Subject – Math(Higher Level)
Topic - Statistics and Probability
Year - Nov 2011 – Nov 2019
Paper -2

Question -1

[Maximum mark: 5]

The number of vehicles passing a particular junction can be modelled using the Poisson distribution. Vehicles pass the junction at an average rate of 300 per hour.

(a) Find the probability that no vehicles pass in a given minute. *[2 marks]*

(b) Find the expected number of vehicles which pass in a given two minute period. *[1 mark]*

(c) Find the probability that more than this expected number actually pass in a given two minute period. *[2 marks]*

Question-2

[Maximum mark: 5]

The probability that the 08:00 train will be delayed on a work day (Monday to Friday) is $\frac{1}{10}$. Assuming that delays occur independently,

(a) find the probability that the 08:00 train is delayed exactly twice during any period of five work days; *[2 marks]*

(b) find the minimum number of work days for which the probability of the 08:00 train being delayed at least once exceeds 90 %. *[3 marks]*

Question 3

[Maximum mark: 15]

Jan and Sia have been selected to represent their country at an international discus throwing competition. Assume that the distance thrown by each athlete is normally distributed. The mean distance thrown by Jan in the past year was 60.33 metres with a standard deviation of 1.95 metres.

(a) In the past year, 80 % of Jan's throws have been longer than x metres. Find x correct to two decimal places. *[2 marks]*

(b) In the past year, 80 % of Sia's throws have been longer than 56.52 metres. If the mean distance of her throws was 59.39 metres, find the standard deviation of her throws. *[3 marks]*

(c) This year, Sia's throws have a mean of 59.50 metres and a standard deviation of 3.00 metres. The mean and standard deviation of Jan's throws have remained the same. In the competition, an athlete must have at least one throw of 65 metres or more in the first round to qualify for the final round. Each athlete is allowed three throws in the first round.

(i) Determine whether Jan or Sia is more likely to qualify for the final on their first throw.

(ii) Find the probability that both athletes qualify for the final. *[10 marks]*

Question -4

[Maximum mark: 7]

A team of 6 players is to be selected from 10 volleyball players, of whom 8 are boys and 2 are girls.

(a) In how many ways can the team be selected? *[2 marks]*

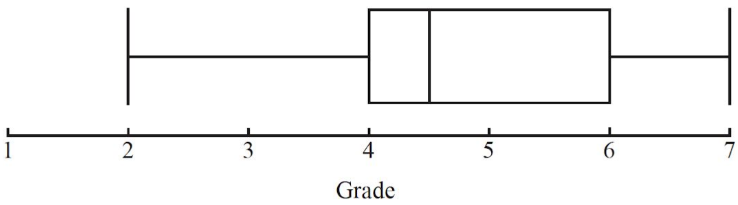
(b) In how many of these selections is exactly one girl in the team? *[3 marks]*

(c) If the selection of the team is made at random, find the probability that exactly one girl is in the team. *[2 marks]*

Question -5

[Maximum mark: 5]

The box and whisker plot below illustrates the IB grades obtained by 100 students.



IB grades can only take integer values.

(a) How many students obtained a grade of more than 4? *[1 mark]*

(b) State, with reasons, the maximum possible number and minimum possible number of students who obtained a 4 in the exam. *[4 marks]*

Question – 6

[Maximum mark: 5]

A fisherman notices that in any hour of fishing, he is equally likely to catch exactly two fish, as he is to catch less than two fish. Assuming the number of fish caught can be modelled by a Poisson distribution, calculate the expected value of the number of fish caught when he spends four hours fishing.

Question -7

[Maximum mark: 22]

A ski resort finds that the mean number of accidents on any given weekday (Monday to Friday) is 2.2. The number of accidents can be modelled by a Poisson distribution.

(a) Find the probability that in a certain week (Monday to Friday only)

(i) there are fewer than 12 accidents;

(ii) there are more than 8 accidents, given that there are fewer than 12 accidents. *[6 marks]*

Due to the increased usage, it is found that the probability of more than 3 accidents in a day at the weekend (Saturday and Sunday) is 0.24.

(b) Assuming a Poisson model,

(i) calculate the mean number of accidents per day at the weekend (Saturday and Sunday);

(ii) calculate the probability that, in the four weekends in February, there will be more than 5 accidents during at least two of the weekends. *[10 marks]*

It is found that 20 % of skiers having accidents are at least 25 years of age and 40 % are under 18 years of age.

(c) Assuming that the ages of skiers having accidents are normally distributed, find the mean age of skiers having accidents. *[6 marks]*

Question -8

[Maximum mark: 5]

The random variable X has the distribution $\mathrm{B}(30,\ p)$. Given that $\mathrm{E}(X) = 10$, find

(a) the value of p; *[1 mark]*

(b) $\mathrm{P}(X = 10)$; *[2 marks]*

(c) $\mathrm{P}(X \geq 15)$. *[2 marks]*

Question -9

[Maximum mark: 5]

The random variable X has the distribution $\mathrm{Po}(m)$.
Given that $\mathrm{P}(X = 5) = \mathrm{P}(X = 3) + \mathrm{P}(X = 4)$, find

(a) the value of m; *[3 marks]*

(b) $\mathrm{P}(X > 2)$. *[2 marks]*

Question 10

[Maximum mark: 5]

The probability density function of a continuous random variable X is given by

$$f(x) = \frac{1}{1 + x^4},\ 0 \le x \le a.$$

(a) Find the value of a. *[3 marks]*

(b) Find the mean of X. *[2 marks]*

Question 11

[Maximum mark: 14]

A market stall sells apples, pears and plums.

(a) The weights of the apples are normally distributed with a mean of 200 grams and a standard deviation of 25 grams.

(i) Given that there are 450 apples on the stall, what is the expected number of apples with a weight of more than 225 grams?

