Subject – Math(Higher Level)
Topic - Vector
Year - Nov 2011 – Nov 2019

Question 1

[Maximum mark: 15]

(a) For non-zero vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, show that

(i) if $|\boldsymbol{a}-\boldsymbol{b}| = |\boldsymbol{a}+\boldsymbol{b}|$, then $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular;

(ii) $|\boldsymbol{a}\times\boldsymbol{b}|^2 = |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a}\bullet\boldsymbol{b})^2$. *[8 marks]*

(b) The points A, B and C have position vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$.

(i) Show that the area of triangle ABC is $\frac{1}{2}|\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{b}\times\boldsymbol{c}+\boldsymbol{c}\times\boldsymbol{a}|$.

(ii) Hence, show that the shortest distance from B to AC is

$$\frac{|\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{b}\times\boldsymbol{c}+\boldsymbol{c}\times\boldsymbol{a}|}{|\boldsymbol{c}-\boldsymbol{a}|}.$$ *[7 marks]*

Question 2

[Maximum mark: 5]

Find the values of x for which the vectors $\begin{pmatrix}1\\2\cos x\\0\end{pmatrix}$ and $\begin{pmatrix}-1\\2\sin x\\1\end{pmatrix}$ are perpendicular, $0 \le x \le \frac{\pi}{2}$.

Question 3

[Maximum mark: 6]

Two boats, A and B, move so that at time t hours, their position vectors, in kilometres, are $\boldsymbol{r}_A = (9t)\boldsymbol{i} + (3-6t)\boldsymbol{j}$ and $\boldsymbol{r}_B = (7-4t)\boldsymbol{i} + (7t-6)\boldsymbol{j}$.

(a) Find the coordinates of the common point of the paths of the two boats. *[4 marks]*

(b) Show that the boats do not collide. *[2 marks]*

Question 4

[Maximum mark: 21]

The vertices of a triangle ABC have coordinates given by $A(-1, 2, 3)$, $B(4, 1, 1)$ and $C(3, -2, 2)$.

(a) (i) Find the lengths of the sides of the triangle.

(ii) Find $\cos B\hat{A}C$. *[6 marks]*

(b) (i) Show that $\overrightarrow{BC} \times \overrightarrow{CA} = -7\boldsymbol{i} - 3\boldsymbol{j} - 16\boldsymbol{k}$.

(ii) Hence, show that the area of the triangle ABC is $\frac{1}{2}\sqrt{314}$. *[5 marks]*

(c) Find the Cartesian equation of the plane containing the triangle ABC. *[3 marks]*

(d) Find a vector equation of (AB). *[2 marks]*

The point D on (AB) is such that $\overrightarrow{OD}$ is perpendicular to $\overrightarrow{BC}$ where O is the origin.

(e) (i) Find the coordinates of D.

(ii) Show that D does not lie between A and B. *[5 marks]*

Question 5

[Maximum mark: 20]

Consider the points $A(1, 0, 0)$, $B(2, 2, 2)$ and $C(0, 2, 1)$.

(a) Find the vector $\overrightarrow{CA} \times \overrightarrow{CB}$. *[4]*

(b) Find an exact value for the area of the triangle ABC. *[3]*

(c) Show that the Cartesian equation of the plane Π_1, containing the triangle ABC, is $2x + 3y - 4z = 2$. *[3]*

A second plane Π_2 is defined by the Cartesian equation $\Pi_2: 4x - y - z = 4$. L_1 is the line of intersection of the planes Π_1 and Π_2.

(d) Find a vector equation for L_1. *[5]*

A third plane Π_3 is defined by the Cartesian equation $16x + \alpha y - 3z = \beta$.

(e) Find the value of α if all three planes contain L_1. *[3]*

(f) Find conditions on α and β if the plane Π_3 does **not** intersect with L_1. *[2]*

Question 6

[Maximum mark: 22]

(a) Show that the points $O(0,0,0)$, $A(6,0,0)$, $B\left(6,-\sqrt{24},\sqrt{12}\right)$, $C\left(0,-\sqrt{24},\sqrt{12}\right)$ form a square. *[3]*

(b) Find the coordinates of M, the mid-point of [OB]. *[1]*

(c) Show that an equation of the plane Π, containing the square OABC, is $y+\sqrt{2}z=0$. *[3]*

(d) Find a vector equation of the line L, through M, perpendicular to the plane Π. *[3]*

(e) Find the coordinates of D, the point of intersection of the line L with the plane whose equation is $y=0$. *[3]*

(f) Find the coordinates of E, the reflection of the point D in the plane Π. *[3]*

(g) (i) Find the angle $O\hat{D}A$.

