1A1*

Note: Award ***A1*** for x^3, ***A1*** for $-6x^2$, ***A1*** for $13x$.

correct substitution of **correct** limits into **their** expression *A1A1*

e.g. $(4^3-6\times 4^2+13\times 4)-(2^3-6\times 2^2+13\times 2)$, $64-96+52-(8-24+26)$

Note: Award ***A1*** for substituting 4, ***A1*** for substituting 2.

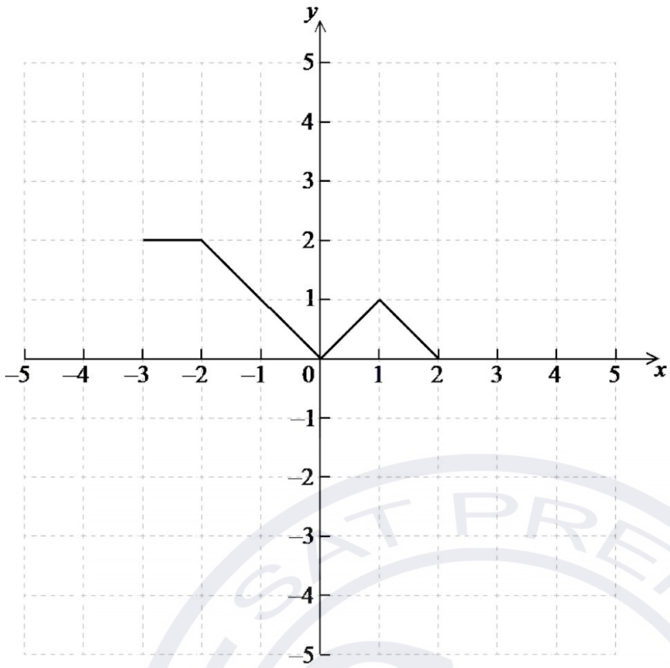
correct working *(A1)*

e.g. $64-96+52-8+24-26$, $20-10$

Area $=10$ *A1* *N3*

Question -7

(a)



A2 *N2*

[2 marks]

(b) $a = -2, b = -1$ *A2A2* *N4*

Note: Award ***A1*** for $a = 2$, ***A1*** for $b = 1$.

[4 marks]

Question -8

evidence of rearranged quadratic equation (may be seen in working) ***A1***

e.g. $x^2 - 3x + k^2 - 4 = 0$, $k^2 - 4$

evidence of discriminant (must be seen explicitly, not in quadratic formula) ***(M1)***

e.g. $b^2 - 4ac$, $\Delta = (-3)^2 - 4(1)(k^2 - 4)$

recognizing that discriminant is greater than zero (seen anywhere, including answer) ***R1***

e.g. $b^2 - 4ac > 0$, $9 + 16 - 4k^2 > 0$

correct working (accept equality) ***A1***

e.g. $25 - 4k^2 > 0$, $4k^2 < 25$, $k^2 = \frac{25}{4}$

both correct values (even if inequality never seen) ***(A1)***

e.g. $\pm\sqrt{\frac{25}{4}}$, ± 2.5

correct interval ***A1*** ***N3***

e.g. $-\frac{5}{2} < k < \frac{5}{2}$, $-2.5 < k < 2.5$

Question -9

(a) $x=1$, $x=-3$ $\left(\text{accept } (1, 0), (-3, 0)\right)$ *A1A1* ***N2***

[2 marks]

(b) **METHOD 1**

attempt to find x-coordinate *(M1)*

eg $\frac{1+-3}{2}$, $x=\frac{-b}{2a}$, $f'(x)=0$

correct value, $x=-1$ (may be seen as a coordinate in the answer) ***A1***

attempt to find **their** y-coordinate *(M1)*

eg $f(-1)$, -2×2, $y=\frac{-D}{4a}$

$y=-4$ ***A1***

vertex $(-1, -4)$ ***N3***

[4 marks]

METHOD 2

attempt to complete the square *(M1)*

eg $x^2+2x+1-1-3$

attempt to put into vertex form *(M1)*

eg $(x+1)^2-4$, $(x-1)^2+4$

vertex $(-1, -4)$ ***A1A1*** ***N3***

[4 marks]

Total [6 marks]

Question-10

(a) **METHOD 1**

attempt to set up equation *(M1)*

eg $2=\sqrt{y-5}$, $2=\sqrt{x-5}$

correct working *(A1)*

eg $4=y-5$, $x=2^2+5$

$f^{-1}(2)=9$ ***A1*** ***N2***

[3 marks]

METHOD 2

interchanging x and y (seen anywhere) *(M1)*

eg $x=\sqrt{y-5}$

correct working *(A1)*

eg $x^2=y-5$, $y=x^2+5$

$f^{-1}(2)=9$ ***A1*** ***N2***

[3 marks]

(b) recognizing $g^{-1}(3)=30$ *(M1)*

eg $f(30)$

correct working *(A1)*

eg $(f\circ g^{-1})(3)=\sqrt{30-5}$, $\sqrt{25}$

$(f\circ g^{-1})(3)=5$ ***A1*** ***N2***

Note: Award ***A0*** for multiple values, *eg* ± 5.

[3 marks]

Total [6 marks]

Question -11

evidence of discriminant **(M1)**

eg $b^2 - 4ac$, $\Delta = 0$

correct substitution into discriminant **(A1)**

eg $(k+2)^2 - 4(2k)$, $k^2 + 4k + 4 - 8k$

correct discriminant **A1**

eg $k^2 - 4k + 4$, $(k-2)^2$

recognizing discriminant is positive **R1**

eg $\Delta > 0$, $(k+2)^2 - 4(2k) > 0$

attempt to solve **their** quadratic in k **(M1)**

eg factorizing, $k = \dfrac{4 \pm \sqrt{16-16}}{2}$

correct working **A1**

eg $(k-2)^2 > 0$, $k = 2$, sketch of positive parabola on the x-axis

correct values **A2**

eg $k \in \mathbb{R}$ and $k \neq 2$, $\mathbb{R} \setminus 2$, $]-\infty, 2[\cup]2, \infty[$

Question -12

(a) interchanging x and y *(M1)*

eg $x = 3y - 2$

$f^{-1}(x) = \frac{x+2}{3}$ $\left(\text{accept } y = \frac{x+2}{3}, \frac{x+2}{3}\right)$ *A1* *N2*

[2 marks]