(ii) Given that 70 % of the apples weigh less than m grams, find the value of m. *[5 marks]*

(b) The weights of the pears are normally distributed with a mean of $\propto$ grams and a standard deviation of σ grams. Given that 8 % of these pears have a weight of more than 270 grams and 15 % have a weight less than 250 grams, find $\propto$ and σ. *[6 marks]*

(c) The weights of the plums are normally distributed with a mean of 80 grams and a standard deviation of 4 grams. 5 plums are chosen at random. What is the probability that exactly 3 of them weigh more than 82 grams? *[3 marks]*

Question 12

[Maximum mark: 6]

A set of 15 observations has mean 11.5 and variance 9.3. One observation of 22.1 is considered unreliable and is removed. Find the mean and variance of the remaining 14 observations.

Question 13

[Maximum mark: 8]

Kathy plays a computer game in which she has to find the path through a maze within a certain time. The first time she attempts the game, the probability of success is known to be 0.75. In subsequent attempts, if Kathy is successful, the difficulty increases and the probability of success is half the probability of success on the previous attempt. However, if she is unsuccessful, the probability of success remains the same. Kathy plays the game three times consecutively.

(a) Find the probability that she is successful in all three games. *[2 marks]*

(b) Assuming that she is successful in the first game, find the probability that she is successful in exactly two games. *[6 marks]*

Question 14

[Maximum mark: 18]

The number of visitors that arrive at a museum every minute can be modelled by a Poisson distribution with mean 2.2.

(a) If the museum is open 6 hours daily, find the expected number of visitors in 1 day. *[2 marks]*

(b) Find the probability that the number of visitors arriving during an hour exceeds 100. *[3 marks]*

(c) Find the probability that the number of visitors in each of the 6 hours the museum is open exceeds 100. *[2 marks]*

The ages of the visitors to the museum can be modelled by a normal distribution with mean μ and variance σ^2. The records show that 29 % of the visitors are under 35 years of age and 23 % are at least 55 years of age.

(d) Find the values of μ and σ. *[6 marks]*

(e) One day, 100 visitors under 35 years of age come to the museum. Estimate the number of visitors under 50 years of age that were at the museum on that day. *[5 marks]*

Question 15

[Maximum mark: 4]

The marks obtained by a group of students in a class test are shown below.

Marks	Frequency
5	6
6	k
7	3
8	1
9	2
10	1

Given the mean of the marks is 6.5, find the value of k.

Question 16

[Maximum mark: 5]

Emily walks to school every day. The length of time this takes can be modelled by a normal distribution with a mean of 11 minutes and a standard deviation of 3 minutes. She is late if her journey takes more than 15 minutes.

(a) Find the probability she is late next Monday. *[2 marks]*

(b) Find the probability she is late at least once during the next week (Monday to Friday). *[3 marks]*

Question 17

[Maximum mark: 7]

A ferry carries cars across a river. There is a fixed time of T minutes between crossings. The arrival of cars at the crossing can be assumed to follow a Poisson distribution with a mean of one car every four minutes. Let X denote the number of cars that arrive in T minutes.

(a) Find T, to the nearest minute, if $P(X \le 3) = 0.6$. *[3 marks]*

It is now decided that the time between crossings, T, will be 10 minutes. The ferry can carry a maximum of three cars on each trip.

(b) One day all the cars waiting at 13:00 get on the ferry. Find the probability that all the cars that arrive in the next 20 minutes will get on either the 13:10 or the 13:20 ferry. *[4 marks]*

Question 18

[Maximum mark: 5]

It is believed that the lifespans of Manx cats are normally distributed with a mean of 13.5 years and a variance of 9.5 years^2.

(a) Calculate the range of lifespans of Manx cats whose lifespans are within one standard deviation of the mean. *[2 marks]*

(b) Estimate the number of Manx cats in a population of 10 000 that will have a lifespan of less than 10 years. Give your answer to the nearest whole number. *[3 marks]*

Question 19

[Maximum mark: 7]

The length, X metres, of a species of fish has the probability density function

$$f(x)=\begin{cases} ax^2, & \text{for } 0 \le x \le 0.5 \\ 0.5a(1-x), & \text{for } 0.5 \le x \le 1 \\ 0, & \text{otherwise .} \end{cases}$$

(a) Show that $a = 9.6$. *[3 marks]*

(b) Sketch the graph of the distribution. *[2 marks]*

(c) Find $\mathrm{P}(X < 0.6)$. *[2 marks]*

Question 20

[Maximum mark: 7]

A small car hire company has two cars. Each car can be hired for one whole day at a time. The rental charge is US$60 per car per day. The number of requests to hire a car for one whole day may be modelled by a Poisson distribution with mean 1.2.

(a) Find the probability that on a particular weekend, three requests are received on Saturday and none are received on Sunday. *[2 marks]*

Over a weekend of two days, it is given that a total of three requests are received.

(b) Find the expected total rental income for the weekend. *[5 marks]*

Question 21

[Maximum mark: 6]

The duration of direct flights from London to Singapore in a particular year followed a normal distribution with mean μ and standard deviation σ .

92% of flights took under 13 hours, while only 12% of flights took under 12 hours 35 minutes.

Find μ and σ to the nearest minute.

Question 22

[Maximum mark: 5]

At the start of each week, Eric and Marina pick a night at random on which they will watch a movie.

If they choose a Saturday night, the probability that they watch a French movie is $\frac{7}{9}$ and if they choose any other night the probability that they watch a French movie is $\frac{4}{9}$.

(a) Find the probability that they watch a French movie. *[3]*

(b) Given that last week they watched a French movie, find the probability that it was on a Saturday night. *[2]*

Question 23

[Maximum mark: 18]

(a) The number of cats visiting Helena's garden each week follows a Poisson distribution with mean $\lambda = 0.6$.

