Subject – Math(Higher Level)
Topic - Statistics and Probability
Year - Nov 2011 – Nov 2017
Paper -2

Question -1

(a) $m = \frac{300}{60} = 5$ ***(A1)***

$P(X = 0) = 0.00674$ ***A1***

or e^{-5}

(b) $E(X) = 5 \times 2 = 10$ ***A1***

(c) $P(X > 10) = 1 - P(X \leq 10)$ ***(M1)***

$= 0.417$ ***A1***

[5 marks]

Question-2

(a) $X \sim B(5, 0.1)$ ***(M1)***

$P(X = 2) = 0.0729$ ***A1***

(b) $P(X \geq 1) = 1 - P(X = 0)$ ***(M1)***

$0.9 < 1 - \left(\frac{9}{10}\right)^n$ ***(M1)***

$n > \frac{\ln 0.1}{\ln 0.9}$

$n = 22$ days ***A1***

[5 marks]

Question 3

(a) $X \sim \mathrm{N}(60.33, 1.95^2)$

$\mathrm{P}(X < x) = 0.2 \Rightarrow x = 58.69$ m **(M1)A1**

[2 marks]

(b) $z = -0.8416\ldots$ **(A1)**

$-0.8416 = \dfrac{56.52 - 59.39}{\sigma}$ **(M1)**

$\sigma \approx 3.41$ **A1**

[3 marks]

(c) Jan $X \sim \mathrm{N}(60.33, 1.95^2)$; Sia $X \sim \mathrm{N}(59.50, 3.00^2)$

(i) Jan: $\mathrm{P}(X > 65) \approx 0.00831$ **(M1)A1**

Sia: $\mathrm{P}(Y > 65) \approx 0.0334$ **A1**

Sia is more likely to qualify **R1**

Note: Only award ***R1*** if ***(M1)*** has been awarded.

(ii) Jan: $\mathrm{P}(X \ge 1) = 1 - \mathrm{P}(X = 0)$ **(M1)**

$= 1 - (1 - 0.00831\ldots)^3 \approx 0.0247$ **(M1)A1**

Sia: $\mathrm{P}(Y \ge 1) = 1 - \mathrm{P}(Y = 0) = 1 - (1 - 0.0334\ldots)^3 \approx 0.0968$ **A1**

Note: Accept 0.0240 and 0.0969.

hence, $\mathrm{P}(X \ge 1 \text{ and } Y \ge 1) = 0.0247 \times 0.0968 = 0.00239$ **(M1)A1**

[10 marks]

Total [15 marks]

Question -4

(a) $\binom{10}{6} = 210$ **(M1)A1**

[2 marks]

(b) $2 \times \binom{8}{5} = 112$ **(M1)A1A1**

Note: Accept 210 – 28 – 70 = 112

[3 marks]

(c) $\dfrac{112}{210} \left(= \dfrac{8}{15} = 0.533 \right)$ **(M1)A1**

[2 marks]

Total [7 marks]

Question -5

(a) 50 **A1**

[1 mark]

(b) Lower quartile is 4 so at least 26 obtained a 4 **R1**
Lower bound is 26 **A1**

Minimum is 2 but the rest could be 4 **R1**

So upper bound is 49 **A1**

Note: Do not allow follow through for ***A*** marks.

Note: If answers are incorrect award ***R0A0***; if argument is correct but no clear lower/upper bound is stated award ***R1A0;*** award ***R0A1*** for correct answer without explanation or incorrect explanation.

[4 marks]

Total [5 marks]

Question – 6

$X \sim \text{Po}(m)$

$\text{P}(X=2)=\text{P}(X<2)$ ***(M1)***

$\frac{1}{2}m^2\text{e}^{-m}=\text{e}^{-m}(1+m)$ ***(A1)(A1)***

$m=2.73 \quad \left(1+\sqrt{3}\right)$ ***A1***

in four hours the expected value is 10.9 $\left(4+4\sqrt{3}\right)$ ***A1***

te: Value of m does not need to be rounded.

[5 marks]

Question -7

(a) (i) $X \sim \text{Po}(11)$ ***(M1)***

$\text{P}(X \le 11)=0.579$ ***(M1)A1***

(ii) $\text{P}(X>8 \mid X<12)=$ ***(M1)***

$=\frac{\text{P}(8<X<12)}{\text{P}(X<12)} \left(\text{or } \frac{\text{P}(X\le 11)-\text{P}(X\le 8)}{\text{P}(X\le 11)} \text{ or } \frac{0.3472...}{0.5792...}\right)$ ***A1***

$=0.600$ ***A1*** ***N2***

[6 marks]

(b) (i) $Y \sim \text{Po}(m)$

$\text{P}(Y>3)=0.24$ ***(M1)***

$\text{P}(Y \le 3)=0.76$ ***(M1)***

$\text{e}^{-m}\left(1+m+\frac{1}{2}m^2+\frac{1}{6}m^3\right)=0.76$ ***(A1)***

Note: At most two of the above lines can be implied.

Attempt to solve equation with GDC ***(M1)***
$m=2.49$ ***A1***

(ii) $A \sim \text{Po}(4.98)$

$\text{P}(A > 5) = 1 - \text{P}(A \le 5) = 0.380\ldots$ **M1A1**

$W \sim \text{B}(4, 0.380\ldots)$ ***(M1)***

$\text{P}(W \ge 2) = 1 - \text{P}(W \le 1) = 0.490$ **M1A1**

[10 marks]

(c) $\text{P}(A < 25) = 0.8$, $\text{P}(A < 18) = 0.4$

$\frac{25 - \mu}{\sigma} = 0.8416\ldots$ ***(M1)(A1)***

$\frac{18 - \mu}{\sigma} = -0.2533\ldots$ (or -0.2534 from tables) ***(M1)(A1)***

solving these equations ***(M1)***

$\mu = 19.6$ ***A1***

Note: Accept just 19.6, 19 or 20; award A0 to any other final answer.

[6 marks]

Total [22 marks]

Question -8

(a) $\text{E}(X) = np$

$\Rightarrow 10 = 30p$

$\Rightarrow p = \frac{1}{3}$ *A1*

[1 mark]

(b) $\text{P}(X = 10) = \binom{30}{10}\left(\frac{1}{3}\right)^{10}\left(\frac{2}{3}\right)^{20} = 0.153$ *(M1)A1*

[2 marks]

(c) $\text{P}(X \ge 15) = 1 - \text{P}(X \le 14)$ *(M1)*

$= 1 - 0.9565\ldots = 0.0435$ *A1*

[2 marks]

Total [5 marks]

Question -9

(a) $\mathrm{P}(X=5)=\mathrm{P}(X=3)+\mathrm{P}(X=4)$

$\frac{e^{-m}m^5}{5!}=\frac{e^{-m}m^3}{3!}+\frac{e^{-m}m^4}{4!}$ ***M1(A1)***

$m^2-5m-20=0$

$\Rightarrow m=\frac{5+\sqrt{105}}{2}=(7.62)$ ***A1***

[3 marks]

(b) $\mathrm{P}(X>2)=1-\mathrm{P}(X\le 2)$ ***(M1)***

$=1-0.018\ldots$

$=0.982$ ***A1***

[2 marks]

Total [5 marks]

Question 10

(a) $\int_0^a \frac{1}{1+x^4}\,dx=1$ ***M2***

$a=1.40$ ***A1***

[3 marks]

(b) $\mathrm{E}(X)=\int_0^a \frac{x}{1+x^4}\,dx$ ***M1***

$\left(=\frac{1}{2}\arctan(a^2)\right)$

$=0.548$ ***A1***

[2 marks]

Total [5 marks]

Question 11

(a) (i) $P(X > 225) = 0.158\ldots$ *(M1)(A1)*

expected number $= 450 \times 0.158\ldots = 71.4$ *A1*

(ii) $P(X < m) = 0.7$ *(M1)*

$\Rightarrow m = 213$ (grams) *A1*

[5 marks]

(b) $\dfrac{270-\mu}{\sigma} = 1.40\ldots$ *(M1)A1*

$\dfrac{250-\mu}{\sigma} = -1.03\ldots$ *A1*

Note: These could be seen in graphical form.

solving simultaneously *(M1)*

$\mu = 258,\ \sigma = 8.19$ *A1A1*

[6 marks]

(c)

$X \sim N(80, 4^2)$

$P(X > 82) = 0.3085\ldots$ *A1*

recognition of the use of binomial distribution. *(M1)*

$X \sim B(5, 0.3085\ \ldots)$

$P(X = 3) = 0.140$ *A1*

[3 marks]

.

Total [14 marks]

Question 12

$$\frac{\sum_{i=1}^{15} x_i}{15} = 11.5 \Rightarrow \sum_{i=1}^{15} x_i = 172.5 \qquad \textit{(A1)}$$

$$\text{new mean} = \frac{172.5 - 22.1}{14} \qquad \textit{(M1)}$$

$$= 10.7428\ldots = 10.7 \text{ (3sf)} \qquad \textit{A1}$$

$$\frac{\sum_{i=1}^{15} x_i^2}{15} - 11.5^2 = 9.3 \qquad \textit{(M1)}$$

$$\Rightarrow \sum_{i=1}^{15} x_i^2 = 2123.25$$

$$\text{new variance} = \frac{2123.25 - 22.1^2}{14} - (10.7428\ldots)^2 \qquad \textit{(M1)}$$

$$= 1.37 \text{ (3sf)} \qquad \textit{A1}$$

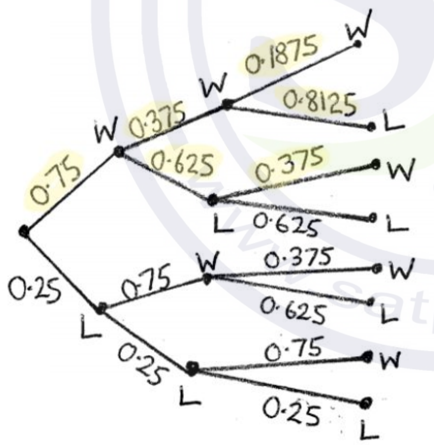
[6 marks]

Question 13

(a) $\text{P(WWW)} = 0.75 \times 0.375 \times 0.1875 = 0.0527 \text{ (3sf)} \left(\frac{3}{4} \times \frac{3}{8} \times \frac{3}{16} = \frac{27}{512}\right)$ ***(M1)A1***

[2 marks]

(b)



(M1)(A1)

Note:	Award ***M1*** for any reasonable attempt to use a tree diagram showing that three games were played (do not award ***M1*** for tree diagrams that only show the first two games) and ***A1*** for the highlighted probabilities.

$$\text{P(wins 2 games} \mid \text{wins first game)} = \frac{\text{P(WWL, WLW)}}{\text{P(wins first game)}} \qquad \textit{(M1)}$$

$$= \frac{0.75 \times 0.375 \times 0.8125 + 0.75 \times 0.625 \times 0.375}{0.75} \qquad \textit{(A1)(A1)}$$

$$= 0.539 \text{ (3sf)} \left(\text{or } \frac{69}{128}\right) \qquad \textit{A1}$$

[4 marks]

Total [6 marks]

Question 14

(a) $2.2 \times 6 \times 60 = 792$ ***(M1)A1***

[2 marks]

(b) $V \sim \text{Po}(2.2 \times 60)$ ***(M1)***

$\text{P}(V > 100) = 0.998$ ***(M1)A1***

[3 marks]

(c) $(0.997801\ldots)^6 = 0.987$ ***(M1)A1***

[2 marks]

(d) $A \sim \text{N}(\mu, \sigma^2)$

$\text{P}(A < 35) = 0.29$ and $\text{P}(A > 55) = 0.23 \Rightarrow \text{P}(A < 55) = 0.77$

$\text{P}\left(Z < \frac{35-\mu}{\sigma}\right) = 0.29$ and $\text{P}\left(Z < \frac{55-\mu}{\sigma}\right) = 0.77$ ***(M1)***

use of inverse normal ***(M1)***

$\frac{35-\mu}{\sigma} = -0.55338\ldots$ and $\frac{55-\mu}{\sigma} = 0.738846\ldots$ ***(A1)***

solving simultaneously ***(M1)***

$\mu = 43.564\ldots$ and $\sigma = 15.477\ldots$ ***A1A1***

$\mu = 43.6$ and $\sigma = 15.5 \text{(3sf)}$

[6 marks]

(e) $0.29n = 100 \Rightarrow n = 344.82\ldots$ ***(M1)(A1)***

$\text{P}(A < 50) = 0.66121\ldots$ ***(A1)***

expected number of visitors under $50 = 228$ ***(M1)A1***

[5 marks]

Total [18 marks]

Question 15

$$\frac{5\times6+6k+7\times3+8\times1+9\times2+10\times1}{13+k} = 6.5 \text{ (or equivalent)}$$ ***(M1)(A1)(A1)***

ote: Award ***(M1)(A1)*** for correct numerator, and ***(A1)*** for correct denominator.