(b) attempt to form composite (in any order) *(M1)*

eg $g\left(\frac{x+2}{3}\right)$, $\frac{\frac{5}{3x}+2}{3}$

correct substitution *A1*

eg $\frac{5}{3\left(\frac{x+2}{3}\right)}$

$\left(g \circ f^{-1}\right)(x) = \frac{5}{x+2}$ *AG* *N0*

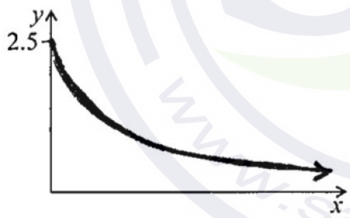
[2 marks]

(c) (i) valid approach *(M1)*

eg $h(0)$, $\frac{5}{0+2}$

$y = \frac{5}{2}$ (accept $(0, 2.5)$) *A1* *N2*

(ii)



A1A2 *N3*

(d) (i) $x = \frac{5}{2}$ (accept $(2.5, 0)$) *A1* *N1*

(ii) $x = 0$ (must be an equation) *A1* *N1*

[2 marks]

(e) **METHOD 1**

attempt to substitute 3 into h (seen anywhere) *(M1)*

eg $h(3)$, $\frac{5}{3+2}$

correct equation *(A1)*

eg $a = \frac{5}{3+2}$, $h(3) = a$

$a = 1$ *A1* *N2*

[3 marks]

METHOD 2

attempt to find inverse (may be seen in (d)) *(M1)*

eg $x = \frac{5}{y+2}$, $h^{-1} = \frac{5}{x} - 2$, $\frac{5}{x} + 2$

correct equation, $\frac{5}{x} - 2 = 3$ *(A1)*

$a = 1$ *A1* *N2*

[3 marks]

Total [14 marks]

Question 13

(a) interchanging x and y (seen anywhere) *(M1)*

eg $x = 4y - 2$

evidence of correct manipulation *(A1)*

eg $x + 2 = 4y$

$f^{-1}(x) = \dfrac{x+2}{4}$ $\left(\text{accept } y = \dfrac{x+2}{4}, \dfrac{x+2}{4}, f^{-1}(x) = \dfrac{1}{4}x + \dfrac{1}{2}\right)$ ***A1*** ***N2***

[3 marks]

(b) **METHOD 1**

attempt to substitute 1 into $g(x)$ *(M1)*

eg $g(1) = -2 \times 1^2 + 8$

$g(1) = 6$ *(A1)*

$f(6) = 22$ ***A1*** ***N3***

METHOD 2

attempt to form composite function (in any order) *(M1)*

eg $(f \circ g)(x) = 4(-2x^2 + 8) - 2$ $(= -8x^2 + 30)$

correct substitution

eg $(f \circ g)(1) = 4(-2 \times 1^2 + 8) - 2,\ -8 + 30$ *(A1)*

$f(6) = 22$ ***A1*** ***N3***

[3 marks]

Total [6 marks]

Question 14

(a) (i) $f(2)=3$ *A1* *N1*

(ii) $f^{-1}(-1)=0$ *A2* *N2*

[3 marks]

(b) **EITHER**

attempt to draw $y=x$ on grid ***(M1)***

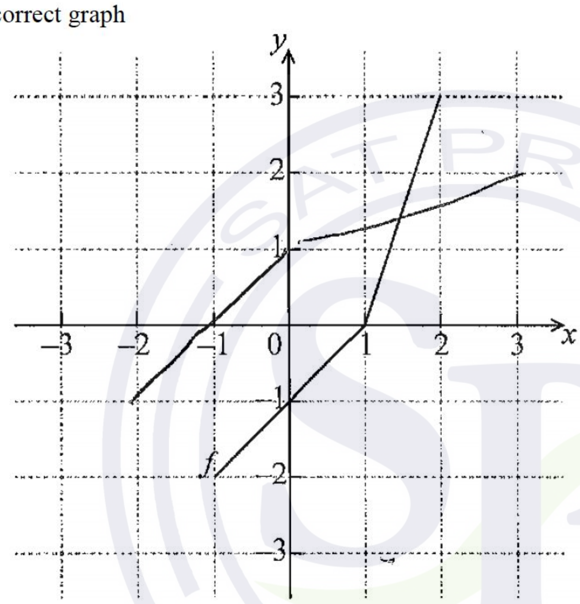
OR

attempt to reverse x and y coordinates ***(M1)***

eg writing or plotting **at least two** of the points $(-2,-1)$, $(-1,0)$, $(0,1)$, $(3,2)$

THEN

correct graph *A2* *N3*



[3 marks]

Total [6 marks]

Question 15

(a) $h=2, k=3$ *A1A1* *N2*

[2 marks]

(b) attempt to substitute $(1, 7)$ in any order into **their** $f(x)$ ***(M1)***

eg $7=a(1-2)^2+3$, $7=a(1-3)^2+2$, $1=a(7-2)^2+3$

correct equation ***(A1)***

eg $7=a+3$

$a=4$ *A1* *N2*

[3 marks]

Total [5 marks]

Question 16

(a) (i) $f(-3) = -1$ *A1* *N1*

(ii) $f^{-1}(1) = 0$ (accept $y = 0$) *A1* *N1*

[2 marks]

(b) domain of f^{-1} is range of f ***(R1)***

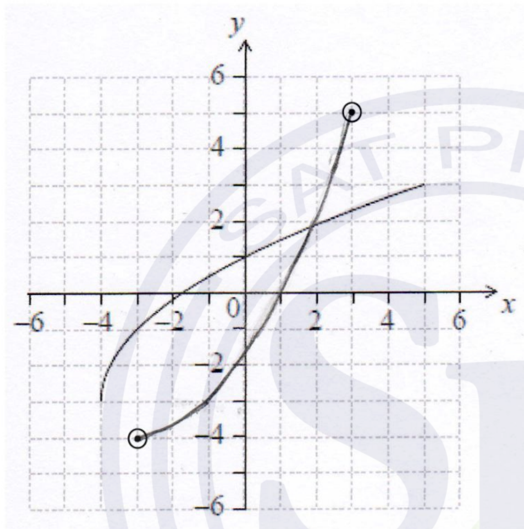
eg $\mathrm{R}f = \mathrm{D}f^{-1}$

correct answer *A1* *N2*

eg $-3 \le x \le 3$, $x \in [-3, 3]$ (accept $-3 < x < 3$, $-3 \le y \le 3$)

[2 marks]

(c)



A1A1 *N2*

Question 17

(a)	(i)	correct value 0, or $36-12p$	*A2*	*N2*
	(ii)	correct equation which clearly leads to $p=3$ *eg* $36-12p=0$, $36=12p$	*A1*	
		$p=3$	*AG*	*N0*
				[3 marks]