Find the probability that

(i) in a particular week no cats will visit Helena's garden;

(ii) in a particular week at least three cats will visit Helena's garden;

(iii) over a four-week period no more than five cats in total will visit Helena's garden;

(iv) over a twelve-week period there will be exactly four weeks in which at least one cat will visit Helena's garden. *[9]*

(b) A continuous random variable X has probability distribution function f given by

$$f(x) = k \ln x \quad 1 \le x \le 3$$
$$f(x) = 0 \quad \text{otherwise}$$

(i) Find the value of k to six decimal places.

(ii) Find the value of $\mathrm{E}(X)$.

(iii) State the mode of X.

(iv) Find the median of X. *[9]*

Question 24

[Maximum mark: 5]

A student sits a national test and is told that the marks follow a normal distribution with mean 100. The student receives a mark of 124 and is told that he is at the 68^{th} percentile. Calculate the variance of the distribution.

Question 25

[Maximum mark: 8]

(a) Find the term in x^5 in the expansion of $(3x + A)(2x + B)^6$. *[4]*

Mina and Norbert each have a fair cubical die with faces labelled 1, 2, 3, 4, 5 and 6; they throw it to decide if they are going to eat a cookie.

Mina throws her die just once and she eats a cookie if she throws a four, a five or a six. Norbert throws his die six times and each time eats a cookie if he throws a five or a six.

(b) Calculate the probability that five cookies are eaten. *[4]*

Question 26

[Maximum mark: 7]

The number of birds seen on a power line on any day can be modelled by a Poisson distribution with mean 5.84.

(a) Find the probability that during a certain seven-day week, more than 40 birds have been seen on the power line. *[2]*

(b) On Monday there were more than 10 birds seen on the power line. Show that the probability of there being more than 40 birds seen on the power line from that Monday to the following Sunday, inclusive, can be expressed as:

$$\frac{\mathrm{P}(X > 40) + \sum_{r=11}^{40} \mathrm{P}(X = r)\mathrm{P}(Y > 40 - r)}{\mathrm{P}(X > 10)} \text{ where } X \sim \mathrm{Po}(5.84) \text{ and } Y \sim \mathrm{Po}(35.04).$$ *[5]*

Question 27

[Maximum mark: 21]

A random variable X has probability density function

$$f(x)=\begin{cases} ax+b, & 2\le x\le 3 \\ 0, & \text{otherwise} \end{cases},\ a,b\in\mathbb{R}.$$

(a) Show that $5a+2b=2$. *[4]*

Let $\mathrm{E}(X)=\mu$.

(b) (i) Show that $a=12\mu-30$.

(ii) Find a similar expression for b in terms of μ. *[7]*

Let the median of the distribution be 2.3.

(c) (i) Find the value of μ.

(ii) Find the value of the standard deviation of X. *[10]*

Question 28

[Maximum mark: 7]

Six customers wait in a queue in a supermarket. A customer can choose to pay with cash or a credit card. Assume that whether or not a customer pays with a credit card is independent of any other customers' methods of payment.

It is known that 60% of customers choose to pay with a credit card.

(a) Find the probability that:

(i) the first three customers pay with a credit card and the next three pay with cash;

(ii) exactly three of the six customers pay with a credit card. *[4]*

There are n customers waiting in another queue in the same supermarket. The probability that at least one customer pays with cash is greater than 0.995.

(b) Find the minimum value of n. *[3]*

Question 29

[Maximum mark: 4]

The random variable X has a Poisson distribution with mean μ.

Given that $\mathrm{P}(X=2)+\mathrm{P}(X=3)=\mathrm{P}(X=5)$,

(a) find the value of μ; *[2]*

(b) find the probability that X lies within one standard deviation of the mean. *[2]*

Question 30

[Maximum mark: 13]

The probability density function of a random variable X is defined as:

$$f(x)=\begin{cases} a\,x\cos x, & 0\le x\le \dfrac{\pi}{2}, \text{ where } a\in\mathbb{R}. \\ 0, & \text{elsewhere} \end{cases}$$

(a) Show that $a=\dfrac{2}{\pi-2}$. *[5]*

(b) Find $\mathrm{P}\left(X<\dfrac{\pi}{4}\right)$. *[2]*

(c) Find:

(i) the mode of X;

(ii) the median of X. *[4]*

(d) Find $\mathrm{P}\left(X<\dfrac{\pi}{8} \mid X<\dfrac{\pi}{4}\right)$. *[2]*

Question 31

[Maximum mark: 5]

The wingspans of a certain species of bird can be modelled by a normal distribution with mean 60.2 cm and standard deviation 2.4 cm.

According to this model, 99% of wingspans are greater than x cm.

(a) Find the value of x. *[2]*

In a field experiment, a research team studies a large sample of these birds. The wingspans of each bird are measured correct to the nearest 0.1 cm.

(b) Find the probability that a randomly selected bird has a wingspan measured as 60.2 cm. *[3]*

Question 32

[Maximum mark: 6]

Consider the data set $\{2, x, y, 10, 17\}$, $x, y \in \mathbb{Z}^+$ and $x < y$.

The mean of the data set is 8 and its variance is 27.6.

Find the value of x and the value of y.

Question 33

[Maximum mark: 10]

The number of complaints per day received by customer service at a department store follows a Poisson distribution with a mean of 0.6.

(a) On a randomly chosen day, find the probability that

(i) there are no complaints;

(ii) there are at least three complaints. *[3]*

(b) In a randomly chosen five-day week, find the probability that there are no complaints. *[2]*

(c) On a randomly chosen day, find the most likely number of complaints received. Justify your answer. *[3]*

The department store introduces a new policy to improve customer service. The number of complaints received per day now follows a Poisson distribution with mean λ.