$0.5k = 2.5 \Rightarrow k = 5$ ***A1***

[4 marks]

Question 16

Let X represent the length of time a journey takes on a particular day.

(a) $P(X > 15) = 0.0912112819\ldots = 0.0912$ ***(M1)A1***

(b) Use of correct Binomial distribution ***(M1)***

$N \sim B(5, 0.091\ldots)$

$1 - 0.0912112819\ldots = 0.9087887181\ldots$

$1 - (0.9087887181\ldots)^5 = 0.380109935\ldots = 0.380$ ***(M1)A1***

Note: Allow answers to be given as percentages.

[5 marks]

Question 17

(a) $X \sim \text{Po}(0.25\text{T})$ ***(A1)***

Attempt to solve $\text{P}(X \le 3) = 0.6$ ***(M1)***

$T = 12.8453\ldots = 13$ (minutes) ***A1***

Note: Award ***A1M1A0*** if T found correctly but not stated to the nearest minute.

[3 marks]

(b) let X_1 be the number of cars that arrive during the first interval and X_2 be the number arriving during the second.

X_1 and X_2 are $\text{Po}(2.5)$ ***(A1)***

P (all get on) $= \text{P}(X_1 \le 3) \times \text{P}(X_2 \le 3) + \text{P}(X_1 = 4) \times \text{P}(X_2 \le 2)$

$+ \text{P}(X_1 = 5) \times \text{P}(X_2 \le 1) + \text{P}(X_1 = 6) \times \text{P}(X_2 = 0)$ ***(M1)***

$= 0.573922\ldots + 0.072654\ldots + 0.019192\ldots + 0.002285\ldots$ ***(M1)***

$= 0.668\ (053\ldots)$ ***A1***

[4 marks]

Total [7 marks]

Question 18

(a) $X \sim N(13.5, 9.5)$

$13.5 - \sqrt{9.5} < X < 13.5 + \sqrt{9.5}$ ***(M1)***

$10.4 < X < 16.6$ ***A1***

Note: Accept 6.16.

[2 marks]

(b) $\text{P}(X < 10) = 0.12807\ldots$ ***(M1)(A1)***

estimate is 1281 (correct to the nearest whole number). ***A1***

Note: Accept 1280.

[3 marks]

Total [5 marks]

Question 19

(a) $\int_0^{0.5} ax^2\,dx + \int_{0.5}^{1} 0.5a(1-x)\,dx = 1$ **_M1A1_**

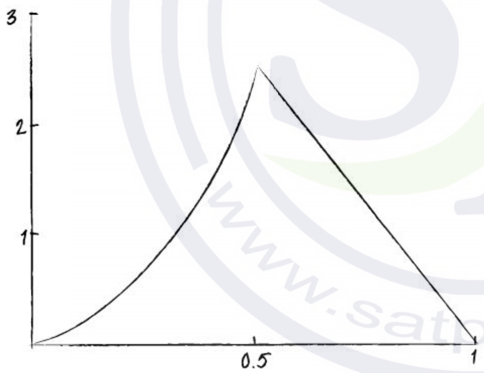
$\frac{5a}{48}$ (or equivalent) or $a \times 0.104... = 1$ **_A1_**

Note:	Award ***M1*** for considering two definite integrals. Award ***A1*** for equating to 1. Award ***A1*** for a correct equation. The ***A1A1*** can be awarded in any order.

$a = 9.6$ **_AG_**

[3 marks]

(b)



correct shape for $0 \le x \le 0.5$ and $f(0.5) \approx 2.4$ **_A1_**

correct shape for $0.5 \le x \le 1$ and $f(1) = 0$ **_A1_**

[2 marks]

(c) attempting to find $P(X < 0.6)$ ***(M1)***

direct GDC use or *eg* $P(0 \le X \le 0.5) + P(0.5 \le X \le 0.6)$ or $1 - P(0.6 \le X \le 1)$

$P(X < 0.6) = 0.616 \left(= \frac{77}{125}\right)$ **_A1_**

[2 marks]

Total [7 marks]

Question 20

(a) $X \sim \text{Po}(1.2)$

$\text{P}(X=3) \times \text{P}(X=0)$ ***(M1)***

$= 0.0867\ldots \times 0.3011\ldots$

$= 0.0261$ ***A1***

[2 marks]

(b) Three requests over two days can occur as $(3, 0), (0, 3), (2, 1)$ or $(1, 2)$. ***R1***

using conditional probability, for example

$\dfrac{\text{P}(3,0)}{\text{P}(3 \text{ requests}, m = 2.4)} = 0.125$ or $\dfrac{\text{P}(2,1)}{\text{P}(3 \text{ requests}, m = 2.4)} = 0.375$ ***M1A1***

expected income is

$2 \times 0.125 \times \text{US\$}120 + 2 \times 0.375 \times \text{US\$}180$ ***M1***

Note: Award ***M1*** for attempting to find the expected income including both $(3, 0)$ and $(2, 1)$ cases.

$= \text{US\$}30 + \text{US\$}135$

$= \text{US\$}165$ ***A1***

[5 marks]

Total [7 marks]

Question 21

$\text{P}\left(Z < \dfrac{780-\mu}{\sigma}\right) = 0.92$ and $\text{P}\left(Z < \dfrac{755-\mu}{\sigma}\right) = 0.12$ ***(M1)***

use of inverse normal ***(M1)***

$\Rightarrow \dfrac{780-\mu}{\sigma} = 1.405\ldots$ and $\dfrac{755-\mu}{\sigma} = -1.174\ldots$ ***(A1)***

solving simultaneously ***(M1)***

Note: Award ***M1*** for attempting to solve an incorrect pair of equations *eg,* inverse normal not used.

$\mu = 766.385$

$\sigma = 9.6897$

$\mu = 12$ hrs 46 mins $(= 766$ mins$)$ ***A1***

$\sigma = 10$ mins ***A1***

Total [6 marks]

Question 22

(a) $\mathrm{P}(F)=\left(\frac{1}{7}\times\frac{7}{9}\right)+\left(\frac{6}{7}\times\frac{4}{9}\right)$ ***(M1)(A1)***

Note: Award ***M1*** for the sum of two products.

$=\frac{31}{63}\ (=0.4920\ldots)$ ***A1***

[3 marks]

(b) Use of $\mathrm{P}(S\mid F)=\frac{\mathrm{P}(S\cap F)}{\mathrm{P}(F)}$ to obtain $\mathrm{P}(S\mid F)=\frac{\frac{1}{7}\times\frac{7}{9}}{\frac{31}{63}}$. ***M1***

Note: Award ***M1*** only if the numerator results from the product of two probabilities.

$=\frac{7}{31}\ (=0.2258\ldots)$ ***A1***

[2 marks]

Total [5 marks]

Question 23

(a) (i) $X \sim \text{Po}(0.6)$

$\text{P}(X = 0) = 0.549 \ \left(= e^{-0.6}\right)$ ***A1***

(ii) $\text{P}(X \geq 3) = 1 - \text{P}(X \leq 2)$ ***(M1)(A1)***

$= 1 - \left(e^{-0.6} + e^{-0.6} \times 0.6 + e^{-0.6} \times \frac{0.6^2}{2}\right)$

$= 0.0231$ ***A1***

(iii) $Y \sim \text{Po}(2.4)$ ***(M1)***

$\text{P}(Y \leq 5) = 0.964$ ***A1***

(iv) $Z \sim \text{B}(12,\ 0.451...)$ ***(M1)(A1)***

Note: Award ***M1*** for recognising binomial and ***A1*** for using correct parameters.

$\text{P}(Z = 4) = 0.169$ ***A1***

[9 marks]

(b) (i) $k\int_1^3 \ln x \, dx = 1$ ***(M1)***

$(k \times 1.2958... = 1)$

$k = 0.771702$ ***A1***

(ii) $\text{E}(X) = \int_1^3 kx \ln x \, dx$ ***(A1)***

attempting to evaluate their integral ***(M1)***

$= 2.27$ ***A1***

(iii) $x = 3$ ***A1***

(iv) $\int_1^m k \ln x \, dx = 0.5$ ***(M1)***

$k[x \ln x - x]_1^m = 0.5$

attempting to solve for m ***(M1)***

$m = 2.34$ ***A1***

[9 marks]

Total [18 marks]

Question 24

$X \sim \mathrm{N}(100, \sigma^2)$

$\mathrm{P}(X < 124) = 0.68$ ***(M1)(A1)***

$\frac{24}{\sigma} = 0.4676....$ ***(M1)***

$\sigma = 51.315...$ ***(A1)***

variance $= 2630$ ***A1***

[5 marks]

Notes: Accept use of $\mathrm{P}(X < 124.5) = 0.68$ leading to variance $= 2744$.

Question 25

(a) $\left(A\binom{6}{5}2^5 B + 3\binom{6}{4}2^4 B^2 \right)x^5$ ***M1A1A1***

$= \left(192AB + 720B^2\right)x^5$ ***A1***

[4 marks]

(b) **METHOD 1**

$x = \frac{1}{6},\ A = \frac{3}{6}\left(= \frac{1}{2}\right),\ B = \frac{4}{6}\left(= \frac{2}{3}\right)$ ***A1A1A1***

probability is $\frac{4}{81}$ $(= 0.0494)$ ***A1***

METHOD 2

$\mathrm{P}(5 \text{ eaten}) = \mathrm{P}(\text{M eats } 1)\ \mathrm{P}(\text{N eats } 4) + \mathrm{P}(\text{M eats } 0)\ \mathrm{P}(\text{N eats } 5)$ ***(M1)***

$= \frac{1}{2}\binom{6}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2 + \frac{1}{2}\binom{6}{5}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)$ ***(A1)(A1)***

$= \frac{4}{81}$ $(= 0.0494)$ ***A1***

[4 marks]

Total [8 marks]

Question 26

(a) mean for week is 40.88 ***(A1)***

$P(S > 40) = 1 - P(S \le 40) = 0.513$ ***A1***

[2 marks]

(b) $\frac{\text{probability there were more than 10 on Monday AND more than 40 over the week}}{\text{probability there were more than 10 on Monday}}$ ***M1***

possibilities for the numerator are:

there were more than 40 birds on the power line on Monday ***R1***
11 on Monday and more than 29 over the course of the next 6 days ***R1***
12 on Monday and more than 28 over the course of the next 6 days … until
40 on Monday and more than 0 over the course of the next 6 days ***R1***
hence if X is the number on the power line on Monday and Y, the number on the power line Tuesday – Sunday then the numerator is ***M1***

$$P(X > 40) + P(X = 11) \times P(Y > 29) + P(X = 12) \times P(Y > 28) + \ldots$$

$$+ P(X = 40) \times P(Y > 0)$$

$$= P(X > 40) + \sum_{r=11}^{40} P(X = r) P(Y > 40 - r)$$

hence solution is $\dfrac{P(X > 40) + \sum_{r=11}^{40} P(X = r) P(Y > 40 - r)}{P(X > 10)}$ ***AG***

[5 marks]

Total [7 marks]

Question 27

(a) $\int_2^3 (ax+b)\,dx\,(=1)$ **M1A1**

$\left[\frac{1}{2}ax^2+bx\right]_2^3 (=1)$ **A1**

$\frac{5}{2}a+b=1$ **M1**

$5a+2b=2$ **AG**

[4 marks]

(b) (i) $\int_2^3 \left(ax^2+bx\right)dx\,(=\mu)$ **M1A1**

$\left[\frac{1}{3}ax^3+\frac{1}{2}bx^2\right]_2^3 (=\mu)$ **A1**

$\frac{19}{3}a+\frac{5}{2}b=\mu$ **A1**

eliminating b **M1**

eg

$\frac{19}{3}a+\frac{5}{2}\left(1-\frac{5}{2}a\right)=\mu$ **A1**

$\frac{1}{12}a+\frac{5}{2}=\mu$

$a=12\mu-30$ **AG**

Note: Elimination of b could be at different stages.