(b) **METHOD 1**

valid approach *(M1)*

eg $x=-\frac{b}{2a}$

correct working *A1*

eg $-\frac{(-6)}{2(3)}$, $x=\frac{6}{6}$

correct answers *A1A1* *N2*

eg $x=1$, $y=0$; $(1, 0)$

METHOD 2

valid approach *(M1)*

eg $f(x)=0$, factorisation, completing the square

correct working *A1*

eg $x^2-2x+1=0$, $(3x-3)(x-1)$, $f(x)=3(x-1)^2$

correct answers *A1A1* *N2*

eg $x=1$, $y=0$; $(1, 0)$

METHOD 3

valid approach using derivative *(M1)*

eg $f'(x)=0$, $6x-6$

correct equation *A1*

eg $6x-6=0$

correct answers *A1A1* *N2*

eg $x=1$, $y=0$; $(1, 0)$

[4 marks]

(c) $x = 1$ *A1* *N1*

[1 mark]

(d) (i) $a = 3$ *A1* *N1*

(ii) $h = 1$ *A1* *N1*

(iii) $k = 0$ *A1* *N1*

[3 marks]

(e) attempt to apply vertical reflection *(M1)*

eg $-f(x)$, $-3(x-1)^2$, sketch

attempt to apply vertical shift 6 units up *(M1)*

eg $-f(x)+6$, vertex $(1, 6)$

transformations performed correctly (in correct order) *(A1)*

eg $-3(x-1)^2+6$, $-3x^2+6x-3+6$

$g(x) = -3x^2+6x+3$ *A1* *N3*

[4 marks]

Total [15 marks]

Question 18

(a) y-intercept is -6, $(0, -6)$, $y = -6$ *A1* *N1*

[1 mark]

(b) valid attempt to solve *(M1)*

eg $(x-2)(x+3) = 0$, $x = \frac{-1 \pm \sqrt{1+24}}{2}$, one correct answer

$x = 2$, $x = -3$ *A1A1* *N3*

[3 marks]

Question 19

(a) $q = 3$ *A1* *N1*

[1 mark]

(b) correct expression for $f(0)$ *(A1)*

eg $p + \dfrac{9}{0-3}$, $4 = p + \dfrac{9}{-q}$

recognizing that $f(0) = 4$ (may be seen in equation) *(M1)*

correct working *(A1)*

eg $4 = p - 3$

$p = 7$ *A1* *N3*

[4 marks]

(c) $y = 7$ (must be an equation, do not accept $p = 7$) *A1* *N1*

[1 mark]

Total [6 marks]

Question 20

(a) valid approach ***(M1)***

eg horizontal line on graph at -1, $f(a) = -1$, $(-1, 5)$

$f^{-1}(-1) = 5$ ***A1*** ***N2***

[2 marks]

(b) attempt to find $f(-1)$ ***(M1)***

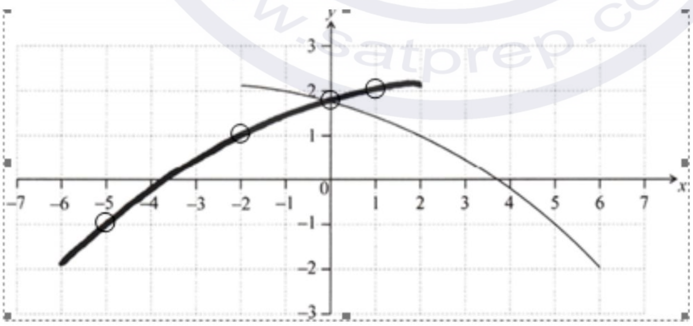
eg line on graph

$f(-1) = 2$ ***(A1)***

$(f \circ f)(-1) = 1$ ***A1*** ***N3***

[3 marks]

(c)



A1A1 ***N2***

Question 21

(a) correct substitution into $b^2 - 4ac$ — ***A1***

eg $(10-p)^2 - 4(p)\left(\frac{5}{4}p - 5\right)$

correct expansion of each term — ***A1A1***

eg $100 - 20p + p^2 - 5p^2 + 20p$, $100 - 20p + p^2 - \left(5p^2 - 20p\right)$

$100 - 4p^2$ — ***AG*** ***N0***

[3 marks]

(b) recognizing discriminant is zero for equal roots — ***(R1)***

eg $D = 0$, $4p^2 = 100$

correct working — ***(A1)***

eg $p^2 = 25$, 1 correct value of p

both correct values $p = \pm 5$ — ***A1*** ***N2***

[3 marks]

Total [6 marks]

Question 22

(a) (i) recognizing intercepts occur when $f(x) = 0$ — ***(M1)***

eg $p = 1$, $q = -3$

$p = -3$, $q = 1$ — ***A1A1*** ***N3***

(ii) attempt to substitute $(0, 12)$ into **their** f to find a — ***(M1)***

eg $f(0) = 12$

correct working — ***(A1)***

eg $12 = a(3)(-1)$

$a = -4$ — ***A1*** ***N2***

[6 marks]

(b) attempt to find x-value — ***(M1)***

eg $\frac{p+q}{2}$, $-\frac{b}{2a}$, $f'(x) = 0$

correct working — ***(A1)***

eg $\frac{-3+1}{2}$, $\frac{8}{2(-4)}$, -1, $-8x - 8 = 0$

$x = -1$ (must be equation) — ***A1*** ***N3***

[3 marks]

(c) **METHOD 1**

substituting **their** x to find y-value *(M1)*

eg $f(-1)$, $-4(-1+3)(-1-1)$

correct calculation *(A1)*

eg $-4(2)(-2)$

largest value is 16 ***A1*** ***N2***

METHOD 2

valid attempt to complete the square *(M1)*

eg $-4\left(x^2+2x+1\right)+12+4$, $-4\left(x^2+2x+1\right)+12-1$

correct vertex form *(A1)*

eg $-4(x+1)^2+16$

largest value is 16 ***A1*** ***N2***

METHOD 3

valid approach (may be seen in (b)) *(M1)*

eg $f'(x)=0$, $-8x-8=0$

substituting $x=-1$ into $f(x)$ *(A1)*

eg $-4(-1)^2-8(-1)+12$

largest value is 16 ***A1*** ***N2***

[3 marks]