On a randomly chosen day, the probability that there are no complaints is now 0.8.

(d) Find the value of λ. *[2]*

Question 34

Ava and Barry play a game with a bag containing one green marble and two red marbles. Each player in turn randomly selects a marble from the bag, notes its colour and replaces it. Ava wins the game if she selects a green marble. Barry wins the game if he selects a red marble. Ava starts the game.

Find the probability that

(a) Ava wins on her first turn; *[1]*

(b) Barry wins on his first turn; *[2]*

(c) Ava wins in one of her first three turns; *[4]*

(d) Ava eventually wins. *[4]*

Question 35

[Maximum mark: 4]

The finishing times in a marathon race follow a normal distribution with mean 210 minutes and standard deviation 22 minutes.

(a) Find the probability that a runner finishes the race in under three hours. [2]

The fastest 90% of the finishers receive a certificate.

(b) Find the time, below which a competitor has to complete the race, in order to gain a certificate. [2]

Question 36

[Maximum mark: 5]

A mosaic is going to be created by randomly selecting 1000 small tiles, each of which is either black or white. The probability that a tile is white is 0.1. Let the random variable W be the number of white tiles.

(a) State the distribution of W, including the values of any parameters. [2]

(b) Write down the mean of W. [1]

(c) Find $\mathrm{P}(W > 89)$. [2]

Question 37

[Maximum mark: 7]

The random variable X follows a Poisson distribution with mean $m \neq 0$.

(a) Given that $2\mathrm{P}(X = 4) = \mathrm{P}(X = 5)$, show that $m = 10$. [3]

(b) Given that $X \leq 11$, find the probability that $X = 6$. [4]

Question 38

[Maximum mark: 20]

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} 0 \ , \ x < 0 \\ \dfrac{\sin x}{4} \ , \ 0 \le x \le \pi \\ a(x-\pi), \ \pi < x \le 2\pi \\ 0 \ , \ 2\pi < x \end{cases}.$$

(a) Sketch the graph of $y = f(x)$. [2]

(b) Find $P(X \le \pi)$. [2]

(c) Show that $a = \dfrac{1}{\pi^2}$. [3]

(d) Write down the median of X. [1]

(e) Calculate the mean of X. [3]

(f) Calculate the variance of X. [3]

(g) Find $P\left(\dfrac{\pi}{2} \le X \le \dfrac{3\pi}{2}\right)$. [2]

(h) Given that $\dfrac{\pi}{2} \le X \le \dfrac{3\pi}{2}$ find the probability that $\pi \le X \le 2\pi$. [4]

Question 39

[Maximum mark: 7]

Emma acquires a new cell phone for her birthday and receives texts from her friends. It is assumed that the daily number of texts Emma receives follows a Poisson distribution with mean $m = 5$.

(a) (i) Find the probability that on a certain day Emma receives more than 7 texts.

(ii) Determine the expected number of days in a week on which Emma receives more than 7 texts. [4]

(b) Find the probability that Emma receives fewer than 30 texts during a week. [3]

Question 40

[Maximum mark: 6]

Natasha lives in Chicago and has relatives in Nashville and St. Louis.
Each time she visits her relatives, she either flies or drives.

When travelling to Nashville, the probability that she drives is $\frac{4}{5}$, and when travelling to St. Louis, the probability that she flies is $\frac{1}{3}$.

Given that the probability that she drives when visiting her relatives is $\frac{13}{18}$, find the probability that for a particular trip,

(a) she travels to Nashville; [3]

(b) she is on her way to Nashville, given that she is flying. [3]

Question 41

[Maximum mark: 12]

Farmer Suzie grows turnips and the weights of her turnips are normally distributed with a mean of 122 g and standard deviation of 14.7 g.

(a) (i) Calculate the percentage of Suzie's turnips that weigh between 110 g and 130 g.

(ii) Suzie has 100 turnips to take to market. Find the expected number weighing more than 130 g.

(iii) Find the probability that at least 30 of the 100 turnips weigh more than 130 g. [6]

Farmer Ray also grows turnips and the weights of his turnips are normally distributed with a mean of 144 g. Ray only takes to market turnips that weigh more than 130 g. Over a period of time, Ray finds he has to reject 1 in 15 turnips due to their being underweight.

(b) (i) Find the standard deviation of the weights of Ray's turnips.

(ii) Ray has 200 turnips to take to market. Find the expected number weighing more than 150 g. [6]

Question 42

[Maximum mark: 4]

The events A and B are such that $\mathrm{P}(A) = 0.65$, $\mathrm{P}(B) = 0.48$ and $\mathrm{P}(A \cup B) = 0.818$.

(a) Find $\mathrm{P}(A \cap B)$. [2]

(b) Hence show that the events A and B are independent. [2]

Question 43

[Maximum mark: 6]

The data of the goals scored by players in a football club during a season are given in the following table.

Goals	Frequency
0	4
1	k
2	3
3	2
4	3
8	1

(a) Given that the mean number of goals scored per player is 1.95, find the value of k. [3]

It is discovered that there is a mistake in the data and that the top scorer, who scored 22 goals, has not been included in the table.

(b) (i) Find the correct mean number of goals scored per player.

(ii) Find the correct standard deviation of the number of goals scored per player. [3]

Question 44

[Maximum mark: 6]

Josie has three ways of getting to school. 30% of the time she travels by car, 20% of the time she rides her bicycle and 50% of the time she walks.
When travelling by car, Josie is late 5% of the time. When riding her bicycle she is late 10% of the time. When walking she is late 25% of the time. Given that she was on time, find the probability that she rides her bicycle.

Question 45

[Maximum mark: 6]

The continuous random variable X has the probability distribution function
$f(x) = A\sin(\ln(x))$, $1 \le x \le 5$.