(ii) State what this tells you about the solid OABCDE. *[6]*

Question 7

[Maximum mark: 6]

PQRS is a rhombus. Given that $\overrightarrow{PQ}=\boldsymbol{a}$ and $\overrightarrow{QR}=\boldsymbol{b}$,

(a) express the vectors $\overrightarrow{PR}$ and $\overrightarrow{QS}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$; *[2]*

(b) hence show that the diagonals in a rhombus intersect at right angles. *[4]*

Question 8

[Maximum mark: 18]

Given the points $A(1, 0, 4)$, $B(2, 3, -1)$ and $C(0, 1, -2)$,

(a) find the vector equation of the line L_1 passing through the points A and B. *[2]*

The line L_2 has Cartesian equation $\frac{x-1}{3}=\frac{y+2}{1}=\frac{z-1}{-2}$.

(b) Show that L_1 and L_2 are skew lines. *[5]*

Consider the plane Π_1, parallel to both lines L_1 and L_2. Point C lies in the plane Π_1.

(c) Find the Cartesian equation of the plane Π_1. *[4]*

The line L_3 has vector equation $\boldsymbol{r} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} k \\ 1 \\ -1 \end{pmatrix}$.

The plane Π_2 has Cartesian equation $x + y = 12$.

The angle between the line L_3 and the plane Π_2 is 60°.

(d) (i) Find the value of k.

(ii) Find the point of intersection P of the line L_3 and the plane Π_2. *[7]*

Question 9

[Maximum mark: 5]

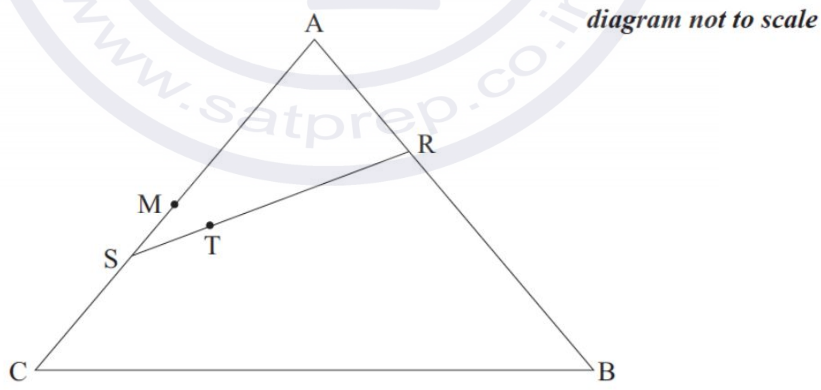
A point P, relative to an origin O, has position vector $\overrightarrow{OP} = \begin{pmatrix} 1+s \\ 3+2s \\ 1-s \end{pmatrix}$, $s \in \mathbb{R}$.

Find the minimum length of $\overrightarrow{OP}$.

Question 10

[Maximum mark: 14]

The position vectors of the points A, B and C are $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ respectively, relative to an origin O. The following diagram shows the triangle ABC and points M, R, S and T.



M is the midpoint of [AC].

R is a point on [AB] such that $\overrightarrow{AR} = \frac{1}{3}\overrightarrow{AB}$.

S is a point on [AC] such that $\overrightarrow{AS} = \frac{2}{3}\overrightarrow{AC}$.

T is a point on [RS] such that $\overrightarrow{RT} = \frac{2}{3}\overrightarrow{RS}$.

(a) (i) Express $\vec{AM}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{c}$.

(ii) Hence show that $\vec{BM} = \frac{1}{2}\boldsymbol{a} - \boldsymbol{b} + \frac{1}{2}\boldsymbol{c}$. *[4]*

(b) (i) Express $\vec{RA}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(ii) Show that $\vec{RT} = -\frac{2}{9}\boldsymbol{a} - \frac{2}{9}\boldsymbol{b} + \frac{4}{9}\boldsymbol{c}$. *[5]*

(c) Prove that T lies on [BM]. *[5]*

Question 11

[Maximum mark: 21]

Two lines l_1 and l_2 are given respectively by the equations $\boldsymbol{r_1} = \vec{OA} + \lambda\boldsymbol{v}$ and $\boldsymbol{r_2} = \vec{OB} + \mu\boldsymbol{w}$ where $\vec{OA} = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$, $\boldsymbol{v} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$, $\vec{OB} = 2\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$, $\boldsymbol{w} = \boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and O is the origin.
Let P be a point on l_1 and let Q be a point on l_2.

