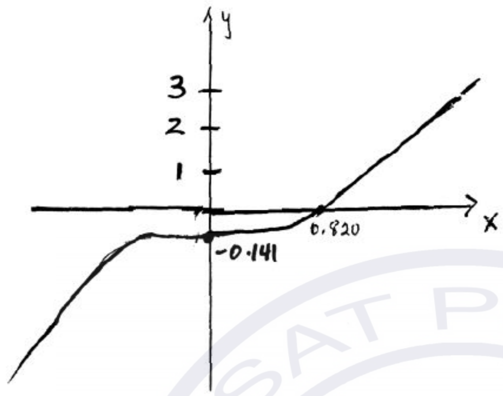
Subject – Math (Higher Level)
Topic - Functions and Equations
Year - Nov 2011 – Nov 2017
Paper -2

Question 1

(a)




***A1A1A1***

> **Note:** Award ***A1*** for shape,
> ***A1*** for $x$-intercept is 0.820, accept sin(–3) or –sin(3)
> ***A1*** for $y$-intercept is $-0.141$.

Question 2

(a) $y = \dfrac{1}{1+e^{-x}}$

$y(1+e^{-x}) = 1$ ***M1***

$1+e^{-x} = \dfrac{1}{y} \Rightarrow e^{-x} = \dfrac{1}{y} - 1$ ***A1***

$\Rightarrow x = -\ln\left(\dfrac{1}{y} - 1\right)$ ***A1***

$f^{-1}(x) = -\ln\left(\dfrac{1}{x} - 1\right) \quad \left(= \ln\left(\dfrac{x}{1-x}\right)\right)$ ***A1***

domain: $0 < x < 1$ ***A1A1***

> **Note:** Award ***A1*** for endpoints and ***A1*** for strict inequalities.

(b) 0.659 ***A1***

***[7 marks]***

Question 3

$h(x) = f(x-3) - 2 = \ln(x-3) - 2$ ***(M1)(A1)***

$g(x) = -h(x) = 2 - \ln(x-3)$ ***M1***

| |
|---|
| **te:** Award **M1** only if it is clear the effect of the reflection in the $x$-axis:<br>the expression is correct ***OR***<br>there is a change of signs of the previous expression ***OR***<br>there's a graph or an explanation making it explicit |

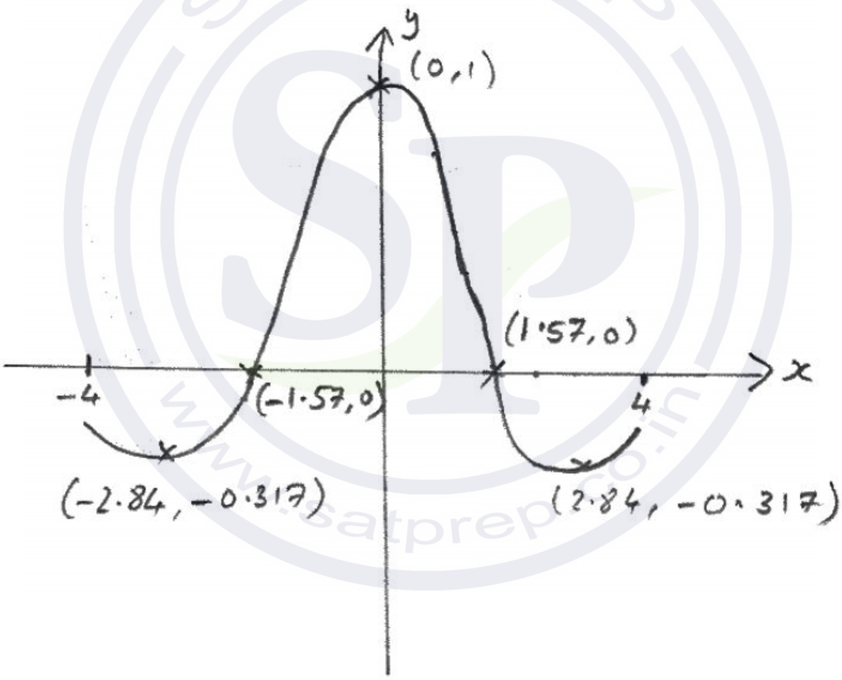
$= \ln e^2 - \ln(x-3)$ ***M1***

$= \ln\left(\frac{e^2}{x-3}\right)$ ***A1***

***[5 marks]***

Question 4

(a)