(a) Find the value of A to three decimal places. [2]

(b) Find the mode of X. [2]

(c) Find the value $\mathrm{P}(X \le 3 \mid X \ge 2)$. [2]

Question 46

[Maximum mark: 18]

A survey is conducted in a large office building. It is found that 30% of the office workers weigh less than 62 kg and that 25% of the office workers weigh more than 98 kg.
The weights of the office workers may be modelled by a normal distribution with mean μ and standard deviation σ.

(a) (i) Determine two simultaneous linear equations satisfied by μ and σ.

(ii) Find the values of μ and σ. [6]

(b) Find the probability that an office worker weighs more than 100 kg. [1]

There are elevators in the office building that take the office workers to their offices.
Given that there are 10 workers in a particular elevator,

(c) find the probability that at least four of the workers weigh more than 100 kg. [2]

Given that there are 10 workers in an elevator and at least one weighs more than 100 kg,

(d) find the probability that there are fewer than four workers exceeding 100 kg. [3]

The arrival of the elevators at the ground floor between $08{:}00$ and $09{:}00$ can be modelled by a Poisson distribution. Elevators arrive on average every 36 seconds.

(e) Find the probability that in any half hour period between $08{:}00$ and $09{:}00$ more than 60 elevators arrive at the ground floor. [3]

An elevator can take a maximum of 10 workers. Given that 400 workers arrive in a half hour period independently of each other,

(f) find the probability that there are sufficient elevators to take them to their offices. [3]

Question 47

[Maximum mark: 6]

The heights of students in a single year group in a large school can be modelled by a normal distribution.

It is given that 40% of the students are shorter than $1.62\,\text{m}$ and 25% are taller than $1.79\,\text{m}$.

Find the mean and standard deviation of the heights of the students.

Question 48

[Maximum mark: 8]

Students sign up at a desk for an activity during the course of an afternoon. The arrival of each student is independent of the arrival of any other student and the number of students arriving per hour can be modelled as a Poisson distribution with a mean of λ.

The desk is open for 4 hours. If exactly 5 people arrive to sign up for the activity during that time find the probability that exactly 3 of them arrived during the first hour.

Question 49

[Maximum mark: 22]

Six balls numbered $1, 2, 2, 3, 3, 3$ are placed in a bag. Balls are taken one at a time from the bag at random and the number noted. Throughout the question a ball is always replaced before the next ball is taken.

(a) A single ball is taken from the bag. Let X denote the value shown on the ball. Find $\mathrm{E}(X)$. [2]

(b) Three balls are taken from the bag. Find the probability that

(i) the total of the three numbers is 5;

(ii) the median of the three numbers is 1. [6]

(c) Ten balls are taken from the bag. Find the probability that less than four of the balls are numbered 2. [3]

(d) Find the least number of balls that must be taken from the bag for the probability of taking out at least one ball numbered 2 to be greater than 0.95. [3]

(e) Another bag also contains balls numbered $1, 2$ or 3.
Eight balls are to be taken from this bag at random. It is calculated that the expected number of balls numbered 1 is 4.8, and the variance of the number of balls numbered 2 is 1.5.

Find the least possible number of balls numbered 3 in this bag. [8]

Question 50

[Maximum mark: 7]

A random variable X is normally distributed with mean 3 and variance 2^2.

(a) Find $\mathrm{P}(0 \le X \le 2)$. [2]

(b) Find $\mathrm{P}(|X| > 1)$. [3]

(c) If $\mathrm{P}(X > c) = 0.44$, find the value of c. [2]

Question 51

[Maximum mark: 8]

A company produces rectangular sheets of glass of area 5 square metres. During manufacturing these glass sheets flaws occur at the rate of 0.5 per 5 square metres. It is assumed that the number of flaws per glass sheet follows a Poisson distribution.

(a) Find the probability that a randomly chosen glass sheet contains at least one flaw. [3]

Glass sheets with no flaws earn a profit of $\$5$. Glass sheets with at least one flaw incur a loss of $\$3$.

(b) Find the expected profit, P dollars, per glass sheet. [3]

This company also produces larger glass sheets of area 20 square metres. The rate of occurrence of flaws remains at 0.5 per 5 square metres.
A larger glass sheet is chosen at random.

(c) Find the probability that it contains no flaws. [2]

Question 52

[Maximum mark: 15]

A continuous random variable T has probability density function f defined by

$$f(t) = \begin{cases} \dfrac{t|\sin 2t|}{\pi}, & 0 \le t \le \pi \\ 0, & \text{otherwise} \end{cases}$$

(a) Sketch the graph of $y = f(t)$. [2]

(b) Use your sketch to find the mode of T. [1]

(c) Find the mean of T. [2]

(d) Find the variance of T. [3]

(e) Find the probability that T lies between the mean and the mode. [2]

(f) (i) Find $\int_0^T f(t)\,\mathrm{d}t$ where $0 \le T \le \dfrac{\pi}{2}$.

(ii) Hence verify that the lower quartile of T is $\dfrac{\pi}{2}$. [5]

Question 53

[Maximum mark: 5]

A random variable X has a probability distribution given in the following table.

x	0.5	1.5	2.5	3.5	4.5	5.5
$P(X=x)$	0.12	0.18	0.20	0.28	0.14	0.08

(a) Determine the value of $E(X^2)$. [2]

(b) Find the value of $\text{Var}(X)$. [3]

Question 54

[Maximum mark: 6]

A discrete random variable X follows a Poisson distribution $\text{Po}(\mu)$.