(ii) $b=1-\frac{5}{2}(12\mu-30)$

$=76-30\mu$ **A1**

Note: This solution may be seen in part (i).

[7 marks]

(c) (i) $\int_2^{2.3} (ax+b)\,dx\,(=0.5)$ **(M1)(A1)**

$\left[\frac{1}{2}ax^2+bx\right]_2^{2.3} (=0.5)$

$0.645a+0.3b\,(=0.5)$ **(A1)**

$0.645(12\mu-30)+0.3(76-30\mu)=0.5$ **M1**

$\mu=2.34\left(=\frac{295}{126}\right)$ **A1**

(ii) $\mathrm{E}\left(X^2\right)=\int_2^3 x^2(ax+b)\,\mathrm{d}x$ ***(M1)***

$a=12\mu-30=-\frac{40}{21},\ b=76-30\mu=\frac{121}{21}$ ***(A1)***

$\mathrm{E}\left(X^2\right)=\int_2^3 x^2\left(-\frac{40}{21}x+\frac{121}{21}\right)\mathrm{d}x=5.539\ldots\left(=\frac{349}{63}\right)$ ***(A1)***

$\mathrm{Var}(X)=5.539\mathrm{K}-(2.341\mathrm{K})^2=0.05813\ldots$ ***(M1)***

$\sigma=0.241$ ***A1***

[10 marks]

Total [21 marks]

Question 28

(a) (i) $0.6^3\times0.4^3$ ***(M1)***

Note: Award ***(M1)*** for use of the product of probabilities.

$=0.0138$ ***A1***

(ii) binomial distribution $X:\ \mathrm{B}(6,\ 0.6)$ ***(M1)***

Note: Award ***(M1)*** for recognizing the binomial distribution.

$\mathrm{P}(X=3)={}^6C_3\,(0.6)^3\,(0.4)^3$

$=0.276$ ***A1***

Note: Award ***(M1)A1*** for ${}^6C_3\times0.0138=0.276$.

(b) $Y:\ \mathrm{B}(n,\ 0.4)$

$\mathrm{P}(Y\geq1)>0.995$

$1-\mathrm{P}(Y=0)>0.995$

$\mathrm{P}(Y=0)<0.005$ ***(M1)***

Note: Award ***(M1)*** for any of the last three lines. Accept equalities.

$0.6^n<0.005$ ***(M1)***

Note: Award ***(M1)*** for attempting to solve $0.6^n<0.005$ using any method, *eg*, logs, graphically, use of solver. Accept an equality.

$n>10.4$

$\therefore n=11$ ***A1***

[3 marks]

Total [7 marks]

Question 29

(a) $\frac{\mu^2 e^{-\mu}}{2!} + \frac{\mu^3 e^{-\mu}}{3!} = \frac{\mu^5 e^{-\mu}}{5!}$ ***(M1)***

$\frac{\mu^2}{2} + \frac{\mu^3}{6} - \frac{\mu^5}{120} = 0$

$\mu = 5.55$ ***A1***

[2 marks]

(b) $\sigma = \sqrt{5.55...} = 2.35598...$ ***(M1)***

$P(3.19 \le X \le 7.9)$

$P(4 \le X \le 7)$

$= 0.607$ ***A1***

[2 marks]

Total [4 marks]

Question 30

(a) $a\int_0^{\frac{\pi}{2}} x\cos x \, dx = 1$ *(M1)*

integrating by parts:

$u = x \quad v' = \cos x$

$u' = 1 \quad v = \sin x$ *M1*

$\int x\cos x \, dx = x\sin x + \cos x$ *A1*

$[x\sin x + \cos x]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 1$ *A1*

$a = \frac{1}{\frac{\pi}{2} - 1}$ *A1*

$= \frac{2}{\pi - 2}$ *AG*

[5 marks]

(b) $\mathrm{P}\left(X < \frac{\pi}{4}\right) = \frac{2}{\pi - 2}\int_0^{\frac{\pi}{4}} x\cos x \, dx = 0.460$ *(M1)A1*

Note: Accept $\frac{2}{\pi - 2}\left(= \frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2} - 1\right)$ or equivalent

[2 marks]

(c) (i) mode = 0.860 *A1*

(x-value of a maximum on the graph over the given domain)

(ii) $\frac{2}{\pi - 2}\int_0^{m} x\cos x \, dx = 0.5$ *(M1)*

$\int_0^{m} x\cos x \, dx = \frac{\pi - 2}{4}$

$m\sin m + \cos m - 1 = \frac{\pi - 2}{4}$ *(M1)*

median $= 0.826$ *A1*

Note: Do not accept answers containing additional solutions.

[4 marks]

(d) $\mathrm{P}\left(X < \frac{\pi}{8} \mid X < \frac{\pi}{4}\right) = \frac{\mathrm{P}\left(X < \frac{\pi}{8}\right)}{\mathrm{P}\left(X < \frac{\pi}{4}\right)}$ *M1*

$= \frac{0.129912}{0.459826}$

$= 0.283$ *A1*

[2 marks]

Total [13 marks]

Question 31

(a) $\mathrm{P}(X > x) = 0.99$ $(= \mathrm{P}(X < x) = 0.01)$ ***(M1)***

$\Rightarrow x = 54.6\,(\text{cm})$ ***A1***

[2 marks]

(b) $\mathrm{P}(60.15 \le X \le 60.25)$ ***(M1)(A1)***

$= 0.0166$ ***A1***

[3 marks]

Total [5 marks]

Question 32

use of $\mu = \dfrac{\sum_{i=1}^{k} f_i x_i}{n}$ to obtain $\dfrac{2+x+y+10+17}{5} = 8$ ***(M1)***

$x + y = 11$ ***A1***

EITHER

use of $\sigma^2 = \dfrac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n}$ to obtain $\dfrac{(-6)^2 + (x-8)^2 + (y-8)^2 + 2^2 + 9^2}{5} = 27.6$ ***(M1)***

$(x-8)^2 + (y-8)^2 = 17$ ***A1***

OR

use of $\sigma^2 = \dfrac{\sum_{i=1}^{k} f_i x_i^2}{n} - \mu^2$ to obtain $\dfrac{2^2 + x^2 + y^2 + 10^2 + 17^2}{5} - 8^2 = 27.6$ ***(M1)***

$x^2 + y^2 = 65$ ***A1***

THEN

attempting to solve the two equations ***(M1)***

$x = 4$ and $y = 7$ (only as $x < y$) ***A1*** ***N4***

Note: Award ***A0*** for $x = 7$ and $y = 4$.

Note: Award ***(M1)A1(M0)A0(M1)A1*** for $x + y = 11 \Rightarrow x = 4$ and $y = 7$.

Total [6 marks]

Question 33

(a) (i) $\mathrm{P}(X=0)=0.549\left(=\mathrm{e}^{-0.6}\right)$ *A1*

(ii) $\mathrm{P}(X \geq 3)=1-\mathrm{P}(X \leq 2)$ *(M1)*

$\mathrm{P}(X \geq 3)=0.0231$ *A1*

[3 marks]

(b) **EITHER**

using $Y \sim \mathrm{Po}(3)$ *(M1)*

OR

using $(0.549)^5$ *(M1)*

THEN

$\mathrm{P}(Y=0)=0.0498\left(=\mathrm{e}^{-3}\right)$ *A1*

[2 marks]

(c) $\mathrm{P}(X=0)$ (most likely number of complaints received is zero) *A1*

EITHER

calculating $\mathrm{P}(X=0)=0.549$ and $\mathrm{P}(X=1)=0.329$ *M1A1*

OR

sketching an appropriate (discrete) graph of $\mathrm{P}(X=x)$ against x *M1A1*

OR

finding $\mathrm{P}(X=0)=e^{-0.6}$ and stating that $\mathrm{P}(X=0)>0.5$ *M1A1*

OR

using $\mathrm{P}(X=x)=\mathrm{P}(X=x-1)\times\frac{\mu}{x}$ where $\mu<1$ *M1A1*

[3 marks]

(d) $\mathrm{P}(X=0)=0.8(\Rightarrow e^{-\lambda}=0.8)$ *(A1)*

$\lambda=0.223\left(=\ln\frac{5}{4},=-\ln\frac{4}{5}\right)$ *A1*

[2 marks]

Total [10 marks]

Question 34

(a) $\mathrm{P}(\text{Ava wins on her first turn}) = \frac{1}{3}$ ***A1***

[1 mark]

(b) $\mathrm{P}(\text{Barry wins on his first turn}) = \left(\frac{2}{3}\right)^2$ ***(M1)***

$= \frac{4}{9} (= 0.444)$ ***A1***

[2 marks]

(c) $\mathrm{P}(\text{Ava wins in one of her first three turns})$

$= \frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\frac{1}{3}$ ***M1A1A1***

> **Note:** Award ***M1*** for adding probabilities, award ***A1*** for a correct second term and award ***A1*** for a correct third term.
> Accept a correctly labelled tree diagram, awarding marks as above.

$= \frac{103}{243} (= 0.424)$ ***A1***

[4 marks]

(d) $\mathrm{P}(\text{Ava eventually wins}) = \frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\frac{1}{3} + \ldots$ ***(A1)***

using $S_\infty = \frac{a}{1-r}$ with $a = \frac{1}{3}$ and $r = \frac{2}{9}$ ***(M1)(A1)***

> **Note:** Award ***(M1)*** for using $S_\infty = \frac{a}{1-r}$ and award ***(A1)*** for $a = \frac{1}{3}$ and $r = \frac{2}{9}$.

$= \frac{3}{7} (= 0.429)$ ***A1***

[4 marks]

Total [11 marks]

Question 35

(a) $X \sim N\left(210, 22^2\right)$

$\mathrm{P}(X < 180) = 0.0863$ ***(M1)A1***

[2 marks]

(b) $\mathrm{P}(X < T) = 0.9 \Rightarrow T = 238$ (mins) ***(M1)A1***

[2 marks]

Total [4 marks]

Question 36

(a) $W \sim B(1000,\ 0.1)$ (accept $C_k^{1000}(0.1)^k(0.9)^{1000-k}$) ***A1A1***

Note: First ***A1*** is for recognizing the binomial, second ***A1*** for both parameters if stated explicitly in this part of the question.

[2 marks]

(b) $\mu(=1000\times 0.1)=100$ ***A1***

[1 mark]

(c) $\mathrm{P}(W>89)=\mathrm{P}(W\ge 90)\ \left(=1-\mathrm{P}(W\le 89)\right)$ ***(M1)***

$=0.867$ ***A1***

Notes: Award ***M0A0*** for 0.889

[2 marks]

Total [5 marks]

Question 37

(a) $2\dfrac{\mathrm{e}^{-m}m^4}{4!}=\dfrac{\mathrm{e}^{-m}m^5}{5!}$ ***M1A1***

$\dfrac{2}{4!}=\dfrac{m}{5!}$ or other simplification ***M1***

Note: accept a labelled graph showing clearly the solution to the equation. Do not accept simple verification that $m=10$ is a solution.

$\Rightarrow m=10$ ***AG***

[3 marks]

(b) $\mathrm{P}(X=6\mid X\le 11)=\dfrac{\mathrm{P}(X=6)}{\mathrm{P}(X\le 11)}$ ***(M1) (A1)***

$=\dfrac{0.063055\ldots}{0.696776\ldots}$ ***(A1)***

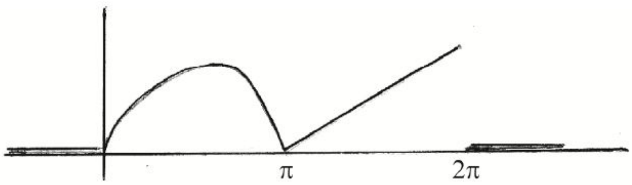
$=0.0905$ ***A1***

[4 marks]

Total [7 marks]

Question 38

(a)



Award ***A1*** for sine curve from 0 to π, award ***A1*** for straight line from π to 2π ***A1A1***

[2 marks]

(b) $\int_0^{\pi} \frac{\sin x}{4} \mathrm{d}x = \frac{1}{2}$ ***(M1)A1***

[2 marks]

(c) **METHOD 1**

require $\frac{1}{2} + \int_{\pi}^{2\pi} a(x-\pi)\,\mathrm{d}x = 1$ ***(M1)***

$\Rightarrow \frac{1}{2} + a\left[\frac{(x-\pi)^2}{2}\right]_{\pi}^{2\pi} = 1$ (or $\frac{1}{2} + a\left[\frac{x^2}{2} - \pi x\right]_{\pi}^{2\pi} = 1$) ***A1***

$\Rightarrow a\frac{\pi^2}{2} = \frac{1}{2}$ ***A1***

$\Rightarrow a = \frac{1}{\pi^2}$ ***AG***

Note: Must obtain the exact value. Do not accept answers obtained with calculator.