(a) Find $\vec{PQ}$, in terms of λ and μ. [2]

(b) Find the value of λ and the value of μ for which $\vec{PQ}$ is perpendicular to the direction vectors of both l_1 and l_2. [5]

(c) Hence find the shortest distance between l_1 and l_2. [3]

(d) Find the Cartesian equation of the plane Π, which contains line l_1 and is parallel to the direction vector of line l_2. [5]

Let $\vec{OT} = \vec{OB} + \eta(\boldsymbol{v} \times \boldsymbol{w})$.

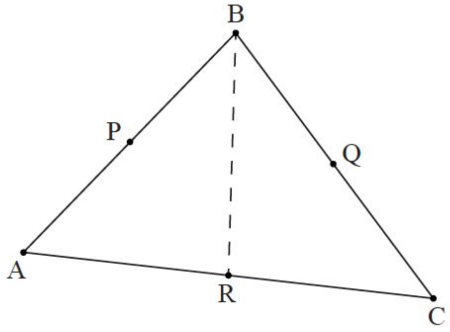
(e) Find the value of η for which the point T lies in the plane Π. [2]

(f) For this value of η, calculate $\left|\vec{BT}\right|$. [2]

(g) State what you notice about your answers to (c) and (f), and give a geometrical interpretation of this result. [2]

Question 12

[Maximum mark: 23]



Consider the triangle ABC. The points P, Q and R are the midpoints of the line segments $[\mathrm{AB}]$, $[\mathrm{BC}]$ and $[\mathrm{AC}]$ respectively.

Let $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$, $\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$ and $\overrightarrow{\mathrm{OC}} = \boldsymbol{c}$.

(a) Find $\overrightarrow{\mathrm{BR}}$ in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. [2]

(b) (i) Find a vector equation of the line that passes through B and R in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ and a parameter λ.

(ii) Find a vector equation of the line that passes through A and Q in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ and a parameter μ.

(iii) Hence show that $\overrightarrow{\mathrm{OG}} = \dfrac{1}{3}(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$ given that G is the point where $[\mathrm{BR}]$ and $[\mathrm{AQ}]$ intersect. [9]

(c) Show that the line segment $[\mathrm{CP}]$ also includes the point G. [3]

The coordinates of the points A, B and C are $(1, 3, 1)$, $(3, 7, -5)$ and $(2, 2, 1)$ respectively.
A point X is such that $[\mathrm{GX}]$ is perpendicular to the plane ABC.

(d) Given that the tetrahedron ABCX has volume 12 units^3, find possible coordinates of X. [9]

Question 13

[Maximum mark: 21]

Two planes have equations

$$\Pi_1 : 4x + y + z = 8 \text{ and } \Pi_2 : 4x + 3y - z = 0$$

(a) Find the cosine of the angle between the two planes in the form $\sqrt{\frac{p}{q}}$ where $p, q \in \mathbb{Z}$. [4]

Let L be the line of intersection of the two planes.

(b) (i) Show that L has direction $\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$.

(ii) Show that the point $A(1, 0, 4)$ lies on both planes.

(iii) Write down a vector equation of L. [6]

B is the point on Π_1 with coordinates $(a, b, 1)$.

(c) Given the vector $\overrightarrow{AB}$ is perpendicular to L find the value of a and the value of b. [5]

(d) Show that $AB = 3\sqrt{2}$. [1]

The point P lies on L and $A\hat{B}P = 45°$.

(e) Find the coordinates of the two possible positions of P. [5]

Question 14

[Maximum mark: 5]

Consider the vectors $\boldsymbol{a} = \boldsymbol{i} - 3\boldsymbol{j} - 2\boldsymbol{k}$, $\boldsymbol{b} = -3\boldsymbol{j} + 2\boldsymbol{k}$.