(d) **METHOD 1**

recognizing coordinates of vertex *(M1)*

eg $(-1, 16)$

$h=-1$, $k=16$ $\left(\text{accept } -4(x+1)^2+16\right)$ ***A1A1*** ***N3***

METHOD 2

valid attempt to complete the square (may be seen in (c)) *(M1)*

eg $-4\left(x^2+2x+1\right)+12+4$, $-4\left(x^2+2x+1\right)+12-1$

$h=-1$, $k=16$ $\left(\text{accept } -4(x+1)^2+16\right)$ ***A1A1*** ***N3***

[3 marks]

Total [15 marks]

Question 23

(a) interchanging x and y (seen anywhere) ***(M1)***

eg $x=(y-5)^3$

evidence of correct manipulation ***(A1)***

eg $y-5=\sqrt[3]{x}$

$f^{-1}(x)=\sqrt[3]{x}+5$ (accept $5+x^{\frac{1}{3}}$, $y=5+\sqrt[3]{x}$) ***A1*** ***N2***

[3 marks]

METHOD 1

attempt to form composite (in any order) ***(M1)***

eg $g\left((x-5)^3\right),\left(g(x)-5\right)^3=8x^6$

correct working ***(A1)***

eg $g-5=2x^2$, $\left(\left(2x^2+5\right)-5\right)^3$

$g(x)=2x^2+5$ ***A1*** ***N2***

METHOD 2

recognising inverse relationship ***(M1)***

eg $f^{-1}\left(8x^6\right)=g(x)$, $f^{-1}(f\circ g)(x)=f^{-1}\left(8x^6\right)$

correct working

eg $g(x)=\sqrt[3]{\left(8x^6\right)}+5$ ***(A1)***

$g(x)=2x^2+5$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

Question 24

(a) $h=1$, $k=-9$ $\left(\text{accept } (x-1)^2-9\right)$ **_A1A1_** **_N2_**

[2 marks]

(b) **METHOD 1**

attempt to substitute $x=0$ into **their** quadratic function **_(M1)_**

eg $f(0)$, $(0-1)^2-9$

$c=-8$ **_A1_** **_N2_**

METHOD 2

attempt to expand **their** quadratic function **_(M1)_**

eg $x^2-2x+1-9$, x^2-2x-8

$c=-8$ **_A1_** **_N2_**

[2 marks]

(c) evidence of correct reflection **_A1_**

eg $-\left((x-1)^2-9\right)$, vertex at $(1,\ 9)$, y-intercept at $(0,\ 8)$

valid attempt to find horizontal shift **_(M1)_**

eg $1+p=3$, $1\to 3$

$p=2$ **_A1_** **_N2_**

valid attempt to find vertical shift **_(M1)_**

eg $9+q=1$, $9\to 1$, $-9+q=1$

$q=-8$ **_A1_** **_N2_**

[5 marks]

(d) valid approach **_M1_**

eg $f(x)=g(x)$, $(x-1)^2-9=-(x-3)^2+1$

correct expansion of both binomials **_(A1)_**

eg x^2-2x+1, x^2-6x+9

correct working **_(A1)_**

eg $x^2-2x-8=-x^2+6x-8$

correct equation **_(A1)_**

eg $2x^2-8x=0$, $2x^2=8x$

correct working **_(A1)_**

eg $2x(x-4)=0$

$x=0$, $x=4$ **_A1A1_** **_N3_**

[7 marks]

Total [16 marks]

Question 25

(a) $g(2) = 8$ **_A1_** **_N1_**

[1 mark]

(b) attempt to form composite (in any order) **_(M1)_**

eg $f(4x)$, $4 \times (8x+3)$

$(f \circ g)(x) = 32x + 3$ **_A1_** **_N2_**

[2 marks]

(c) interchanging x and y (may be seen at any time) **_(M1)_**

eg $x = 8y + 3$

$f^{-1}(x) = \frac{x-3}{8}$ (accept $\frac{x-3}{8}$, $y = \frac{x-3}{8}$) **_A1_** **_N2_**

[2 marks]

Total [5 marks]

Question 26

(a) $h = 3$, $k = -1$ **_A1A1_** **_N2_**

[2 marks]

(b) $a = 2$, $b = 4$ (or $a = 4$, $b = 2$) **_A1A1_** **_N2_**

[2 marks]

(c) attempt to substitute $x = 0$ into their f **_(M1)_**

eg $(0-3)^2 - 1$, $(0-2)(0-4)$

$y = 8$ **_A1_** **_N2_**

[2 marks]

Total [6 marks]

Question 27

(a) attempt to form composite in any order **(M1)**

eg $f\left(g(x)\right)$, $\cos\left(6x\sqrt{1-x^2}\right)$

correct working **(A1)**

eg $6\cos x\sqrt{1-\cos^2 x}$

correct application of Pythagorean identity (do not accept $\sin^2 x+\cos^2 x=1$) **(A1)**

eg $\sin^2 x=1-\cos^2 x$, $6\cos x\sin x$, $6\cos x\left|\sin x\right|$

valid approach (do not accept $2\sin x\cos x=\sin 2x$) **(M1)**

eg $3(2\cos x\sin x)$

$h(x)=3\sin 2x$ **A1** **N3**

[5 marks]

(b) valid approach **(M1)**

eg amplitude $=3$, sketch with max and min y-values labelled, $-3<y<3$

correct range **A1** **N2**

eg $-3\le y\le 3$, $[-3, 3]$, from -3 to 3

Question 28

(a) correct approach **(A1)**

eg $\dfrac{-(-4)}{2}$, $f'(x)=2x-4=0$, $\left(x^2-4x+4\right)+5-4$

$x=2$ (must be an equation) **A1** **N2**

[2 marks]

(b) (i) $h=2$ **A1** **N1**

(ii) **METHOD 1**

valid attempt to find k **(M1)**

eg $f(2)$

correct substitution into **their** function **(A1)**

eg $(2)^2-4(2)+5$

$k=1$ **A1** **N2**

METHOD 2

valid attempt to complete the square **(M1)**

eg x^2-4x+4

correct working **(A1)**

eg $\left(x^2-4x+4\right)-4+5$, $(x-2)^2+1$

$k=1$ **A1** **N2**

[4 marks]

[Total 6 marks]