METHOD 2

0.5 + area of triangle $= 1$ ***R1***

area of triangle $= \frac{1}{2}\pi \times a\pi = 0.5$ ***M1A1***

Note: Award ***M1*** for correct use of area formula $= 0.5$, ***A1*** for $a\pi$.

$a = \frac{1}{\pi^2}$ ***AG***

[3 marks]

(d) median is π ***A1***

[1 mark]

(e) $\mu = \int_0^{\pi} x \cdot \frac{\sin x}{4} \mathrm{d}x + \int_{\pi}^{2\pi} x \cdot \frac{x-\pi}{\pi^2} \mathrm{d}x$ ***(M1)(A1)***

$= 3.40339... = 3.40$ (or $\frac{\pi}{4} + \frac{5\pi}{6} = \frac{13}{12}\pi$) ***A1***

[3 marks]

(f) For $\mu = 3.40339...$

EITHER

$\sigma^2 = \int_0^{\pi} x^2 \cdot \frac{\sin x}{4} \mathrm{d}x + \int_{\pi}^{2\pi} x^2 \cdot \frac{x-\pi}{\pi^2} \mathrm{d}x - \mu^2$ ***(M1)(A1)***

OR

$\sigma^2 = \int_0^{\pi} (x-\mu)^2 \cdot \frac{\sin x}{4} \mathrm{d}x + \int_{\pi}^{2\pi} (x-\mu)^2 \cdot \frac{x-\pi}{\pi^2} \mathrm{d}x$ ***(M1)(A1)***

THEN

$= 3.866277... = 3.87$ ***A1***

[3 marks]

(g) $\int_{\frac{\pi}{2}}^{\pi} \frac{\sin x}{4} \mathrm{d}x + \int_{\pi}^{\frac{3\pi}{2}} \frac{x-\pi}{\pi^2} \mathrm{d}x = 0.375$ (or $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$) ***(M1)A1***

[2 marks]

(h) $\mathrm{P}\left(\pi \le X \le 2\pi \middle| \frac{\pi}{2} \le X \le \frac{3\pi}{2}\right) = \frac{\mathrm{P}\left(\pi \le X \le \frac{3\pi}{2}\right)}{\mathrm{P}\left(\frac{\pi}{2} \le X \le \frac{3\pi}{2}\right)}$ ***(M1)(A1)***

$= \frac{\int_{\pi}^{\frac{3\pi}{2}} \frac{(x-\pi)}{\pi^2} \mathrm{d}x}{0.375} = \frac{0.125}{0.375}$ (or $= \frac{\frac{1}{8}}{\frac{3}{8}}$ from diagram areas) ***(M1)***

$= \frac{1}{3}$ (0.333) ***A1***

[4 marks]

Total [20 marks]

Question 39

(a) (i) $X \sim Po(5)$

$\mathrm{P}(X \ge 8) = 0.133$ ***(M1)A1***

(ii) $7 \times 0.133...$ ***M1***

≈ 0.934 days ***A1***

Note: Accept "1 day".

[4 marks]

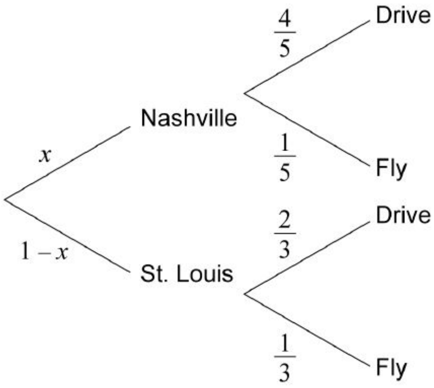
(b) $7 \times 5 = 35$ $(Y \sim Po(35))$ ***(A1)***

$\mathrm{P}(Y \le 29) = 0.177$ ***(M1)A1***

[3 marks]

Total [7 marks]

Question 40



(a) attempt to set up the problem using a tree diagram and/or an equation, with the unknown x ***M1***

$$\frac{4}{5}x+\frac{2}{3}(1-x)=\frac{13}{18}$$ ***A1***

$$\frac{4x}{5}-\frac{2x}{3}=\frac{13}{18}-\frac{2}{3}$$

$$\frac{2x}{15}=\frac{1}{18}$$

$$x=\frac{5}{12}$$ ***A1***

[3 marks]

(b) attempt to set up the problem using conditional probability ***M1***

EITHER

$$\frac{\frac{5}{12}\times\frac{1}{5}}{1-\frac{13}{18}}$$ ***A1***

OR

$$\frac{\frac{5}{12}\times\frac{1}{5}}{\frac{1}{12}+\frac{7}{36}}$$ ***A1***

THEN

$$=\frac{3}{10}$$ ***A1***

Question 41

(a) (i) $P(110 < X < 130) = = 0.49969... = 0.500 = 50.0\%$ ***(M1)A1***

> **Note:** Accept 50

> **Note:** Award ***M1A0*** for 0.50 (0.500)

(ii) $P(X > 130) = (1 - 0.707\ldots) = 0.293\ldots$ ***M1***

expected number of turnips $= 29.3$ ***A1***

> **Note:** Accept 29.

(iii) no of turnips weighing more than 130 is $Y \sim B(100,\ 0.293)$ ***M1***

$P(Y \geq 30) = 0.478$ ***A1***

[6 marks]

(b) (i) $X \sim N\left(144, \sigma^2\right)$

$P(X \leq 130) = \frac{1}{15} = 0.0667$ ***(M1)***

$P\left(Z \leq \frac{130 - 144}{\sigma}\right) = 0.0667$

$\frac{14}{\sigma} = 1.501$ ***(A1)***

$\sigma = 9.33\,g$ ***A1***

(ii) $P(X > 150 \mid X > 130) = \frac{P(X > 150)}{P(X > 130)}$ ***M1***

$= \frac{0.26008\ldots}{1 - 0.06667} = 0.279$ ***A1***

expected number of turnips $= 55.7$ ***A1***

[6 marks]

Total [12 marks]

Question 42

(a) $0.818 = 0.65 + 0.48 - \mathrm{P}(A \cap B)$ ***(M1)***

$\mathrm{P}(A \cap B) = 0.312$ ***A1***

[2 marks]

(b) $\mathrm{P}(A)\ \mathrm{P}(B) = 0.312\left(= 0.48 \times 0.65\right)$ ***A1***

since $\mathrm{P}(A)\ \mathrm{P}(B) = \mathrm{P}(A \cap B)$ then A and B are independent ***R1***

> **e:** Only award the ***R1*** if numerical values are seen. Award ***A1R1*** for a correct conditional probability approach.

[2 marks]

Total [4 marks]

Question 43

(a) $\dfrac{0 \cdot 4 + 1 \cdot k + 2 \cdot 3 + 3 \cdot 2 + 4 \cdot 3 + 8 \cdot 1}{13 + k} = 1.95 \left(\dfrac{k + 32}{k + 13} = 1.95 \right)$ ***(M1)***

attempting to solve for k ***(M1)***

$k = 7$ ***A1***

[3 marks]

(b) (i) $\dfrac{7 + 32 + 22}{7 + 13 + 1} = 2.90 \left(= \dfrac{61}{21} \right)$ ***(M1)A1***

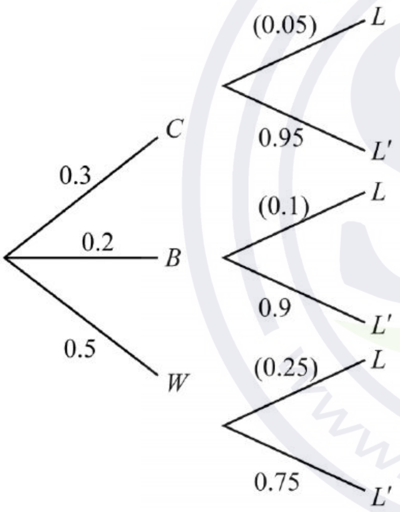
(ii) standard deviation $= 4.66$ ***A1***

: Award ***A0*** for 4.77.

[3 marks]

Total [6 marks]

Question 44



M1A1A1

e: Award ***M1*** for a two-level tree diagram, ***A1*** for correct first level probabilities, and ***A1*** for correct second level probabilities.

OR

$\mathrm{P}(B \mid L') = \dfrac{\mathrm{P}(L' \mid B)\,\mathrm{P}(B)}{\mathrm{P}(L' \mid B)\,\mathrm{P}(B) + \mathrm{P}(L' \mid C)\,\mathrm{P}(C) + \mathrm{P}(L' \mid W)\,\mathrm{P}(W)} \left(= \dfrac{\mathrm{P}(B \cap L')}{\mathrm{P}(L')} \right)$ ***(M1)(A1)(A1)***

THEN

$\mathrm{P}(B \mid L') = \dfrac{0.9 \times 0.2}{0.9 \times 0.2 + 0.95 \times 0.3 + 0.75 \times 0.5} \left(= \dfrac{0.18}{0.84} \right)$ ***M1A1***

$= 0.214 \left(= \dfrac{3}{14} \right)$ ***A1***

[6 marks]

Question 45

(a) $A\int_1^5 \sin(\ln x)\,dx = 1$ **(M1)**

$A = 0.323$ (3 dp only) **A1**

[2 marks]

(b) either a graphical approach or $f'(x) = \dfrac{\cos(\ln x)}{x} = 0$ **(M1)**

$x = 4.81 \left(= e^{\frac{\pi}{2}} \right)$ **A1**

Note: Do not award ***A1FT*** for a candidate working in degrees.

[2 marks]

(c) $P(X \le 3 \mid X \ge 2) = \dfrac{P(2 \le X \le 3)}{P(X \ge 2)} \left(= \dfrac{\int_2^3 \sin(\ln(x))\,dx}{\int_2^5 \sin(\ln(x))\,dx} \right)$ **(M1)**

$= 0.288$ **A1**

Note: Do not award ***A1FT*** for a candidate working in degrees.

[2 marks]

Question 46

(a) (i) let W be the weight of a worker and $W \sim N\left(\mu, \sigma^2\right)$

$P\left(Z < \dfrac{62 - \mu}{\sigma}\right) = 0.3$ and $P\left(Z < \dfrac{98 - \mu}{\sigma}\right) = 0.75$ **(M1)**

$\dfrac{62 - \mu}{\sigma} = \Phi^{-1}(0.3)\ (= -0.524\ldots)$ and

$\dfrac{98 - \mu}{\sigma} = \Phi^{-1}(0.75)\ (= 0.674\ldots)$

or linear equivalents **A1A1**

(ii) attempting to solve simultaneously **(M1)**

$\mu = 77.7,\ \sigma = 30.0$ **A1A1**

[6 marks]

(b) $P(W > 100) = 0.229$ **A1**

[1 mark]

(c) let X represent the number of workers over $100\,\text{kg}$ in a lift of ten passengers

$X \sim B(10,\ 0.229\ldots)$ **(M1)**

$P(X \ge 4) = 0.178$ **A1**

[2 marks]

(d) $P(X < 4 | X \geq 1) = \frac{P(1 \leq X \leq 3)}{P(X \geq 1)}$ ***M1(A1)***

Note: Award the ***M1*** for a clear indication of conditional probability.

$= 0.808$ ***A1***

[3 marks]

(e) $L \sim Po(50)$ ***(M1)***

$P(L > 60) = 1 - P(L \leq 60)$ ***(M1)***

$= 0.0722$ ***A1***

[3 marks]

(f) 400 workers require at least 40 elevators ***(A1)***

$P(L \geq 40) = 1 - P(L \leq 39)$ ***(M1)***

$= 0.935$ ***A1***

[3 marks]

Question 47

let the heights of the students be X

$P(X < 1.62) = 0.4$, $P(X > 1.79) = 0.25$ ***M1***

Note: Award ***M1*** for either of the probabilities above.

$P\left(Z < \frac{1.62 - \mu}{\sigma}\right) = 0.4$, $P\left(Z < \frac{1.79 - \mu}{\sigma}\right) = 0.75$ ***M1***

Note: Award ***M1*** for either of the expressions above.