***A1A1A1A1***

Question 5

$\Delta = (5-k)^2 + 4(k+2)$ ***M1A1***

$= k^2 - 6k + 33$ ***(A1)***

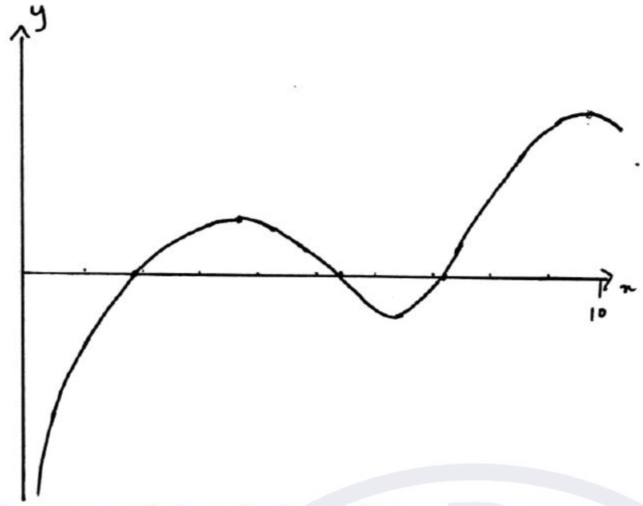
$= (k-3)^2 + 24$ which is positive for all $k$ ***R1***

| |
|---|
| **te:** Accept analytical, graphical or other correct methods. In all cases only award ***R1*** if a reason is given in words or graphically. Award ***M1A1A0R1*** if mistakes are made in the simplification but <u>the argument given is correct</u>. |

***[4 marks]***

Question 6

(a)




A correct graph shape for $0 < x \leq 10$. *A1*

maxima (3.78, 0.882) and (9.70, 1.89) *A1*

minimum (6.22, −0.885) *A1*

$x$-axis intercepts (1.97, 0), (5.24, 0) and (7.11, 0) *A2*

| **Note:** Award ***A1*** if two $x$-axis intercepts are correct. |
|---|

***[5 marks]***

(b) $0 < x \leq 1.97$ *A1*

$5.24 \leq x \leq 7.11$ *A1*

***[2 marks]***

***Total [7 marks]***

Question 7

(a) **METHOD 1**

sketch showing where the lines cross or zeros of $y = x(x+2)^6 - x$ ***(M1)***

$x = 0$ ***(A1)***

$x = -1$ and $x = -3$ ***(A1)***

the solution is $-3 < x < -1$ or $x > 0$ ***A1A1***

**Note:** Do not award either final ***A1*** mark if strict inequalities are not given.

**METHOD 2**

separating into two cases $x > 0$ and $x < 0$ ***(M1)***

if $x > 0$ then $(x+2)^6 > 1 \Rightarrow$ always true ***(M1)***

if $x < 0$ then $(x+2)^6 < 1 \Rightarrow -3 < x < -1$ ***(M1)***

so the solution is $-3 < x < -1$ or $x > 0$ ***A1A1***

**Note:** Do not award either final ***A1*** mark if strict inequalities are not given.

**METHOD 3**

$f(x) = x^7 + 12x^6 + 60x^5 + 160x^4 + 240x^3 + 192x^2 + 64x$ ***(A1)***

solutions to $x^7 + 12x^6 + 60x^5 + 160x^4 + 240x^3 + 192x^2 + 63x = 0$ are ***(M1)***