Question 29

valid approach **(M1)**

eg $f = y$, $m - \frac{1}{x} = x - m$

correct working to eliminate denominator **(A1)**

eg $mx - 1 = x(x - m)$, $mx - 1 = x^2 - mx$

correct quadratic equal to zero **A1**

eg $x^2 - 2mx + 1 = 0$

correct reasoning **R1**

eg for two solutions, $b^2 - 4ac > 0$

correct substitution into the discriminant formula **(A1)**

eg $(-2m)^2 - 4$

correct working **(A1)**

eg $4m^2 > 4$, $m^2 = 1$, sketch of positive parabola on the x-axis

correct interval **A1** **N4**

eg $|m| > 1$, $m < -1$ or $m > 1$

Question 30

(a) interchanging x and y **(M1)**

eg $x = 5y$

$f^{-1}(x) = \frac{x}{5}$ **A1** **N2**

[2 marks]

(b) **METHOD 1**

attempt to substitute 7 into $g(x)$ or $f(x)$ **(M1)**

eg $7^2 + 1$, 5×7

$g(7) = 50$ **(A1)**

$f(50) = 250$ **A1** **N2**

METHOD 2

attempt to form composite function (in any order) **(M1)**

eg $5(x^2 + 1)$, $(5x)^2 + 1$

correct substitution **(A1)**

eg $5 \times (7^2 + 1)$

$(f \circ g)(7) = 250$ **A1** **N2**

[3 marks]

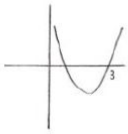
Total [5 marks]

Question 31

(a) **METHOD 1 (using *x*-intercept)**

determining that 3 is an x-intercept **(M1)**

eg $x-3=0$,



valid approach **(M1)**

eg $3-2.5$, $\frac{p+3}{2}=2.5$

$p=2$ **A1** **N2**

METHOD 2 (expanding $f(x)$)

correct expansion (accept absence of a) **(A1)**

eg $ax^2-a(3+p)x+3ap$, $x^2-(3+p)x+3p$

valid approach involving equation of axis of symmetry **(M1)**

eg $\frac{-b}{2a}=2.5$, $\frac{a(3+p)}{2a}=\frac{5}{2}$, $\frac{3+p}{2}=\frac{5}{2}$

$p=2$ **A1** **N2**

METHOD 3 (using derivative)

correct derivative (accept absence of a) **(A1)**

eg $a(2x-3-p)$, $2x-3-p$

valid approach **(M1)**

eg $f'(2.5)=0$

$p=2$ **A1** **N2**

[3 marks]

(b) attempt to substitute $(0,-6)$ **(M1)**

eg $-6=a(0-2)(0-3)$, $0=a(-8)(-9)$, $a(0)^2-5a(0)+6a=-6$

correct working **(A1)**

eg $-6=6a$

$a=-1$ **A1** **N2**

[3 marks]

(c) **METHOD 1 (using discriminant)**

recognizing tangent intersects curve once **(M1)**

recognizing one solution when discriminant $= 0$ **M1**

attempt to set up equation **(M1)**

eg $g = f$, $kx - 5 = -x^2 + 5x - 6$

rearranging their equation to equal zero **(M1)**

eg $x^2 - 5x + kx + 1 = 0$

correct discriminant (if seen explicitly, not just in quadratic formula) **A1**

eg $(k-5)^2 - 4$, $25 - 10k + k^2 - 4$

correct working **(A1)**

eg $k - 5 = \pm 2$, $(k-3)(k-7) = 0$, $\frac{10 \pm \sqrt{100 - 4 \times 21}}{2}$

$k = 3, 7$ **A1A1** **N0**

METHOD 2 (using derivatives)

attempt to set up equation **(M1)**

eg $g = f$, $kx - 5 = -x^2 + 5x - 6$

recognizing derivative/slope are equal **(M1)**

eg $f' = m_T$, $f' = k$

correct derivative of f **(A1)**

eg $-2x + 5$

attempt to set up equation in terms of either x or k **M1**

eg $(-2x+5)x - 5 = -x^2 + 5x - 6$, $k\left(\frac{5-k}{2}\right) - 5 = -\left(\frac{5-k}{2}\right)^2 + 5\left(\frac{5-k}{2}\right) - 6$

rearranging their equation to equal zero **(M1)**

eg $x^2 - 1 = 0$, $k^2 - 10k + 21 = 0$

correct working **(A1)**

eg $x = \pm 1$, $(k-3)(k-7) = 0$, $\frac{10 \pm \sqrt{100 - 4 \times 21}}{2}$

$k = 3, 7$ **A1A1** **N0**

[8 marks]

Question 32

(a) correct range (do not accept $0 \le x \le 7$) ***A1*** ***N1***

eg $[0, 7]$, $0 \le y \le 7$

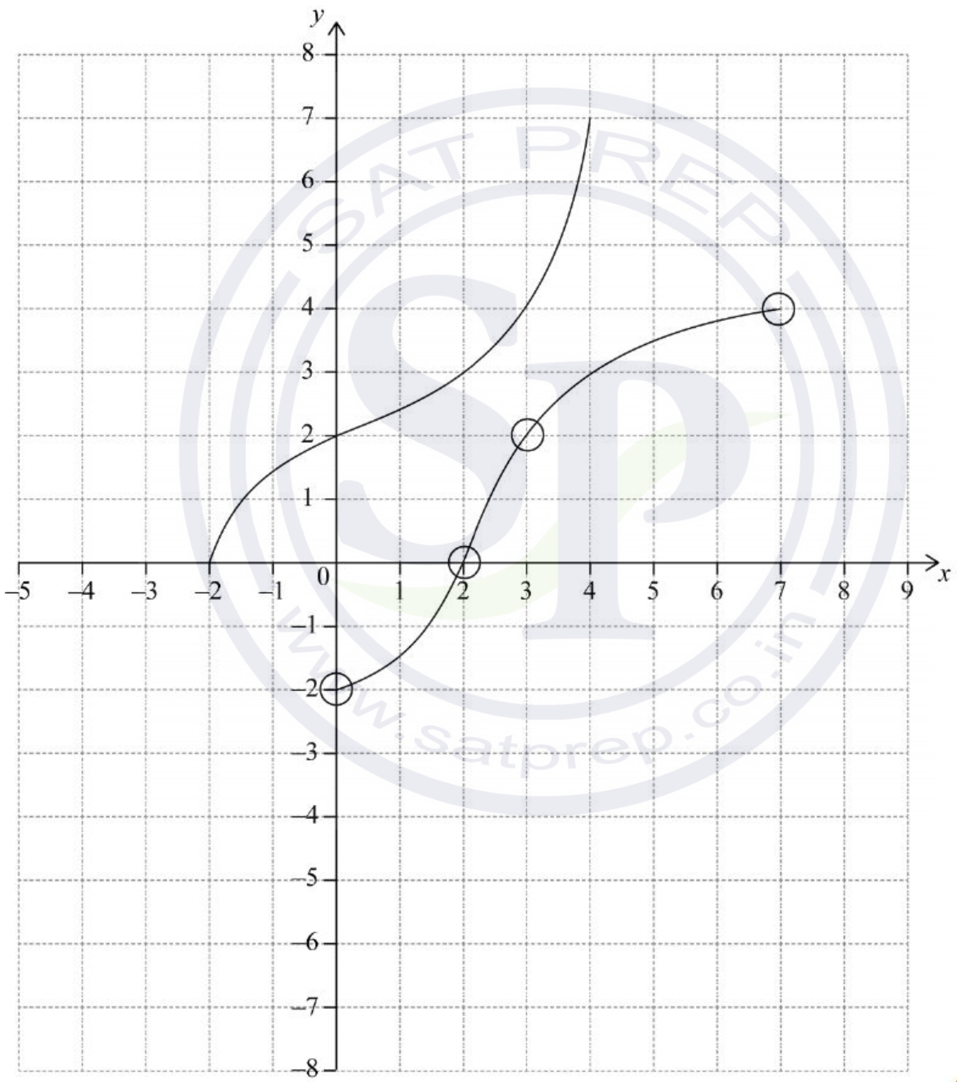
[1 mark]