$\frac{1.62 - \mu}{\sigma} = -0.2533\ldots$, $\frac{1.79 - \mu}{\sigma} = 0.6744\ldots$ ***M1A1***

Note: ***A1*** for both values correct.

$\mu = 1.67(\text{m})$, $\sigma = 0.183(\text{m})$ ***A1A1***

Note: Accept answers that round to 1.7(m) and 0.18(m).

Note: Accept answers in centimetres.

[6 marks]

Question 48

P(3 in the first hour) $= \frac{\lambda^3 e^{-\lambda}}{3!}$ **A1**

number to arrive in the four hours follows $Po(4\lambda)$ **M1**

P(5 arrive in total) $= \frac{(4\lambda)^5 e^{-4\lambda}}{5!}$ **A1**

attempt to find P(2 arrive in the next three hours) **M1**

$= \frac{(3\lambda)^2 e^{-3\lambda}}{2!}$ **A1**

use of conditional probability formula **M1**

P(3 in the first hour given 5 in total) $= \frac{\frac{\lambda^3 e^{-\lambda}}{3!} \times \frac{(3\lambda)^2 e^{-3\lambda}}{2!}}{\frac{(4\lambda)^5 e^{-4\lambda}}{5!}}$ **A1**

$\frac{\left(\frac{9}{2!3!}\right)}{\left(\frac{4^5}{5!}\right)} = \frac{45}{512} = 0.0879$ **A1**

[8 marks]

Question 49

(a) $E(X) = 1 \times \frac{1}{6} + 2 \times \frac{2}{6} + 3 \times \frac{3}{6} = \frac{14}{6}\left(= \frac{7}{3} = 2.33\right)$ **(M1)A1**

[2 marks]

(b) (i) $3 \times P(113) + 3 \times P(122)$ **(M1)**

$3 \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{2} + 3 \times \frac{1}{6} \times \frac{1}{3} \times \frac{1}{3} = \frac{7}{72} (= 0.0972)$ **A1**

Note: Award ***M1*** for attempt to find at least four of the cases.

(ii) recognising 111 as a possibility (implied by $\frac{1}{216}$) **(M1)**

recognising 112 and 113 as possibilities (implied by $\frac{2}{216}$ and $\frac{3}{216}$) **(M1)**

seeing the three arrangements of 112 and 113 **(M1)**

$P(111) + 3 \times P(112) + 3 \times P(113)$

$= \frac{1}{216} + \frac{6}{216} + \frac{9}{216} = \frac{16}{216}\left(= \frac{2}{27} = 0.0741\right)$ **A1**

[6 marks]

(c) let the number of twos be X, $X \sim B\left(10, \frac{1}{3}\right)$ *(M1)*

$\mathrm{P}(X < 4) = \mathrm{P}(X \le 3) = 0.559$ *(M1)A1*

[3 marks]

(d) let n be the number of balls drawn

$\mathrm{P}(X \ge 1) = 1 - \mathrm{P}(X = 0)$ *M1*

$= 1 - \left(\frac{2}{3}\right)^n > 0.95$ *M1*

$\left(\frac{2}{3}\right)^n < 0.05$

$n = 8$ *A1*

[3 marks]

(e) $8p_1 = 4.8 \Rightarrow p_1 = \frac{3}{5}$ *(M1)A1*

$8p_2(1 - p_2) = 1.5$ *(M1)*

$p_2^2 - p_2 - 0.1875 = 0$ *(M1)*

$p_2 = \frac{1}{4}$ $\left(\text{or } \frac{3}{4}\right)$ *A1*

reject $\frac{3}{4}$ as it gives a total greater than one

$\mathrm{P}(1 \text{ or } 2) = \frac{17}{20}$ or $\mathrm{P}(3) = \frac{3}{20}$ *(A1)*

recognising LCM as 20 so min total number is 20 *(M1)*

the least possible number of 3's is 3 *A1*

[8 marks]

Total [22 marks]

Question 50

(a) $\mathrm{P}(0 \le X \le 2) = 0.242$ *(M1)A1*

[2 marks]

(b) **METHOD 1**

$\mathrm{P}(|X| > 1) = \mathrm{P}(X < -1) + \mathrm{P}(X > 1)$ *(M1)*

$= 0.02275\ldots + 0.84134\ldots$ *(A1)*

$= 0.864$ *A1*

METHOD 2

$\mathrm{P}(|X| > 1) = 1 - \mathrm{P}(-1 < X < 1)$ *(M1)*

$= 1 - 0.13590\ldots$ *(A1)*

$= 0.864$ *A1*

[3 marks]

(c) $c = 3.30$ *(M1)A1*

[2 marks]

Total [7 marks]

Question 51

(a) $X \sim \text{Po}(0.5)$ *(A1)*

$\text{P}(X \geq 1) = 0.393 \ \left(=1-\text{e}^{-0.5}\right)$ *(M1)A1*

[3 marks]

(b) $\text{P}(X = 0) = 0.607\ldots$ *(A1)*

$\text{E}(P) = (0.607\ldots \times 5) - (0.393\ldots \times 3)$ *(M1)*

the expected profit is $\$1.85$ per glass sheet ***A1***

[3 marks]

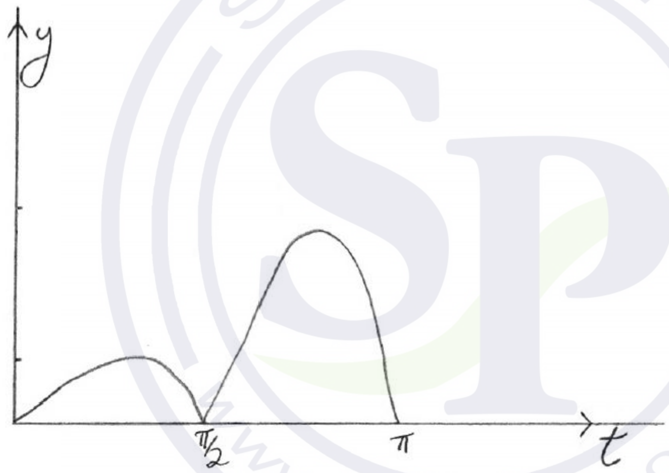
(c) $Y \sim \text{Po}(2)$ *(M1)*

$\text{P}(Y = 0) = 0.135 \ \left(=\text{e}^{-2}\right)$ ***A1***

[2 marks]

Total [8 marks]

Question 52



(a) two enclosed regions ($0 \leq t \leq \dfrac{\pi}{2}$ and $\dfrac{\pi}{2} \leq t \leq \pi$) bounded by the curve and the t-axis ***A1***

correct non-symmetrical shape for $0 \leq t \leq \dfrac{\pi}{2}$ and

$\dfrac{\pi}{2} <$ mode of $T < \pi$ clearly apparent ***A1***

[2 marks]

(b) $\text{mode} = 2.46$ ***A1***

[1 mark]

(c) $\text{E}(T) = \dfrac{1}{\pi}\displaystyle\int_0^{\pi} t^2 \left|\sin 2t\right| \text{d}t$ *(M1)*

$= 2.04$ ***A1***

[2 marks]

(d) **EITHER**

$$\text{Var}(T) = \int_0^{\pi} (t - 2.03788\ldots)^2 \left(\frac{t|\sin 2t|}{\pi}\right) \text{d}t$$ ***(M1)(A1)***

OR

$$\text{Var}(T) = \int_0^{\pi} t^2 \left(\frac{t|\sin 2t|}{\pi}\right) \text{d}t - (2.03788\ldots)^2$$ ***(M1)(A1)***

THEN

$\text{Var}(T) = 0.516$ ***A1***

[3 marks]

(e) $\frac{1}{\pi}\int_{2.03788\ldots}^{2.456590\ldots} t|\sin 2t| \,\text{d}t = 0.285$ ***(M1)A1***

[2 marks]

(f) (i) attempting integration by parts ***(M1)***

($u = t$, $\text{d}u = \text{d}t$, $\text{d}v = \sin 2t \,\text{d}t$ and $v = -\frac{1}{2}\cos 2t$)

$$\frac{1}{\pi}\left[t\left(-\frac{1}{2}\cos 2t\right)\right]_0^T - \frac{1}{\pi}\int_0^T \left(-\frac{1}{2}\cos 2t\right) \text{d}t$$ ***A1***

Note: Award ***A1*** if the limits are not included.

$$= \frac{\sin 2T}{4\pi} - \frac{T\cos 2T}{2\pi}$$ ***A1***

(ii) $\frac{\sin \pi}{4\pi} - \frac{\frac{\pi}{2}\cos \pi}{2\pi} = \frac{1}{4}$ ***A1***

as $\text{P}\left(0 \le T \le \frac{\pi}{2}\right) = \frac{1}{4}$ (or equivalent), then the lower quartile of T is $\frac{\pi}{2}$ ***R1AG***

[5 marks]

Total [15 marks]

Question 53

(a) $\mathrm{E}\left(X^2\right) = \sum x^2 \cdot \mathrm{P}(X = x) = 10.37 \ (= 10.4 \text{ 3 sf})$ ***(M1)A1***

[2 marks]

(b) **METHOD 1**

$\mathrm{sd}(X) = 1.44069\ldots$ ***(M1)(A1)***

$\mathrm{Var}(X) = 2.08 \ (= 2.0756)$ ***A1***

METHOD 2

$\mathrm{E}(X) = 2.88 \ (= 0.06 + 0.27 + 0.5 + 0.98 + 0.63 + 0.44)$ ***(A1)***

use of $\mathrm{Var}(X) = \mathrm{E}\left(X^2\right) - \left(\mathrm{E}(X)\right)^2$ ***(M1)***

Note: Award ***(M1)*** only if $\left(\mathrm{E}(X)\right)^2$ is used correctly.

$\left(\mathrm{Var}(X) = 10.37 - 8.29\right)$

$\mathrm{Var}(X) = 2.08 \ (= 2.0756)$ ***A1***

Note: Accept 2.11.

METHOD 3

$\mathrm{E}(X) = 2.88 \ (= 0.06 + 0.27 + 0.5 + 0.98 + 0.63 + 0.44)$ ***(A1)***

use of $\mathrm{Var}(X) = \mathrm{E}\left(\left(X - \mathrm{E}(X)\right)^2\right)$ ***(M1)***

$\left(0.679728 + \ldots + 0.549152\right)$

$\mathrm{Var}(X) = 2.08 \ (= 2.0756)$ ***A1***

[3 marks]

Total [5 marks]

Question 54

(a) **METHOD 1**

$$P(X = x + 1) = \frac{\mu^{x+1}}{(x+1)!}e^{-\mu}$$ ***A1***

$$= \frac{\mu}{x+1} \times \frac{\mu^{x}}{x!}e^{-\mu}$$ ***M1A1***

$$= \frac{\mu}{x+1} \times P(X = x)$$ ***AG***

METHOD 2

$$\frac{\mu}{x+1} \times P(X = x) = \frac{\mu}{x+1} \times \frac{\mu^{x}}{x!}e^{-\mu}$$ ***A1***

$$= \frac{\mu^{x+1}}{(x+1)!}e^{-\mu}$$ ***M1A1***

$$= P(X = x + 1)$$ ***AG***

METHOD 3

$$\frac{P(X = x + 1)}{P(X = x)} = \frac{\frac{\mu^{x+1}}{(x+1)!}e^{-\mu}}{\frac{\mu^{x}}{x!}e^{-\mu}}$$ ***(M1)***

$$= \frac{\mu^{x+1}}{\mu^{x}} \times \frac{x!}{(x+1)!}$$ ***A1***

$$= \frac{\mu}{x+1}$$ ***A1***

and so $P(X = x + 1) = \frac{\mu}{x+1} \times P(X = x)$ ***AG***

[3 marks]

(b) $P(X = 3) = \frac{\mu}{3} \cdot P(X = 2)$ $\left(0.112777 = \frac{\mu}{3} \cdot 0.241667\right)$ ***A1***

attempting to solve for μ ***(M1)***

$\mu = 1.40$ ***A1***

[3 marks]

Total [6 marks]

Question 55

(a) $\mathrm{P}(X < 42.52) = 0.6940$ **(M1)**

either $\mathrm{P}\left(Z < \frac{30.31-\mu}{\sigma}\right) = 0.1180$ or $\mathrm{P}\left(Z < \frac{42.52-\mu}{\sigma}\right) = 0.6940$ **(M1)**

$\frac{30.31-\mu}{\sigma} = \underbrace{\Phi^{-1}(0.1180)}_{-1.1850\ldots}$ **(A1)**

$\frac{42.52-\mu}{\sigma} = \underbrace{\Phi^{-1}(0.6940)}_{0.5072\ldots}$ **(A1)**

attempting to solve simultaneously **(M1)**

$\mu = 38.9$ and $\sigma = 7.22$ **A1**

[6 marks]

(b) $\mathrm{P}(\mu - 1.2\sigma < X < \mu + 1.2\sigma)$ (or equivalent *eg.* $2\mathrm{P}(\mu < X < \mu + 1.2\sigma)$) **(M1)**

$= 0.770$ **A1**

Note: Award ***(M1)A1*** for $\mathrm{P}(-1.2 < Z < 1.2) = 0.770$.