(a) Find $\boldsymbol{a} \times \boldsymbol{b}$. [2]

(b) Hence find the Cartesian equation of the plane containing the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, and passing through the point $(1, 0, -1)$. [3]

Question 15

[Maximum mark: 6]

Consider the lines l_1 and l_2 defined by

$$l_1 : \boldsymbol{r} = \begin{pmatrix} -3 \\ -2 \\ a \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \text{ and } l_2 : \frac{6-x}{3} = \frac{y-2}{4} = 1 - z \text{ where } a \text{ is a constant.}$$

Given that the lines l_1 and l_2 intersect at a point P,

(a) find the value of a; [4]

(b) determine the coordinates of the point of intersection P. [2]

Question 16

[Maximum mark: 7]

ABCD is a parallelogram, where $\overrightarrow{\mathrm{AB}} = -\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$ and $\overrightarrow{\mathrm{AD}} = 4\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$.

(a) Find the area of the parallelogram ABCD. [3]

(b) By using a suitable scalar product of two vectors, determine whether $\mathrm{A\hat{B}C}$ is acute or obtuse. [4]

Question 17

[Maximum mark: 6]

The points A and B are given by $\mathrm{A}(0, 3, -6)$ and $\mathrm{B}(6, -5, 11)$.

The plane Π is defined by the equation $4x - 3y + 2z = 20$.

(a) Find a vector equation of the line L passing through the points A and B. [3]

(b) Find the coordinates of the point of intersection of the line L with the plane Π. [3]

Question 18

[Maximum mark: 5]

(a) Show that the following system of equations has an infinite number of solutions. *[2]*

$$\begin{aligned} x + y + 2z &= -2 \\ 3x - y + 14z &= 6 \\ x + 2y &= -5 \end{aligned}$$

The system of equations represents three planes in space.

(b) Find the parametric equations of the line of intersection of the three planes. *[3]*

Question 19

[Maximum mark: 6]

The following system of equations represents three planes in space.

$$x + 3y + z = -1$$
$$x + 2y - 2z = 15$$
$$2x + y - z = 6$$

Find the coordinates of the point of intersection of the three planes.

Question 20

[Maximum mark: 5]

O, A, B and C are distinct points such that $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$, $\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$ and $\overrightarrow{\mathrm{OC}} = \boldsymbol{c}$.
It is given that $\boldsymbol{c}$ is perpendicular to $\overrightarrow{\mathrm{AB}}$ and $\boldsymbol{b}$ is perpendicular to $\overrightarrow{\mathrm{AC}}$.

Prove that $\boldsymbol{a}$ is perpendicular to $\overrightarrow{\mathrm{BC}}$.

Question 21

[Maximum mark: 18]

A line L has equation $\frac{x-2}{p} = \frac{y-q}{2} = z - 1$ where $p, q \in \mathbb{R}$.

A plane Π has equation $x + y + 3z = 9$.

(a) Show that L is not perpendicular to Π. [3]

(b) Given that L lies in the plane Π, find the value of p and the value of q. [4]

Consider the different case where the acute angle between L and Π is θ where $\theta = \arcsin\left(\frac{1}{\sqrt{11}}\right)$.

(c) (i) Show that $p = -2$.

(ii) If L intersects Π at $z = -1$, find the value of q. [11]

Question 22

[Maximum mark: 5]

Find the coordinates of the point of intersection of the planes defined by the equations $x + y + z = 3$, $x - y + z = 5$ and $x + y + 2z = 6$.

Question 23

[Maximum mark: 6]

Consider the lines l_1 and l_2 defined by

$$l_1 : \boldsymbol{r} = \begin{pmatrix} -3 \\ -2 \\ a \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \text{ and } l_2 : \frac{6-x}{3} = \frac{y-2}{4} = 1 - z \text{ where } a \text{ is a constant.}$$

Given that the lines l_1 and l_2 intersect at a point P,

(a) find the value of a; [4]

(b) determine the coordinates of the point of intersection P. [2]

Question 24

[Maximum mark: 7]

Consider the following equations, where $a, b \in \mathbb{R}$:

$$x + 3y + (a-1)z = 1$$
$$2x + 2y + (a-2)z = 1$$
$$3x + y + (a-3)z = b.$$

(a) If each of these equations defines a plane, show that, for any value of a, the planes do not intersect at a unique point. *[3 marks]*

(b) Find the value of b for which the intersection of the planes is a straight line. *[4 marks]*

Question 25

[Maximum mark: 6]

Consider the points $\mathrm{A}(1, 2, 3)$, $\mathrm{B}(1, 0, 5)$ and $\mathrm{C}(2, -1, 4)$.