(b) (i) $f(2) = 3$ ***A1*** ***N1***

(ii) $f^{-1}(2) = 0$ ***A1*** ***N1***

[2 marks]

(c)



A1A1A1 ***N3***

Notes: Award ***A1*** for both end points within circles,
A1 for images of $(2, 3)$ and $(0, 2)$ within circles,
A1 for approximately correct reflection in $y = x$, concave up then concave down shape (do not accept line segments).

[3 marks]

Question 33

(a) attempt to form composite **(M1)**

eg $g(1+e^{-x})$

correct function **A1 N2**

eg $(g \circ f)(x) = 2+b+2e^{-x}$, $2(1+e^{-x})+b$

[2 marks]

(b) evidence of $\lim_{x\to\infty}(2+b+2e^{-x}) = 2+b+\lim_{x\to\infty}(2e^{-x})$ **(M1)**

eg $2+b+2e^{-\infty}$, graph with horizontal asymptote when $x \to \infty$

Note: Award ***M0*** if candidate clearly has incorrect limit, such as $x \to 0$, e^{∞}, $2e^{0}$.

evidence that $e^{-x} \to 0$ (seen anywhere) **(A1)**

eg $\lim_{x\to\infty}(e^{-x}) = 0$, $1+e^{-x} \to 1$, $2(1)+b=-3$, $e^{\text{large negative number}} \to 0$, graph of $y=e^{-x}$ or $y=2e^{-x}$ with asymptote $y=0$, graph of composite function with asymptote $y=-3$

correct working **(A1)**

eg $2+b=-3$

$b=-5$ **A1 N2**

[4 marks]

Total [6 marks]

Question 34

METHOD 1 – using discriminant

correct equation without logs **(A1)**

eg $6x-3x^2=k^2$

valid approach **(M1)**

eg $-3x^2+6x-k^2=0$, $3x^2-6x+k^2=0$

recognizing discriminant must be zero (seen anywhere) **M1**

eg $\Delta=0$

correct discriminant **(A1)**

eg $6^2-4(-3)(-k^2)$, $36-12k^2=0$

correct working **(A1)**

eg $12k^2=36$, $k^2=3$

$k=\sqrt{3}$ **A2** **N2**

METHOD 2 – completing the square

correct equation without logs **(A1)**

eg $6x-3x^2=k^2$

valid approach to complete the square **(M1)**

eg $3\left(x^2-2x+1\right)=-k^2+3$, $x^2-2x+1-1+\frac{k^2}{3}=0$

correct working **(A1)**

eg $3(x-1)^2=-k^2+3$, $(x-1)^2-1+\frac{k^2}{3}=0$

recognizing conditions for one solution **M1**

eg $(x-1)^2=0$, $-1+\frac{k^2}{3}=0$

correct working **(A1)**

eg $\frac{k^2}{3}=1$, $k^2=3$

$k=\sqrt{3}$ **A2** **N2**

Question 35

(a)	$f(1)=3$	*A1*	*N1*
			[1 mark]
(b)	attempt to form the composite (including value)	*(M1)*	
	eg $g(3)$, $g(f(1))$		
	$(g \circ f)(1)=5$	*A1*	*N2*
			[2 marks]
(c)	valid approach	*(M1)*	
	eg $g(x)=-2$		
	$g^{-1}(-2)=1$	*A1*	*N2*
			[2 marks]

Total [5 marks]