[2 marks]

Total [8 marks]

Question 56

(a) $\mathrm{P}(X = 3) = (0.1)^3$ **A1**

$= 0.001$ **AG**

$\mathrm{P}(X = 4) = \mathrm{P}\left(VV\bar{V}V\right) + \mathrm{P}\left(V\bar{V}VV\right) + \mathrm{P}\left(\bar{V}VVV\right)$ **(M1)**

$= 3 \times (0.1)^3 \times 0.9$ (or equivalent) **A1**

$= 0.0027$ **AG**

[3 marks]

(b) **METHOD 1**

attempting to form equations in a and b **M1**

$\frac{9+3a+b}{2000} = \frac{1}{1000}$ $(3a + b = -7)$ **A1**

$\frac{16+4a+b}{2000} \times \frac{9}{10} = \frac{27}{10000}$ $(4a + b = -10)$ **A1**

attempting to solve simultaneously **(M1)**

$a = -3,\ b = 2$ **A1**

METHOD 2

$\mathrm{P}(X = n) = \binom{n-1}{2} \times 0.1^3 \times 0.9^{n-3}$ **M1**

$= \frac{(n-1)(n-2)}{2000} \times 0.9^{n-3}$ ***(M1)A1***

$= \frac{n^2 - 3n + 2}{2000} \times 0.9^{n-3}$ **A1**

$a = -3,\ b = 2$ **A1**

Note: Condone the absence of 0.9^{n-3} in the determination of the values of a and b.

[5 marks]

(c) **METHOD 1**

EITHER

$$\mathrm{P}(X = n) = \frac{n^2 - 3n + 2}{2000} \times 0.9^{n-3}$$ ***(M1)***

OR

$$\mathrm{P}(X = n) = \binom{n-1}{2} \times 0.1^3 \times 0.9^{n-3}$$ ***(M1)***

THEN

$$= \frac{(n-1)(n-2)}{2000} \times 0.9^{n-3}$$ ***A1***

$$\mathrm{P}(X = n-1) = \frac{(n-2)(n-3)}{2000} \times 0.9^{n-4}$$ ***A1***

$$\frac{\mathrm{P}(X = n)}{\mathrm{P}(X = n-1)} = \frac{(n-1)(n-2)}{(n-2)(n-3)} \times 0.9$$ ***A1***

$$= \frac{0.9(n-1)}{n-3}$$ ***AG***

METHOD 2

$$\frac{\mathrm{P}(X = n)}{\mathrm{P}(X = n-1)} = \frac{\frac{n^2 - 3n + 2}{2000} \times 0.9^{n-3}}{\frac{(n-1)^2 - 3(n-1) + 2}{2000} \times 0.9^{n-4}}$$ ***(M1)***

$$= \frac{0.9\left(n^2 - 3n + 2\right)}{\left(n^2 - 5n + 6\right)}$$ ***A1A1***

Note: Award ***A1*** for a correct numerator and ***A1*** for a correct denominator.

$$= \frac{0.9(n-1)(n-2)}{(n-2)(n-3)}$$ ***A1***

$$= \frac{0.9(n-1)}{n-3}$$ ***AG***

[4 marks]

(d) (i) attempting to solve $\frac{0.9(n-1)}{n-3} = 1$ for n ***M1***

$n = 21$ ***A1***

$\frac{0.9(n-1)}{n-3} < 1 \Rightarrow n > 21$ ***R1***

$\frac{0.9(n-1)}{n-3} > 1 \Rightarrow n < 21$ ***R1***

X has two modes ***AG***

Note: Award ***R1R1*** for a clearly labelled graphical representation of the two inequalities (using $\frac{\mathrm{P}(X = n)}{\mathrm{P}(X = n-1)}$).

(ii) the modes are 20 and 21 ***A1***

[5 marks]

(e) **METHOD 1**

$Y \sim \mathrm{B}(x, 0.1)$ ***(A1)***

attempting to solve $\mathrm{P}(Y \geq 3) > 0.5$ (or equivalent *eg* $1 - \mathrm{P}(Y \leq 2) > 0.5$) for x ***(M1)***

Note: Award ***(M1)*** for attempting to solve an equality (obtaining $x = 26.4$).

$x = 27$ ***A1***

METHOD 2

$\sum_{n=0}^{x} \mathrm{P}(X = n) > 0.5$ ***(A1)***

attempting to solve for x ***(M1)***

$x = 27$ ***A1***

[3 marks]

Total [20 marks]

Question 57

(a) use of $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***M1***

$0.5 = k + 3k - k^2$ ***A1***

$k^2 - 4k + 0.5 = 0$

$k = 0.129$ ***A1***

Note: Do not award the final ***A1*** if two solutions are given.

[3 marks]

(b) use of $\mathrm{P}(A' \cap B) = \mathrm{P}(B) - \mathrm{P}(A \cap B)$ or alternative ***(M1)***

$\mathrm{P}(A' \cap B) = 3k - k^2$ ***(A1)***

$= 0.371$ ***A1***

[3 marks]

Total [6 marks]

Question 58

(a) $\lambda = 4 \times 0.5$ ***(M1)***

$\lambda = 2$ ***(A1)***

$\mathrm{P}(X \leq 2) = 0.677$ ***A1***

[3 marks]

(b) $Y \sim B(10, 0.677)$ ***(M1)(A1)***

$\mathrm{P}(Y = 7) = 0.263$ ***A1***

Note: Award ***M1*** for clear recognition of binomial distribution.

[3 marks]

Total [6 marks]

Question 59

(a) $T \sim N\left(196, 24^2\right)$

$P(T < 180) = 0.252$ ***(M1)A1***

[2 marks]

(b) $P\left(T < T_1\right) = 0.05$ ***(M1)***

$T_1 = 157$ ***A1***

[2 marks]

(c) $F \sim N\left(210, \sigma^2\right)$

$P(F < 235) = 0.79$ ***(M1)***

$\frac{235 - 210}{\sigma} = 0.806421$ or equivalent ***(M1)(A1)***

$\sigma = 31.0$ ***A1***

[4 marks]

Total [8 marks]

Question 60

(a) $P(5 \text{ or more}) = \frac{29}{75}(= 0.387)$ ***(M1)A1***

[2 marks]

(b) mean score $= \frac{2\times3 + 3\times15 + 4\times28 + 5\times17 + 6\times9 + 7\times3}{75}$ ***(M1)***

$= \frac{323}{75}(= 4.31)$ ***A1***

[2 marks]

Total [4 marks]

Question 61

(a) $P(X < 250) = 0.0228$ ***(M1)A1***

[2 marks]

(b) $\frac{250 - \mu}{1.5} = -2.878\ldots$ ***(M1)(A1)***

$\Rightarrow \mu = 254.32$ ***A1***

> **Notes:** Only award ***A1*** here if the correct 2dp answer is seen.
> Award ***M0*** for use of 1.5^2.

[3 marks]

(c) $\frac{250 - 253}{\sigma} = -2.878\ldots$ ***(A1)***

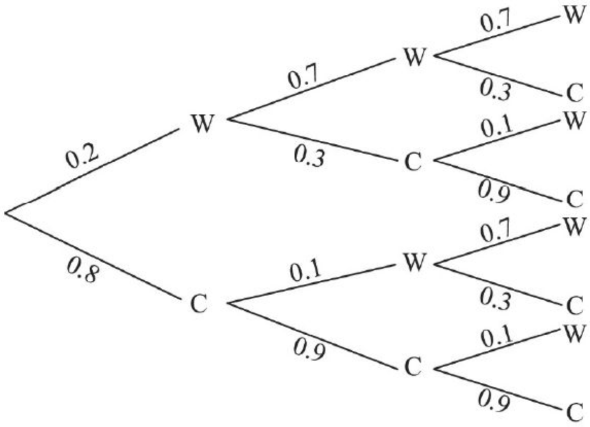
$\Rightarrow \sigma = 1.04$ ***A1***

[2 marks]

Total [7 marks]

Question 62

(a)



M1A2

Note: Award ***M1*** for 3 stage tree-diagram, ***A2*** for 0.8,0.9,0.3 probabilities correctly placed.

[3 marks]

(b) $0.2 \times 0.7 \times 0.3 + 0.2 \times 0.3 \times 0.9 + 0.8 \times 0.1 \times 0.3 + 0.8 \times 0.9 \times 0.9 = 0.768$

(M1)A1

[2 marks]

(c) $\text{P(1st July is calm | 3rd July is windy)} = \dfrac{\text{P(1st July is calm and 3rd July is windy)}}{\text{P(3rd July is windy)}}$

(M1)

$= \dfrac{0.8 \times 0.1 \times 0.7 + 0.8 \times 0.9 \times 0.1}{1 - 0.768}$

OR $\dfrac{0.8 \times 0.1 \times 0.7 + 0.8 \times 0.9 \times 0.1}{0.2 \times 0.7 \times 0.7 + 0.2 \times 0.3 \times 0.1 + 0.8 \times 0.1 \times 0.7 + 0.8 \times 0.9 \times 0.1}$

OR $\dfrac{0.128}{0.232}$ ***(A1)(A1)***

Note: Award ***A1*** for correct numerator, ***A1*** for correct denominator.

$= 0.552$ ***A1***

[4 marks]

Total [9 marks]

Question 64

(a) $\int_0^4 \left(\frac{x^2}{a}+b\right)dx = 1 \Rightarrow \left[\frac{x^3}{3a}+bx\right]_0^4 = 1 \Rightarrow \frac{64}{3a}+4b=1$ ***M1A1***

$\int_2^4 \left(\frac{x^2}{a}+b\right)dx = 0.75 \Rightarrow \frac{56}{3a}+2b=0.75$ ***M1A1***

Note: $\int_0^2 \left(\frac{x^2}{a}+b\right)dx = 0.25 \Rightarrow \frac{8}{3a}+2b=0.25$ could be seen/used in place of either of the above equations.

evidence of an attempt to solve simultaneously
(or check given a, b values are consistent) ***M1***

$a=32,\ b=\frac{1}{12}$ ***AG***

[5 marks]

(b) $E(X)=\int_0^4 x\left(\frac{x^2}{32}+\frac{1}{12}\right)dx$ ***(M1)***

$E(X)=\frac{8}{3}(=2.67)$ ***A1***

[2 marks]

(c) $E(X^2)=\int_0^4 x^2\left(\frac{x^2}{32}+\frac{1}{12}\right)dx$ ***(M1)***

$\text{Var(X)}=E(X^2)-[E(X)]^2=\frac{16}{15}(=1.07)$ ***A1***

[2 marks]

(d) $\int_0^m \left(\frac{x^2}{32}+\frac{1}{12}\right)dx = 0.5$ ***(M1)***

$\frac{m^3}{96}+\frac{m}{12}=0.5(\Rightarrow m^3+8m-48=0)$ ***(A1)***

$m=2.91$ ***A1***

[3 marks]

(e) $Y \sim B(8, 0.75)$ ***(M1)***

$E(Y)=8\times 0.75=6$ ***A1***

[2 marks]

(f) $P(Y \geq 3) = 0.996$ ***A1***

[1 mark]

Total [15 marks]

Question 64

(a) $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

$\Rightarrow 0.75 = \dfrac{0.6}{\mathrm{P}(B)}$ *(M1)*

$\Rightarrow \mathrm{P}(B) \left(= \dfrac{0.6}{0.75} \right) = 0.8$ *A1*

[2 marks]

(b) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\Rightarrow 0.95 = \mathrm{P}(A) + 0.8 - 0.6$ *(M1)*

$\Rightarrow \mathrm{P}(A) = 0.75$ *A1*

[2 marks]

(c) **METHOD 1**

$\mathrm{P}(A' \mid B) = \dfrac{\mathrm{P}(A' \cap B)}{\mathrm{P}(B)} = \dfrac{0.2}{0.8} = 0.25$ *A1*

$\mathrm{P}(A' \mid B) = \mathrm{P}(A')$ *R1*

hence A' and B are independent *AG*

Question 65

let X be the random variable "amount of caffeine content in coffee"

$\mathrm{P}(X > 120) = 0.2$, $\mathrm{P}(X > 110) = 0.6$ *(M1)*

$(\Rightarrow \mathrm{P}(X < 120) = 0.8$, $\mathrm{P}(X < 110) = 0.4)$

ote: Award ***M1*** for at least one correct probability statement.