(a) Show that $P(X = x+1) = \frac{\mu}{x+1} \times P(X = x), x \in \mathbb{N}$. [3]

(b) Given that $P(X=2) = 0.241667$ and $P(X=3) = 0.112777$, use part (a) to find the value of μ. [3]

Question 55

[Maximum mark: 8]

A random variable X is normally distributed with mean μ and standard deviation σ, such that $P(X < 30.31) = 0.1180$ and $P(X > 42.52) = 0.3060$.

(a) Find μ and σ. [6]

(b) Find $P(|X - \mu| < 1.2\sigma)$. [2]

Question 56

[Maximum mark: 20]

A Chocolate Shop advertises free gifts to customers that collect three vouchers.
The vouchers are placed at random into 10% of all chocolate bars sold at this shop.
Kati buys some of these bars and she opens them one at a time to see if they contain a voucher. Let $\mathrm{P}(X=n)$ be the probability that Kati obtains her third voucher on the nth bar opened.

(It is assumed that the probability that a chocolate bar contains a voucher stays at 10% throughout the question.)

(a) Show that $\mathrm{P}(X=3)=0.001$ and $\mathrm{P}(X=4)=0.0027$. [3]

It is given that $\mathrm{P}(X=n)=\dfrac{n^2+an+b}{2000}\times 0.9^{n-3}$ for $n \geq 3$, $n \in \mathbb{N}$.

(b) Find the values of the constants a and b. [5]

(c) Deduce that $\dfrac{\mathrm{P}(X=n)}{\mathrm{P}(X=n-1)}=\dfrac{0.9(n-1)}{n-3}$ for $n>3$. [4]

(d) (i) Hence show that X has two modes m_1 and m_2.

(ii) State the values of m_1 and m_2. [5]

Kati's mother goes to the shop and buys x chocolate bars. She takes the bars home for Kati to open.

(e) Determine the minimum value of x such that the probability Kati receives at least one free gift is greater than 0.5. [3]

Question 57

[Maximum mark: 6]

Consider two events A and B such that $\mathrm{P}(A)=k$, $\mathrm{P}(B)=3k$, $\mathrm{P}(A\cap B)=k^2$ and $\mathrm{P}(A\cup B)=0.5$.

(a) Calculate k; [3]

(b) Find $\mathrm{P}(A'\cap B)$. [3]

Question 58

[Maximum mark: 6]

When carpet is manufactured, small faults occur at random. The number of faults in Premium carpets can be modelled by a Poisson distribution with mean 0.5 faults per $20\,\mathrm{m}^2$.

Mr Jones chooses Premium carpets to replace the carpets in his office building. The office building has 10 rooms, each with the area of $80\,\mathrm{m}^2$.

(a) Find the probability that the carpet laid in the first room has fewer than three faults. [3]

(b) Find the probability that exactly seven rooms will have fewer than three faults in the carpet. [3]

Question 59

[Maximum mark: 8]

The times taken for male runners to complete a marathon can be modelled by a normal distribution with a mean 196 minutes and a standard deviation 24 minutes.

(a) Find the probability that a runner selected at random will complete the marathon in less than 3 hours. [2]

It is found that 5% of the male runners complete the marathon in less than T_1 minutes.

(b) Calculate T_1. [2]

The times taken for female runners to complete the marathon can be modelled by a normal distribution with a mean 210 minutes. It is found that 58% of female runners complete the marathon between 185 and 235 minutes.

(c) Find the standard deviation of the times taken by female runners. [4]

Question 60

[Maximum mark: 4]

There are 75 players in a golf club who take part in a golf tournament. The scores obtained on the 18th hole are as shown in the following table.

Score	2	3	4	5	6	7
Frequency	3	15	28	17	9	3

(a) One of the players is chosen at random. Find the probability that this player's score was 5 or more. [2]

(b) Calculate the mean score. [2]

Question 61

[Maximum mark: 7]

Packets of biscuits are produced by a machine. The weights X, in grams, of packets of biscuits can be modelled by a normal distribution where $X \sim N(\mu, \sigma^2)$. A packet of biscuits is considered to be underweight if it weighs less than 250 grams.

(a) Given that $\mu = 253$ and $\sigma = 1.5$ find the probability that a randomly chosen packet of biscuits is underweight. [2]

The manufacturer makes the decision that the probability that a packet is underweight should be 0.002. To do this μ is increased and σ remains unchanged.

(b) Calculate the new value of μ giving your answer correct to two decimal places. [3]

The manufacturer is happy with the decision that the probability that a packet is underweight should be 0.002, but is unhappy with the way in which this was achieved. The machine is now adjusted to reduce σ and return μ to 253.

(c) Calculate the new value of σ. [2]

Question 62

[Maximum mark: 9]

John likes to go sailing every day in July. To help him make a decision on whether it is safe to go sailing he classifies each day in July as windy or calm. Given that a day in July is calm, the probability that the next day is calm is 0.9. Given that a day in July is windy, the probability that the next day is calm is 0.3. The weather forecast for the 1st July predicts that the probability that it will be calm is 0.8.

(a) Draw a tree diagram to represent this information for the first three days of July. [3]

(b) Find the probability that the 3rd July is calm. [2]

(c) Find the probability that the 1st July was calm given that the 3rd July is windy. [4]

Question 63

[Maximum mark: 15]

A continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \dfrac{x^2}{a} + b, & 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$ where a and b are positive constants.

It is given that $P(X \ge 2) = 0.75$.

(a) Show that $a = 32$ and $b = \dfrac{1}{12}$. [5]

(b) Find $E(X)$. [2]

(c) Find $\text{Var}(X)$. [2]

(d) Find the median of X. [3]

Eight independent observations of X are now taken and the random variable Y is the number of observations such that $X \ge 2$.

(e) Find $E(Y)$. [2]

(f) Find $P(Y \ge 3)$. [1]

Question 64

[Maximum mark: 6]

Events A and B are such that $P(A \cup B) = 0.95$, $P(A \cap B) = 0.6$ and $P(A \mid B) = 0.75$.