(a) Find $\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}$. *[4 marks]*

(b) Hence find the area of the triangle ABC. *[2 marks]*

Question 26

[Maximum mark: 24]

The points A, B, C and D have position vectors $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ and $\boldsymbol{d}$, relative to the origin O.
It is given that $\overrightarrow{AB} = \overrightarrow{DC}$.

(a) (i) Explain why ABCD is a parallelogram.

(ii) Using vector algebra, show that $\overrightarrow{AD} = \overrightarrow{BC}$. [4]

The position vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ are given by

$$\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$$
$$\boldsymbol{b} = 3\boldsymbol{i} - \boldsymbol{j} + p\boldsymbol{k}$$
$$\boldsymbol{c} = q\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$$
$$\boldsymbol{d} = -\boldsymbol{i} + r\boldsymbol{j} - 2\boldsymbol{k}$$

where p, q and r are constants.

(b) Show that $p = 1$, $q = 1$ and $r = 4$. [5]

(c) Find the area of the parallelogram ABCD. [4]

The point where the diagonals of ABCD intersect is denoted by M.

(d) Find the vector equation of the straight line passing through M and normal to the plane Π containing ABCD. [4]

(e) Find the Cartesian equation of Π. [3]

The plane Π cuts the x, y and z axes at X, Y and Z respectively.

(f) (i) Find the coordinates of X, Y and Z.

(ii) Find YZ. [4]

Question 27

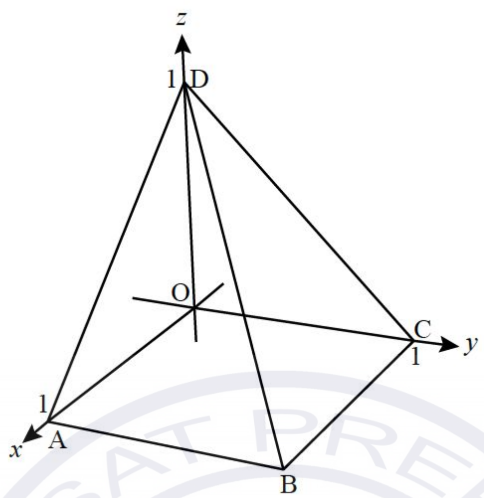
[Maximum mark: 4]

The acute angle between the vectors $3\boldsymbol{i} - 4\boldsymbol{j} - 5\boldsymbol{k}$ and $5\boldsymbol{i} - 4\boldsymbol{j} + 3\boldsymbol{k}$ is denoted by θ.
Find $\cos\theta$.

Question 28

[Maximum mark: 19]

The following figure shows a square based pyramid with vertices at $O(0, 0, 0)$, $A(1, 0, 0)$, $B(1, 1, 0)$, $C(0, 1, 0)$ and $D(0, 0, 1)$.



(a) Find the Cartesian equation of the plane Π_1, passing through the points A, B and D. [3]

The Cartesian equation of the plane Π_2, passing through the points B, C and D, is $y + z = 1$.

(b) Find the angle between the faces ABD and BCD. [4]

The plane Π_3 passes through O and is normal to the line BD.

(c) Find the Cartesian equation of Π_3. [3]

Π_3 cuts AD and BD at the points P and Q respectively.

(d) Show that P is the midpoint of AD. [4]

(e) Find the area of the triangle OPQ. [5]

Question 29

[Maximum mark: 15]

Consider a triangle OAB such that O has coordinates $(0, 0, 0)$, A has coordinates $(0, 1, 2)$ and B has coordinates $(2b, 0, b-1)$ where $b < 0$.

(a) Find, in terms of b, a Cartesian equation of the plane Π containing this triangle. [5]

Let M be the midpoint of the line segment [OB].

(b) Find, in terms of b, the equation of the line L which passes through M and is perpendicular to the plane Π. [3]

(c) Show that L does not intersect the y-axis for any negative value of b. [7]

Question 30

[Maximum mark: 6]

The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are defined by $\boldsymbol{a} = \begin{pmatrix} 1 \\ 1 \\ t \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} 0 \\ -t \\ 4t \end{pmatrix}$, where $t \in \mathbb{R}$.