$\dfrac{120 - \mu}{\sigma} = 0.84162\ldots$, $\dfrac{110 - \mu}{\sigma} = -0.253347\ldots$ *(M1)(A1)(A1)*

ote: Award ***M1*** for attempt to find at least one appropriate z-value.

$120 - \mu = 0.84162\sigma$, $110 - \mu = -0.253347\sigma$

attempt to solve simultaneous equations *(M1)*

$\mu = 112$, $\sigma = 9.13$ *A1*

[6 marks]

Question 66

(a) let X be the number of bananas eaten in one day

$X \sim \text{Po}(0.2)$

$\text{P}(X \geq 1) = 1 - \text{P}(X = 0)$ *(M1)*

$= 0.181\left(= 1 - e^{-0.2}\right)$ ***A1***

[2 marks]

(b) **EITHER**

let Y be the number of bananas eaten in one week

$Y \sim \text{Po}(1.4)$ *(A1)*

$\text{P}(Y = 0) = 0.246596\ldots\left(= e^{-1.4}\right)$ *(A1)*

OR

let Z be the number of days in one week at least one banana is eaten

$Z \sim \text{B}(7, 0.181\ldots)$ *(A1)*

$\text{P}(Z = 0) = 0.246596\ldots$ *(A1)*

THEN

$52 \times 0.246596\ldots$ *(M1)*

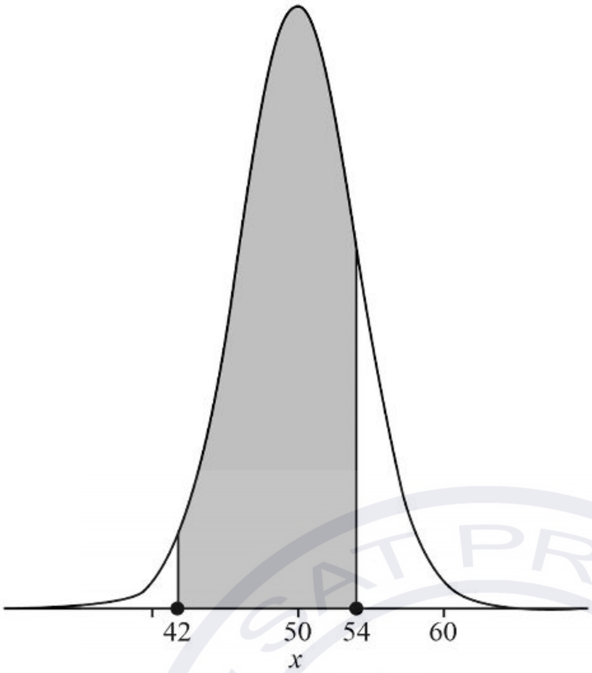
$= 12.8\left(= 52e^{-1.4}\right)$ ***A1***

[4 marks]

Total [6 marks]

Question 67

(a)



normal curve centred on 50 *A1*

vertical lines at $x = 42$ and $x = 54$, with shading in between *A1*

[2 marks]

(b) $\mathrm{P}(42 < X < 54)$ $\left(= \mathrm{P}(-2 < Z < 1)\right)$ *(M1)*

$= 0.819$ *A1*

[2 marks]

(c) $\mathrm{P}(\mu - k\sigma < X < \mu + k\sigma) = 0.5 \Rightarrow \mathrm{P}(X < \mu + k\sigma) = 0.75$ *(M1)*

$k = 0.674$ *A1*

Note: Award ***M1A0*** for $k = -0.674$.

[2 marks]

Total [6 marks]

Question 68

(a) $np = 3.5$ *(A1)*

$p \leq 1 \Rightarrow$ least $n = 4$ *A1*

[2 marks]

(b) $(1-p)^n + np(1-p)^{n-1} = 0.09478$ *M1A1*

attempt to solve above equation with $np = 3.5$ *(M1)*

$n = 12,\ p = \frac{7}{24}\ (= 0.292)$ *A1A1*

Note: Do not accept n as a decimal.

[5 marks]

Total [7 marks]

Question 69

(a) (i) $X \sim \text{Po}(5.3)$

$P(X = 4) = e^{-5.3}\dfrac{5.3^4}{4!}$ *(M1)*

$= 0.164$ ***A1***

(ii) **METHOD 1**

listing probabilities (table or graph) ***M1***

mode $X = 5$ (with probability 0.174) ***A1***

Note: Award ***M0A0*** for 5 (taxis) or mode = 5 with no justification.

METHOD 2

mode is the integer part of mean ***R1***

$E(X) = 5.3 \Rightarrow \text{mode} = 5$ ***A1***

Note: Do not allow ***R0A1***.

(iii) attempt at conditional probability *(M1)*

$\dfrac{P(X = 7)}{P(X \geq 6)}$ or equivalent $\left(= \dfrac{0.1163...}{0.4365...}\right)$ ***A1***

$= 0.267$ ***A1***

[7 marks]

(b) **METHOD 1**

the possible arrivals are $(2,0)$, $(1,1)$, $(0,2)$ *(A1)*

$Y \sim \text{Po}(0.65)$ ***A1***

attempt to compute, using sum and product rule, *(M1)*

$0.070106\ldots \times 0.52204\ldots + 0.026455\ldots \times 0.33932\ldots + 0.0049916\ldots \times 0.11028\ldots$

(A1)(A1)

Note: Award ***A1*** for one correct product and ***A1*** for two other correct products.

$= 0.0461$ ***A1***

[6 marks]

METHOD 2

recognising a sum of 2 independent Poisson variables *eg* $Z = X + Y$ ***R1***

$\lambda = 5.3 + \dfrac{1.3}{2}$ ***A1***

$P(Z = 2) = 0.0461$ *(M1)**A3***

[6 marks]

Total [13 marks]

Question 70

(a) $P(L \geq 5) = 0.910$ *(M1)**A1***

[2 marks]

(b) X is the number of wolves found to be at least 5 years old

recognising binomial distribution ***M1***

$X \sim B(8, 0.910\ldots)$

$P(X > 6) = 1 - P(X \leq 6)$ *(M1)*

$= 0.843$ ***A1***

Note: Award ***M1A0*** for finding $P(X \geq 6)$.

[3 marks]

Total [5 marks]

Question 71

X is number of squirrels in reserve

$X \sim \text{Po}(179.2)$ *A1*

e: Award ***A1*** if 179.2 or 56×3.2 seen or implicit in future calculations.

recognising conditional probability *M1*

$\text{P}\left(X > 190 \mid X \geq 168 \right)$

$= \frac{\text{P}(X > 190)}{\text{P}(X \geq 168)} \left(= \frac{0.19827\ldots}{0.80817\cdots} \right)$ *(A1)(A1)*

$= 0.245$ *A1*

[5 marks]

Question 72

METHOD 1

let p have no pets, q have one pet and r have two pets *(M1)*

$p + q + r + 2 = 25$ *(A1)*

$0p + 1q + 2r + 6 = 18$ *A1*

e: Accept a statement that there are a total of 12 pets.

attempt to use variance equation, or evidence of trial and error *(M1)*

$\frac{0p + 1q + 4r + 18}{25} - \left(\frac{18}{25} \right)^2 = \left(\frac{24}{25} \right)^2$ *(A1)*

attempt to solve a system of linear equations *(M1)*

$p = 14$ *A1*

METHOD 2

x	0	1	2	3
$\text{P}(X = x)$	p	q	r	$\frac{2}{25}$

(M1)

$p + q + r + \frac{2}{25} = 1$ *(A1)*

$q + 2r + \frac{6}{25} = \frac{18}{25} \left(\Rightarrow q + 2r = \frac{12}{25} \right)$ *A1*

$q + 4r + \frac{18}{25} - \left(\frac{18}{25} \right)^2 = \frac{576}{625} \left(\Rightarrow q + 4r = \frac{18}{25} \right)$ *(M1)(A1)*

$q = \frac{6}{25},\ r = \frac{3}{25}$ *(M1)*

$p = \frac{14}{25}$ *A1*

so 14 have no pets

[7 marks]

Question 73

(a) $a\left[\int_0^{0.5} 3x\,dx + \int_{0.5}^{2}(2-x)\,dx\right] = 1$ *M1*

Note: Award the ***M1*** for the total integral equalling 1, or equivalent.

$a\left(\frac{3}{2}\right) = 1$ *(M1)A1*

$a = \frac{2}{3}$ *AG*

[3 marks]

(b) **EITHER**

$\int_0^{0.5} 2x\,dx + \frac{2}{3}\int_{0.5}^{1}(2-x)\,dx$ *(M1)(A1)*

$= \frac{2}{3}$ *A1*

OR

$\frac{2}{3}\int_1^{2}(2-x)\,dx = \frac{1}{3}$ *(M1)*

so $P(X<1) = \frac{2}{3}$ *(M1)A1*

[3 marks]

(c) $P(s< X<0.8) = \int_s^{0.5} 2x\,dx + \frac{2}{3}\int_{0.5}^{0.8}(2-x)\,dx$ *M1A1*

$= \left[x^2\right]_s^{0.5} + 0.27$

$0.25 - s^2 + 0.27$ *(A1)*

$P(2s< X<0.8) = \frac{2}{3}\int_{2s}^{0.8}(2-x)\,dx$ *A1*

$= \frac{2}{3}\left[2x - \frac{x^2}{2}\right]_{2s}^{0.8}$

$\frac{2}{3}\left(1.28 - (4s - 2s^2)\right)$

equating

$0.25 - s^2 + 0.27 = \frac{4}{3}\left(1.28 - (4s - 2s^2)\right)$ *(A1)*

attempt to solve for s *(M1)*

$s = 0.274$ *A1*

[7 marks]

Total [13 marks]

Question 74

(a) use of inverse normal (implied by $\pm 0.1509\ldots$ or $\pm 1.554\ldots$) *(M1)*

$P(X < 16) = 0.56$

$\Rightarrow \dfrac{16-\mu}{\sigma} = 0.1509\ldots$ *(A1)*

$P(X < 17) = 0.94$

$\Rightarrow \dfrac{17-\mu}{\sigma} = 1.554\ldots$ *(A1)*

attempt to solve a pair of simultaneous equations *(M1)*

$\mu = 15.9,\ \sigma = 0.712$ ***A1A1***

[6 marks]

(b) correctly shaded diagram or intent to find $P(X \geq 15)$ *(M1)*

$= 0.895$ ***A1***

Note: Accept answers rounding to 0.89 or 0.90. Award ***M1A0*** for the answer 0.9.

[2 marks]

Total [8 marks]

Question 75

(a) (i) $P(X < 60)$

$= P(X \leq 59)$ *(M1)*

$= 0.102$ ***A1***

(ii) standard deviation $= \sqrt{70}\ (= 8.37)$ *(M1)**A1***

[4 marks]

(b) (i) use of midpoints (accept consistent use of 45, 55 etc.) *(M1)*

$$\frac{44.5\times 2 + 54.5\times 15 + 64.5\times 40 + 74.5\times 53 + 94.5 + 104.5\times 3 + 114.5\times 6}{2 + 15 + 40 + 53 + 0 + 1 + 3 + 6}$$

(M1)

$= \dfrac{8530}{120} (= 71.1)$ ***A1***

Note: If 45, 55, etc. are used consistently instead of midpoints (implied by the answer 71.58…) award ***M1M1A0***.