(a) Find $P(B)$. [2]

(b) Find $P(A)$. [2]

(c) Hence show that events A' and B are independent. [2]

Question 65

[Maximum mark: 6]

It is given that one in five cups of coffee contain more than $120\,\text{mg}$ of caffeine.
It is also known that three in five cups contain more than $110\,\text{mg}$ of caffeine.

Assume that the caffeine content of coffee is modelled by a normal distribution.
Find the mean and standard deviation of the caffeine content of coffee.

Question 66

[Maximum mark: 6]

The number of bananas that Lucca eats during any particular day follows a Poisson distribution with mean 0.2.

(a) Find the probability that Lucca eats at least one banana in a particular day. [2]

(b) Find the expected number of weeks in the year in which Lucca eats no bananas. [4]

Question 67

[Maximum mark: 6]

The random variable X has a normal distribution with mean $\mu = 50$ and variance $\sigma^2 = 16$.

(a) Sketch the probability density function for X, and shade the region representing $P(\mu - 2\sigma < X < \mu + \sigma)$. [2]

(b) Find the value of $P(\mu - 2\sigma < X < \mu + \sigma)$. [2]

(c) Find the value of k for which $P(\mu - k\sigma < X < \mu + k\sigma) = 0.5$. [2]

Question 68

[Maximum mark: 7]

The random variable X has a binomial distribution with parameters n and p.
It is given that $E(X) = 3.5$.

(a) Find the least possible value of n. [2]

It is further given that $P(X \le 1) = 0.09478$ correct to 4 significant figures.

(b) Determine the value of n and the value of p. [5]

Question 69

[Maximum mark: 13]

The number of taxis arriving at Cardiff Central railway station can be modelled by a Poisson distribution. During busy periods of the day, taxis arrive at a mean rate of 5.3 taxis every 10 minutes. Let T represent a random 10 minute busy period.

(a) (i) Find the probability that exactly 4 taxis arrive during T.

(ii) Find the most likely number of taxis that would arrive during T.

(iii) Given that more than 5 taxis arrive during T, find the probability that exactly 7 taxis arrive during T. [7]

During quiet periods of the day, taxis arrive at a mean rate of 1.3 taxis every 10 minutes.

(b) Find the probability that during a period of 15 minutes, of which the first 10 minutes is busy and the next 5 minutes is quiet, that exactly 2 taxis arrive. [6]

Question 70

[Maximum mark: 5]

The age, L, in years, of a wolf can be modelled by the normal distribution $L \sim \mathrm{N}(8, 5)$.

(a) Find the probability that a wolf selected at random is at least 5 years old. [2]

Eight wolves are independently selected at random and their ages recorded.

(b) Find the probability that more than six of these wolves are at least 5 years old. [3]

Question 71

[Maximum mark: 5]

The mean number of squirrels in a certain area is known to be 3.2 squirrels per hectare of woodland. Within this area, there is a 56 hectare woodland nature reserve. It is known that there are currently at least 168 squirrels in this reserve.

Assuming the population of squirrels follow a Poisson distribution, calculate the probability that there are more than 190 squirrels in the reserve.

Question 72

[Maximum mark: 7]

Each of the 25 students in a class are asked how many pets they own. Two students own three pets and no students own more than three pets. The mean and standard deviation of the number of pets owned by students in the class are $\frac{18}{25}$ and $\frac{24}{25}$ respectively.

Find the number of students in the class who do not own a pet.

Question 73

[Maximum mark: 13]

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} 3ax & , \quad 0 \le x < 0.5 \\ a(2-x) & , \quad 0.5 \le x < 2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

(a) Show that $a = \frac{2}{3}$. [3]

(b) Find $\mathrm{P}(X < 1)$. [3]

(c) Given that $\mathrm{P}(s < X < 0.8) = 2 \times \mathrm{P}(2s < X < 0.8)$, and that $0.25 < s < 0.4$, find the value of s. [7]

Question 74

[Maximum mark: 8]

It is known that 56 % of Infiglow batteries have a life of less than 16 hours, and 94 % have a life less than 17 hours. It can be assumed that battery life is modelled by the normal distribution $N(\mu, \sigma^2)$.

(a) Find the value of μ and the value of σ. [6]

(b) Find the probability that a randomly selected Infiglow battery will have a life of at least 15 hours. [2]

Question 75

[Maximum mark: 18]

Willow finds that she receives approximately 70 emails per working day.
She decides to model the number of emails received per working day using the random variable X, where X follows a Poisson distribution with mean 70.

(a) Using this distribution model, find

(i) $P(X < 60)$

(ii) the standard deviation of X. [4]

(b) In order to test her model, Willow records the number of emails she receives per working day over a period of 6 months. The results are shown in the following table.

Number of emails received (x)	Number of days
$40 \le x \le 49$	2
$50 \le x \le 59$	15
$60 \le x \le 69$	40
$70 \le x \le 79$	53
$80 \le x \le 89$	0
$90 \le x \le 99$	1
$100 \le x \le 109$	3
$110 \le x \le 119$	6

From the table, calculate

(i) an estimate for the mean number of emails received per working day;

(ii) an estimate for the standard deviation of the number of emails received per working day. [5]

(c) Give one piece of evidence that suggests Willow's Poisson distribution model is not a good fit. [1]

Archie works for a different company and knows that he receives emails according to a Poisson distribution, with a mean of λ emails per day.

Archie works for a different company and knows that he receives emails according to a Poisson distribution, with a mean of λ emails per day.