(a) Find and simplify an expression for $\boldsymbol{a} \bullet \boldsymbol{b}$ in terms of t. [2]

(b) Hence or otherwise, find the values of t for which the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ is obtuse. [4]

Question 31

[Maximum mark: 6]

Three points in three-dimensional space have coordinates $\mathrm{A}(0, 0, 2)$, $\mathrm{B}(0, 2, 0)$ and $\mathrm{C}(3, 1, 0)$.

(a) Find the vector

(i) $\overrightarrow{\mathrm{AB}}$;

(ii) $\overrightarrow{\mathrm{AC}}$. [2]

(b) Hence or otherwise, find the area of the triangle ABC. [4]

Question 32

[Maximum mark: 20]

Two distinct lines, l_1 and l_2, intersect at a point P. In addition to P, four distinct points are marked out on l_1 and three distinct points on l_2. A mathematician decides to join some of these eight points to form polygons.

(a) (i) Find how many sets of four points can be selected which can form the vertices of a quadrilateral.

(ii) Find how many sets of three points can be selected which can form the vertices of a triangle. [6]

The line l_1 has vector equation $\boldsymbol{r}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\lambda \in \mathbb{R}$ and the line l_2 has vector equation

$\boldsymbol{r}_2 = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 6 \\ 2 \end{pmatrix}$, $\mu \in \mathbb{R}$.

The point P has coordinates $(4, 6, 4)$.

(b) Verify that P is the point of intersection of the two lines. [3]

The point A has coordinates $(3, 4, 3)$ and lies on l_1.

(c) Write down the value of λ corresponding to the point A. [1]

The point B has coordinates $(-1, 0, 2)$ and lies on l_2.

(d) Write down $\overrightarrow{PA}$ and $\overrightarrow{PB}$. [2]

Let C be the point on l_1 with coordinates $(1, 0, 1)$ and D be the point on l_2 with parameter $\mu = -2$.

(e) Find the area of the quadrilateral CDBA. [8]

Question 33

[Maximum mark: 4]

Let $\boldsymbol{a} = \begin{pmatrix} 2 \\ k \\ -1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -3 \\ k+2 \\ k \end{pmatrix}$, $k \in \mathbb{R}$.

Given that $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular, find the possible values of k.

Question 34

[Maximum mark: 17]

Points A$(0, 0, 10)$, B$(0, 10, 0)$, C$(10, 0, 0)$, $V(p, p, p)$ form the vertices of a tetrahedron.

(a) (i) Show that $\overrightarrow{AB} \times \overrightarrow{AV} = -10\begin{pmatrix} 10-2p \\ p \\ p \end{pmatrix}$ and find a similar expression for $\overrightarrow{AC} \times \overrightarrow{AV}$.

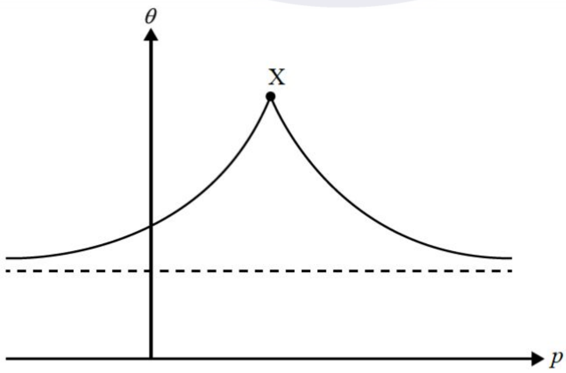
(ii) Hence, show that, if the angle between the faces ABV and ACV is θ, then $\cos\theta = \dfrac{p(3p-20)}{6p^2-40p+100}$. [8]

(b) Consider the case where the faces ABV and ACV are perpendicular.

(i) Find the two possible coordinates of V.

(ii) Comment on the positions of V in relation to the plane ABC. [4]

(c) The following diagram shows the graph of θ against p. The maximum point is shown by X.



(i) At X ,find the value p and the value of θ.

(ii) Find the equation of the horizontal asymptote of the graph. Find the equation of the horizontal asymptote of the graph. [5]

Question 35

[Maximum mark: 6]

A straight line, L_θ, has vector equation $\boldsymbol{r}=\begin{pmatrix}5\\0\\0\end{pmatrix}+\lambda\begin{pmatrix}5\\\sin\theta\\\cos\theta\end{pmatrix}$, $\lambda,\theta\in\mathbb{R}$.

The plane, Π_p, has equation $x=p$, $p\in\mathbb{R}$.

Show that the angle between L_θ and Π_p is independent of both θ and p.