(ii) 13.9 *(M1)**A1***

[5 marks]

(c) valid reason given to include the examples below ***R1***

variance is 192 which is not close to the mean (accept not equal to)

standard deviation too high (using parts (a)(ii) and (b)(ii))

relative frequency of $X \leq 59$ is 0.142 which is too high (using part (a)(i))

Poisson would give a frequency of roughly 14 for $80 \leq X \leq 89$

(d) $P(Y > 10) = 0.99$

$1 - P(Y \leq 10) = 0.99 \Rightarrow P(Y \leq 10) = 0.01$ *(M1)*

attempt to solve a correct equation *(M1)*

$\lambda = 20.1$ ***A1***

[3 marks]

(e) in 1 day, no of emails is $X \sim \text{Po}(\lambda)$

in 2 days, no of emails is $Y \sim \text{Po}(2\lambda)$ ***(A1)***

P(10 on first day | 20 in 2 days) ***(M1)***

$$= \frac{P(X=10) \times P(X=10)}{P(Y=20)}$$ ***(M1)***

$$= \frac{\left(\frac{\lambda^{10}e^{-\lambda}}{10!}\right)^2}{\frac{(2\lambda)^{20}e^{-2\lambda}}{20!}}$$ ***A1***

$$= \frac{\lambda^{20}e^{-2\lambda}}{2^{20}\lambda^{20}e^{-2\lambda}} \times \frac{20!}{(10!)^2}$$ ***A1***

$$= \frac{20!}{2^{20}(10!)^2}$$

which is independent of λ ***AG***

[5 marks]

Total [18 marks]

Question 76

(a) $X \sim \text{N}\left(820, 230^2\right)$ ***(M1)***

Note: Award ***M1*** for an attempt to use normal distribution. Accept labelled normal graph.

$\Rightarrow P\left(X > 1000\right) = 0.217$ ***A1***

[2 marks]

(b) $Y \sim \text{B}\left(24, 0.217...\right)$ ***(M1)***

Note: Award ***M1*** for recognition of binomial distribution with parameters.

$P\left(Y \le 10\right) - P\left(Y \le 4\right)$ ***(M1)***

Note: Award ***M1*** for an attempt to find $P(5 \le Y \le 10)$ or $P\left(Y \le 10\right) - P\left(Y \le 4\right)$.

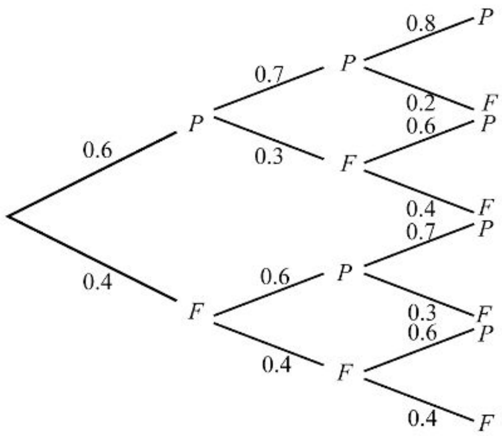
$= 0.613$ ***A1***

[3 marks]

Total [5 marks]

Question 77

(a)



A1A1A1

Note: Award ***A1*** for each correct column of probabilities.

[3 marks]

(b) probability (at least twice) =

EITHER

$(0.6\times 0.7\times 0.8)+(0.6\times 0.7\times 0.2)+(0.6\times 0.3\times 0.6)+(0.4\times 0.6\times 0.7)$ ***(M1)***

OR

$(0.6\times 0.7)\ +\ (0.6\times 0.3\times 0.6)\ +\ (0.4\times 0.6\times 0.7)$ ***(M1)***

Note: Award ***M1*** for summing all required probabilities.

THEN

$=0.696$ ***A1***

[2 marks]

(c) P(passes third paper given only one paper passed before)

$=\frac{\text{P (passes third AND only one paper passed before)}}{\text{P (passes once in first two papers)}}$ ***(M1)***

$=\frac{(0.6\times 0.3\times 0.6)+(0.4\times 0.6\times 0.7)}{(0.6\times 0.3)+(0.4\times 0.6)}$ ***A1***

$=0.657$ ***A1***

[3 marks]

Total [8 marks]

Question 78

(a) $X\sim \text{Po}(2.1)$

$\text{P}(X=0)=0.122\left(=\text{e}^{-2.1}\right)$ ***(M1)A1***

[2 marks]

(b)

y	0	1	2	3	4
$P(Y = y)$	0.122... $\left(= e^{-2.1}\right)$	0.257... $\left(= e^{-2.1}2.1\right)$	0.270... $\left(= \frac{e^{-2.1}2.1^2}{2!}\right)$	0.189... $\left(= \frac{e^{-2.1}2.1^3}{3!}\right)$	0.161...

A1A1A1A1

Note: Award ***A1*** for each correct probability for $Y = 1, 2, 3, 4$. Accept 0.162 for $P(Y = 4)$.

[4 marks]

(c) $E(Y) = \sum y P(Y = y)$ ***(M1)***

$= 1\times 0.257... + 2\times 0.270... + 3\times 0.189... + 4\times 0.161...$ ***(A1)***

$= 2.01$ ***A1***

[3 marks]

(d) let be the no of days per year that Steffi does not visit

$\sim B(365, 0.122...)$ ***(M1)***

require $0.45 \le P(\le n) < 0.55$ ***(M1)***

$P(\le 44) = 0.51$

$= 44$ ***A1***

[3 marks]

(e) **METHOD 1**

let V be the discrete random variable "number of times Steffi is not fed per day"

$E(V) = 1\times P(X = 5) + 2\times P(X = 6) + 3\times P(X = 7) + \cdots$ ***M1***

$= 1\times 0.0416... + 2\times 0.0145... + 3\times 0.00437... + \cdots$ ***A1***

$= 0.083979...$ ***A1***

expected no of occasions per year $> 0.083979...\times 365 = 30.7$ ***A1***

hence Steffi can expect not to be fed on at least 30 occasions ***AG***

Note: Candidates may consider summing more than three terms in their calculation for $E(V)$.

[4 marks]

METHOD 2

$E(X) - E(Y) = 0.0903...$ ***M1A1***

$0.0903... \times 365$ ***M1***

$= 33.0 > 30$ ***A1AG***

[4 marks]

Total [16 marks]

Question 79

(a) (i) 6.75 **A1**

(ii) 2.22 **A1**

[2 marks]

(b) (i) 8.75 **A1**

(ii) 2.22 **A1**

[2 marks]

(c) the order is 3, 4, 6, 7, 7, 8, 9, 10
median is currently 7 **A1**

Note: This can be indicated by a diagram/list, rather than actually stated.

with 9 numbers the middle value (median) will be the 5th value **R1**
which will correspond to 7 regardless of whether the position of the median moves up or down **R1**

Note: Accept answers using data 5, 6, 8, 9, 9, 10, 11, 12 (ie from part (b)).

[3 marks]

Total [7 marks]

Question 80

(a) (i) use of formula or Venn diagram ***(M1)***
$0.72 + 0.45 - 1$ ***(A1)***
$= 0.17$ ***A1***

(ii) $0.72 - 0.17 = 0.55$ ***A1***

[4 marks]

(b) (i) $200 \times 0.45 = 90$ ***A1***

(ii) let X be the number of customers who order cake
$X \sim \mathrm{B}(200, 0.45)$ ***(M1)***
$\mathrm{P}(X > 100) = \mathrm{P}(X \geq 101)\left(= 1 - \mathrm{P}(X \leq 100)\right)$ ***(M1)***
$= 0.0681$ ***A1***

[4 marks]

(c) (i) $0.46 \times 0.8 = 0.368$ ***A1***

(ii) **METHOD 1**

$0.368 + 0.54 \times \mathrm{P}(S \mid F) = 0.72$ ***M1A1A1***

Note: Award ***M1*** for an appropriate tree diagram. Award ***A1*** for LHS, ***A1*** for RHS.

$\mathrm{P}(S \mid F) = 0.652$ ***A1***

METHOD 2

$\mathrm{P}(S \mid F) = \dfrac{\mathrm{P}(S \cap F)}{\mathrm{P}(F)}$ ***(M1)***

$= \dfrac{0.72 - 0.368}{0.54}$ ***A1A1***

Note: Award ***A1*** for numerator, ***A1*** for denominator.

$\mathrm{P}(S \mid F) = 0.652$ ***A1***

[5 marks]

Total [13 marks]

Question 81

(a) $X \sim \text{Po}(1.3)$

$\text{P}(X \geq 2) = 0.373$ ***(M1)A1***

[2 marks]

(b) $V \sim \text{B}(5, 0.373)$ ***(M1)A1***

Note: Award ***(M1)*** for recognition of binomial or equivalent, ***A1*** for correct parameters.

$\text{P}(V = 4) = 0.0608$ ***(M1)A1***

[4 marks]

Total [6 marks]

Question 82

$T \sim \text{N}(11.6, 0.8^2)$

$\text{P}(T < 10.7 \mid T < 11)$ ***(M1)***

$= \dfrac{\text{P}(T < 10.7 \cap T < 11)}{\text{P}(T < 11)}$ ***(M1)***

$= \dfrac{\text{P}(T < 10.7)}{\text{P}(T < 11)}$ ***(M1)***

(A1)

(A1)

$\mid T < 11) = 0.575$ ***A1***

ote: Accept only 0.575.

[6 marks]

Question 83

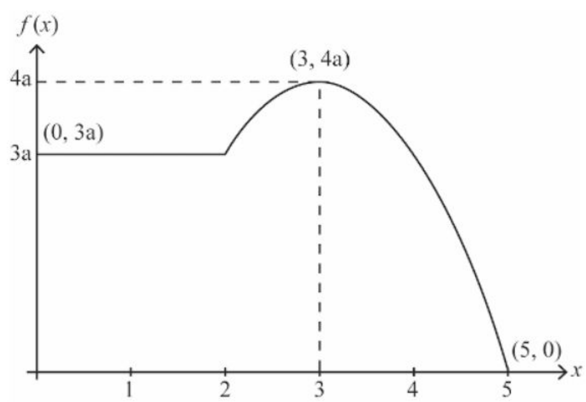
(a) $\left(\text{P}(1 < X < 3) =\right) \int_1^2 3a \, \text{d}x + a\int_2^3 -x^2 + 6x - 5 \, \text{d}x$ ***(M1)(A1)(A1)***

$= 3 + \dfrac{11}{3}$

$= \dfrac{20}{3} (= 6.67a)$ ***A1***

[4 marks]

(b)



A4

award ***A1*** for $(0, 3a)$, ***A1*** for continuity at $(2, 3a)$, ***A1*** for maximum at $(3, 4a)$, ***A1*** for $(5, 0)$

Note: Award ***A3*** if correct four points are not joined by a straight line and a quadratic curve.

[4 marks]

(c) (i) $\mathrm{P}(0 \le X \le 5) = 6a + a\int_2^5 -x^2 + 6x - 5\,\mathrm{d}x$ ***(M1)***

$= 15a$ ***(A1)***

$15a = 1$ ***(M1)***

$\Rightarrow a = \dfrac{1}{15} (= 0.0667)$ ***A1***

(ii) $\mathrm{E}(X) = \dfrac{1}{5}\int_0^2 x\,\mathrm{d}x + \dfrac{1}{15}\int_2^5 -x^3 + 6x^2 - 5x\,\mathrm{d}x$ ***(M1)(A1)***

$= 2.35$ ***A1***

continued…

(iii) attempt to use $\int_0^m f(x)\mathrm{d}x = 0.5$ ***(M1)***

$0.4 + a\int_2^m -x^2 + 6x - 5\,\mathrm{d}x = 0.5$ ***(A1)***

$a\int_2^m -x^2 + 6x - 5\,\mathrm{d}x = 0.1$

attempt to solve integral using GDC and/or analytically ***(M1)***

$\dfrac{1}{15}\left[-\dfrac{1}{3}x^3 + 3x^2 - 5x\right]_2^m = 0.1$

$m = 2.44$ ***A1***

[11 marks]

Total [19 marks]

(iii) attempt to use $\int_0^m f(x)\mathrm{d}x = 0.5$ ***(M1)***

$0.4 + a\int_2^m -x^2 + 6x - 5\,\mathrm{d}x = 0.5$ ***(A1)***

$a\int_2^m -x^2 + 6x - 5\,\mathrm{d}x = 0.1$

attempt to solve integral using GDC and/or analytically ***(M1)***

$\dfrac{1}{15}\left[-\dfrac{1}{3}x^3 + 3x^2 - 5x\right]_2^m = 0.1$

$m = 2.44$ ***A1***

[11 marks]

Total [19 marks]