(d) Suppose that the probability of Archie receiving more than 10 emails in total on any one day is 0.99. Find the value of λ. [3]

(e) Now suppose that Archie received exactly 20 emails in total in a consecutive two day period. Show that the probability that he received exactly 10 of them on the first day is independent of λ . [5]

Question 76

[Maximum mark: 5]

Timmy owns a shop. His daily income from selling his goods can be modelled as a normal distribution, with a mean daily income of \$820, and a standard deviation of \$230. To make a profit, Timmy's daily income needs to be greater than \$1000.

(a) Calculate the probability that, on a randomly selected day, Timmy makes a profit. [2]

The shop is open for 24 days every month.

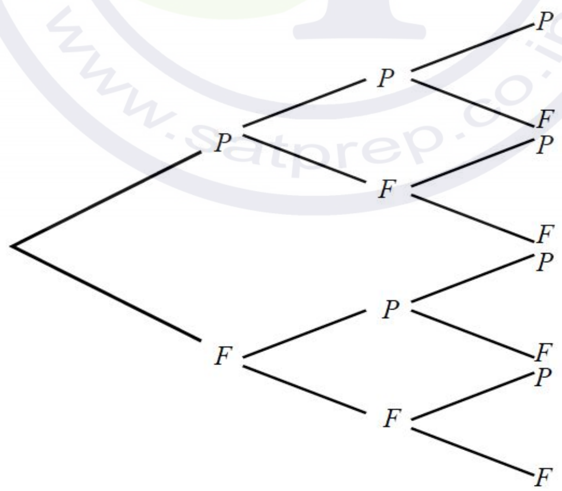
(b) Calculate the probability that, in a randomly selected month, Timmy makes a profit on between 5 and 10 days (inclusive). [3]

Question 77

[Maximum mark: 8]

Iqbal attempts three practice papers in mathematics. The probability that he passes the first paper is 0.6. Whenever he gains a pass in a paper, his confidence increases so that the probability of him passing the next paper increases by 0.1. Whenever he fails a paper the probability of him passing the next paper is 0.6.

(a) Complete the given probability tree diagram for Iqbal's three attempts, labelling each branch with the correct probability. [3]



(b) Calculate the probability that Iqbal passes at least two of the papers he attempts. [2]

(c) Find the probability that Iqbal passes his third paper, given that he passed only one previous paper. [3]

Question 78

[Maximum mark: 16]

Steffi the stray cat often visits Will's house in search of food. Let X be the discrete random variable "the number of times per day that Steffi visits Will's house".
The random variable X can be modelled by a Poisson distribution with mean 2.1.

(a) Find the probability that on a randomly selected day, Steffi does not visit Will's house. [2]

Let Y be the discrete random variable "the number of times per day that Steffi is fed at Will's house". Steffi is only fed on the first four occasions that she visits each day.

(b) Copy and complete the probability distribution table for Y. [4]

y	0	1	2	3	4
$P(Y=y)$					

(c) Hence find the expected number of times per day that Steffi is fed at Will's house. [3]

(d) In any given year of 365 days, the probability that Steffi does not visit Will for at most n days in total is 0.5 (to one decimal place). Find the value of n. [3]

(e) Show that the expected number of occasions per year on which Steffi visits Will's house and is not fed is at least 30. [4]

Question 79

[Maximum mark: 7]

The marks achieved by eight students in a class test are given in the following list.

8	4	7	6	10	9	7	3

(a) Find

(i) the mean;

(ii) the standard deviation. [2]

(b) The teacher increases all the marks by 2. Write down the new value for

(i) the mean;

(ii) the standard deviation. [2]

A ninth student also takes the test.

(c) Explain why the median is unchanged. [3]

Question 80

[Maximum mark: 13]

A café serves sandwiches and cakes. Each customer will choose one of the following three options; buy only a sandwich, buy only a cake or buy both a sandwich and a cake.
The probability that a customer buys a sandwich is 0.72 and the probability that a customer buys a cake is 0.45.

(a) Find the probability that a customer chosen at random will buy

(i) both a sandwich and a cake;

(ii) only a sandwich. [4]

On a typical day 200 customers come to the café.

(b) Find

(i) the expected number of cakes sold on a typical day;

(ii) the probability that more than 100 cakes will be sold on a typical day. [4]

It is known that 46 % of the customers who come to the café are male, and that 80 % of these buy a sandwich.

(c) (i) A customer is selected at random. Find the probability that the customer is male and buys a sandwich.

(ii) A female customer is selected at random. Find the probability that she buys a sandwich. [5]

Question 81

[Maximum mark: 6]

The number of marathons that Audrey runs in any given year can be modelled by a Poisson distribution with mean 1.3.

(a) Calculate the probability that Audrey will run at least two marathons in a particular year. [2]

(b) Find the probability that she will run at least two marathons in exactly four out of the following five years. [4]

Question 82

[Maximum mark: 6]

Runners in an athletics club have season's best times for the 100m, which can be modelled by a normal distribution with mean 11.6 seconds and standard deviation 0.8 seconds.
To qualify for a particular competition a runner must have a season's best time of under 11 seconds. A runner from this club who has qualified for the competition is selected at random. Find the probability that he has a season's best time of under 10.7 seconds.

Question 83

[Maximum mark: 19]

A random variable X has probability density function

$$f(x)=\begin{cases} 3a & , \quad 0\le x<2 \\ a(x-5)(1-x) & , \quad 2\le x\le b \\ 0 & , \quad \text{otherwise} \end{cases} \qquad a,\, b\in\mathbb{R}^{+},\, 3<b\le 5.$$

(a) Find, in terms of a, the probability that X lies between 1 and 3. [4]

Consider the case where $b=5$.

(b) Sketch the graph of f. State the coordinates of the end points and any local maximum or minimum points, giving your answers in terms of a. [4]

(c) Find the value of

(i) a;

(ii) $\mathrm{E}(X)$;

(iii) the median of X. [11]