Question 36

(a) (i) $f(0) = -\frac{1}{2}$ **A1** **N1**

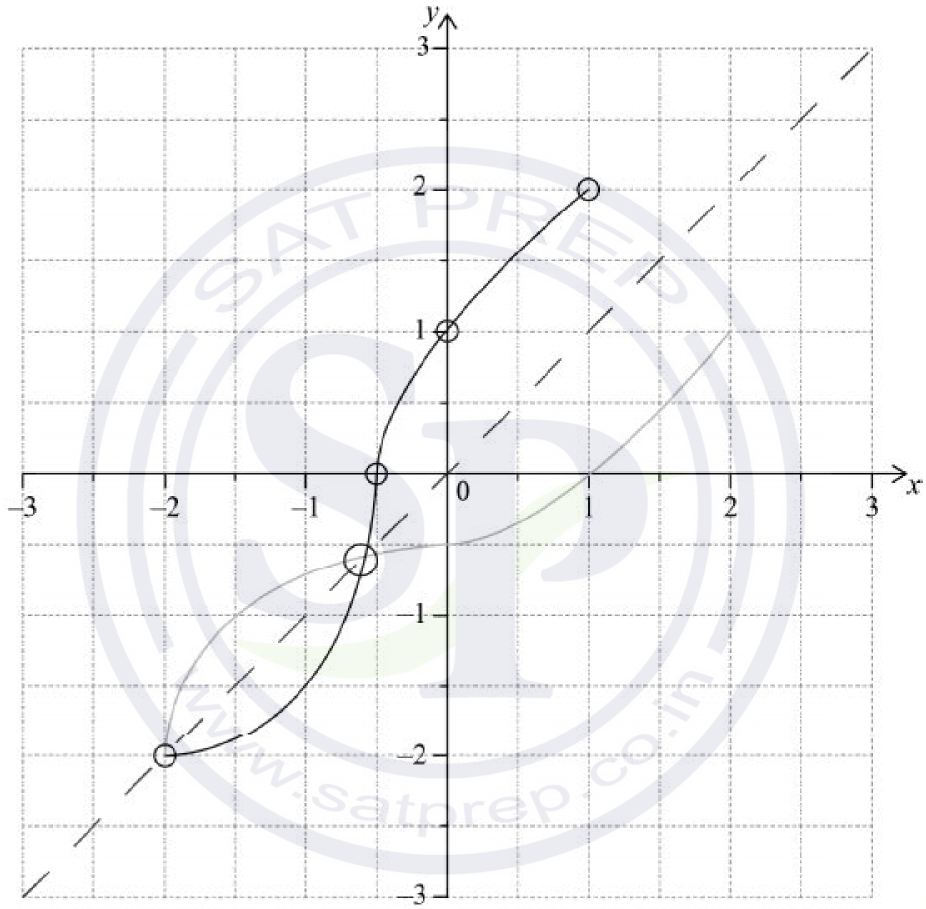
(ii) $f^{-1}(1) = 2$ **A1** **N1**

[2 marks]

(b) $-2 \le y \le 2$, $y \in [-2, 2]$ (accept $-2 \le x \le 2$) **A1** **N1**

[1 mark]

(c)



A1

A1A1A1 **N4**

Note:	Award **A1** for evidence of approximately correct reflection in $y = x$ with correct curvature. ($y = x$ does not need to be explicitly seen) Only if this mark is awarded, award marks as follows: **A1** for both correct invariant points in circles, **A1** for the three other points in circles, **A1** for correct domain.

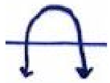
[4 marks]

Total [7 marks]

Question 37

(a) **METHOD 1** (using symmetry to find p)

(i) valid approach **(M1)**

eg $\frac{-1+3}{2}$,  , 

$p=1$ **A1** **N2**

> **Note:** Award no marks if they work backwards by substituting $a=2$ into $-\frac{b}{2a}$ to find p.
>
> Do not accept $p=\frac{2}{a}$.

(ii) valid approach **M1**

eg $-\frac{b}{2a}, \frac{4}{2a}$ (might be seen in (i)), $f'(1)=0$

correct equation **A1**

eg $\frac{4}{2a}=1,\ 2a(1)-4=0$

$a=2$ **AG** **N0**

METHOD 2 (calculating a first)

(i) & (ii) valid approach to calculate a **M1**

eg $a+4-c=a(3^2)-4(3)-c$, $f(-1)=f(3)$

correct working **A1**

eg $8a=16$

$a=2$ **AG** **N0**

valid approach to find p **(M1)**

eg $-\frac{b}{2a}, \frac{4}{2(2)}$

$p=1$ **A1** **N2**

[4 marks]

(b) valid approach **(M1)**

eg $f(-1)=5$, $f(3)=5$

correct working **(A1)**

eg $2+4-c=5$, $18-12-c=5$

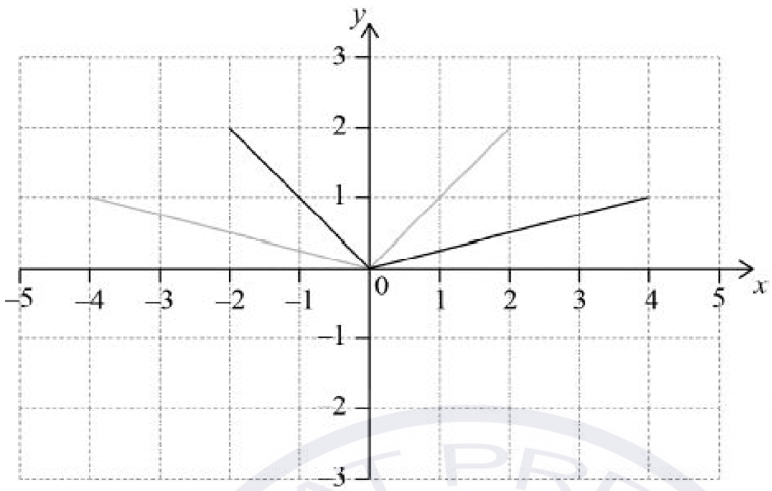
$c=1$ **A1** **N2**

[3 marks]

Total [7 marks]

Question 38

(a)



A2 **N2**
[2 marks]

(b) recognizing horizontal shift/translation of 1 unit **(M1)**
eg $b=1$, moved 1 right

recognizing vertical stretch/dilation with scale factor 2 **(M1)**
eg $a=2$, $y\times(-2)$

$a=-2$, $b=-1$ **A1A1** **N2N2**

[4 marks]

[Total: 6 marks]

Question 39

(a) $f(1)=3$ **A1** **N1**
[1 mark]

(b) attempt to form the composite (including value) **(M1)**
eg $g(3)$, $g\left(f(1)\right)$

$(g\circ f)(1)=5$ **A1** **N2**
[2 marks]

(c) valid approach **(M1)**
eg $g(x)=-2$

$g^{-1}(-2)=1$ **A1** **N2**
[2 marks]

Total [5 marks]

Question 40

(a) (i) $f(0) = -\frac{1}{2}$ **A1** **N1**

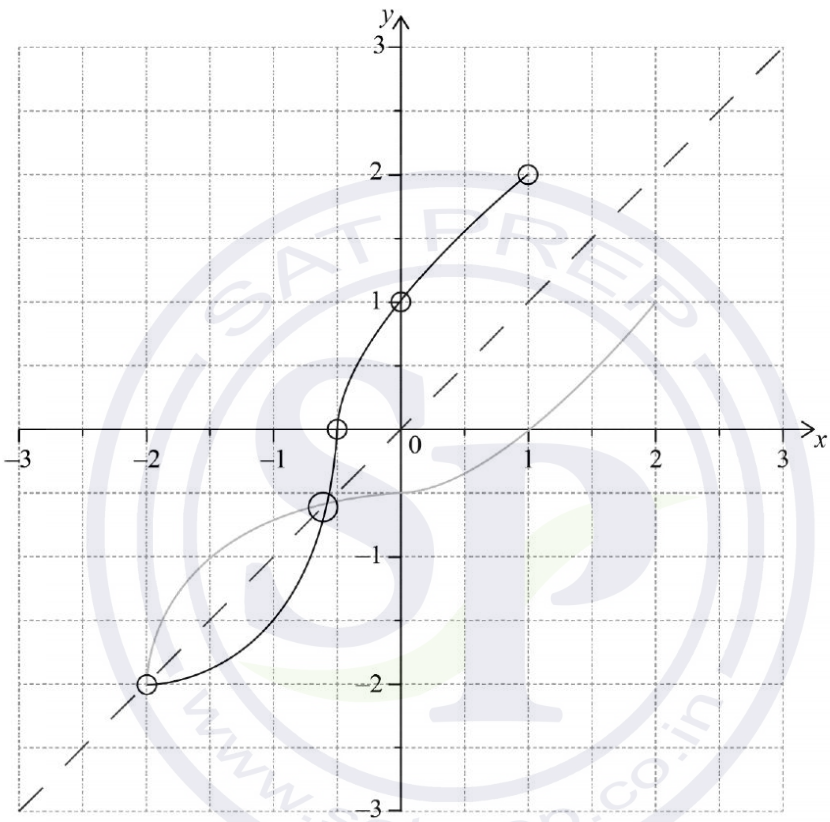
(ii) $f^{-1}(1) = 2$ **A1** **N1**

[2 marks]

(b) $-2 \le y \le 2$, $y \in [-2, 2]$ (accept $-2 \le x \le 2$) **A1** **N1**

[1 mark]

(c)



A1

A1A1A1 **N4**

Note:	Award **A1** for evidence of approximately correct reflection in $y = x$ with correct curvature. ($y = x$ does not need to be explicitly seen) Only if this mark is awarded, award marks as follows: **A1** for both correct invariant points in circles, **A1** for the three other points in circles, **A1** for correct domain.

[4 marks]

Total [7 marks]

Question 41

(a) $f(14)=4$ **A1** **N1**

[1 mark]

(b) attempt to substitute **(M1)**

eg $g(4)$, $3\times 4-7$

5 **A1** **N2**

[2 marks]

(c) interchanging x and y (seen anywhere) **(M1)**

eg $x=3y-7$

evidence of correct manipulation **(A1)**

eg $x+7=3y$

$g^{-1}(x)=\frac{x+7}{3}$ **A1** **N3**

[3 marks]

Total [6 marks]

Question 42

correct substitution into discriminant (do not accept only in quadratic formula) **(A1)**

eg $1-4(1-k)k$

correct expansion of discriminant (do not accept only in quadratic formula) **A1**

eg $1-4k+4k^2$, $4k^2-4k=-1$

recognizing discriminant equals 0 (seen anywhere) **M1**

eg $\Delta=0$, $b^2-4ac=0$

valid attempt to solve **their** quadratic in k **(M1)**

eg factorizing equation, use of quadratic formula, completing the square, recognizing vertex on x-axis

correct working **(A1)**

eg $(2k-1)^2$, $\frac{-(-4)\pm\sqrt{16-4(4)(1)}}{2(4)}$, $\left(k-\frac{1}{2}\right)^2=0$, $k=\frac{-(-4)}{2(4)}$

$k=\frac{1}{2}$ **A1** **N2**

[6 marks]

Question 43

(a) (i) y-intercept is 11 (accept $(0, 11)$) *A1* *N1*

(ii) valid approach *(M1)*

eg $f(4 \times 0) = f(0)$, recognizing stretch of $\frac{1}{4}$ in x-direction

y-intercept is 8 (accept $(0, 8)$) *A1* *N2*

[3 marks]

(b) x-intercept is $\frac{5}{2}$ $(= 2.5)$ (accept $\left(\frac{5}{2}, 0\right)$ or $(2.5, 0)$) *A2* *N2*

[2 marks]

(c) correct name, correct magnitude **and** direction *A1A1* *N2*

eg name: translation, (horizontal) shift (do not accept move)

eg magnitude and direction: 1 unit to the left, $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$, horizontal by -1

[2 marks]

Total [7 marks]

Question 44

(a) (i) $x = 2$ (must be an equation) ***A1*** ***N1***

(ii) valid approach ***(M1)***

eg $3+\frac{7}{x-2}$, $x \to \infty$, $\frac{3x}{x}$, $\frac{3}{1}$, $\frac{3+\frac{1}{x}}{1-\frac{2}{x}}$, $\frac{3(x-2)+7}{x-2}$

$y = 3$ (must be an equation) ***A1*** ***N2***

[3 marks]

(b) **METHOD 1**

attempt to substitute 1 into $g(x)$ or $f(x)$ ***(M1)***

eg 1^2+4, $\frac{3+1}{1-2}$

$g(1) = 5$ ***(A1)***

$(f \circ g)(1) = \frac{16}{3}$ ***A1*** ***N2***

METHOD 2

attempt to form composite function (in any order) ***(M1)***

eg $\frac{3(x^2+4)+1}{x^2+4-2}$, $\left(\frac{3x+1}{x-2}\right)^2+4$

correct substitution ***(A1)***

eg $\frac{3(5)+1}{5-2}$

$(f \circ g)(1) = \frac{16}{3}$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

Question 45

(a) $x = -3$ (must be an equation) **A1** **N1**

[1 mark]

(b) interchanging x and y (seen anywhere) **(M1)**

eg $x = \frac{2y-1}{y+3}$, $x(y+3) = 2y-1$

evidence of correct manipulation **(A1)**

eg $yx + 3x = 2y - 1$, $y(x-2) = -3x - 1$, $2 - \frac{7}{y+3}$

$f^{-1}(x) = \frac{-3x-1}{x-2}$ $\left(= \frac{3x+1}{2-x}, \frac{7}{2-x} - 3\right)$ (accept $y =$) **A1** **N3**

[3 marks]

(c) valid approach to find horizontal asymptote **(M1)**

eg $\frac{-3}{1}$, vert.asymp of f becomes horiz.asymp of f^{-1}, $\frac{-3(x-2)+5}{x-2}$, $x \to \infty$

$y = -3$ (must be an equation) **A1** **N2**

[2 marks]

Total [6 marks]

Question 46

(a) valid attempt to substitute coordinates *(M1)*

eg $g(-1)=8$

correct substitution *(A1)*

eg $(-1)^2+b(-1)+11=8,\ 1-b+11=8$

$b=4$ **A1** **N2**

[3 marks]

(b) valid attempt to solve *(M1)*

eg $\left(x^2+4x+4\right)+7$, $h=\dfrac{-4}{2}$, $k=g(-2)$

correct working **A1**

eg $(x+2)^2+7$, $h=-2$, $k=7$

translation or shift (do not accept move) of vector $\begin{pmatrix}-2\\7\end{pmatrix}$ (accept left by 2 and up by 7)

A1A1 **N2**

[4 marks]

Total [7 marks]