$x = 0$, $x = -1$ and $x = -3$ ***(A1)***

so the solution is $-3 < x < -1$ or $x > 0$ ***A1A1***

**Note:** Do not award either final ***A1*** mark if strict inequalities are not given.

**METHOD 4**

$f(x) = x$ when $x(x+2)^6 = x$

either $x = 0$ or $(x+2)^6 = 1$ ***(A1)***

if $(x+2)^6 = 1$ then $x + 2 = \pm 1$ so $x = -1$ or $x = -3$ ***(M1)(A1)***

the solution is $-3 < x < -1$ or $x > 0$ ***A1A1***

**Note:** Do not award either final ***A1*** mark if strict inequalities are not given.

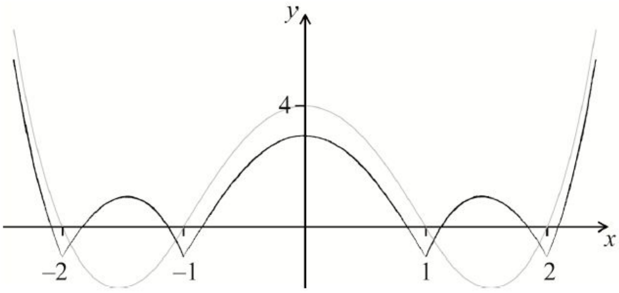
***[5 marks]***

Question 8

(a) (i) $f(0) = -1$ ***(M1)A1***

(ii) $(f \circ g)(0) = f(4) = 3$ ***A1***

(iii)

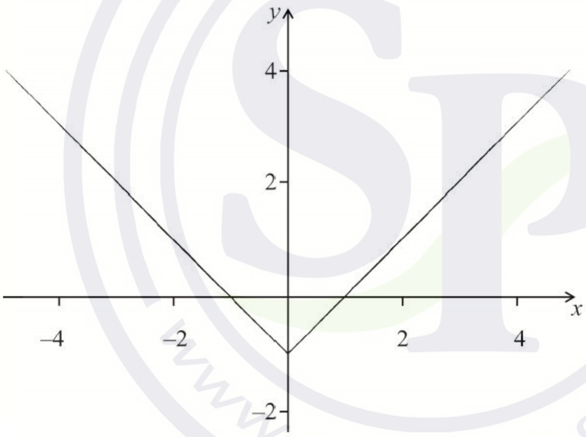



***(M1)A1***

| **Note:** Award ***M1*** for evidence that the lower part of the graph has been reflected and ***A1*** correct shape with $y$-intercept below 4. |
|---|

***[5 marks]***

(b) (i)




***(M1)A1***

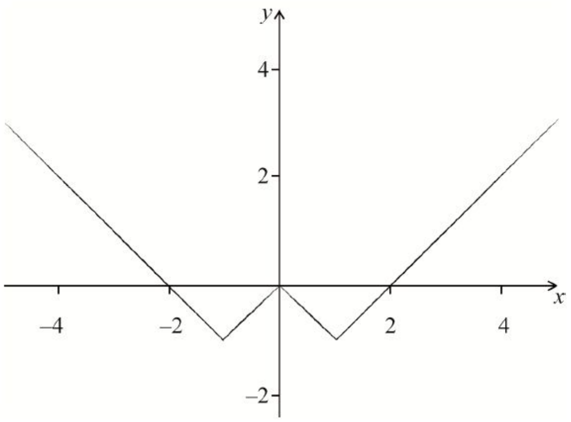
| **Note:** Award ***M1*** for any translation of $y = \lvert x \rvert$. |
|---|

(ii) $\pm 1$ ***A1***

| **Note:** Do not award the ***A1*** if coordinates given, but do not penalise in the rest of the question |
|---|

***[3 marks]***

(c) (i)




***(M1)A1***

> **Note:** Award ***M1*** for evidence that lower part of (b) has been reflected in the $x$-axis and translated.

(ii) $0, \pm 2$ ***A1***

***[3 marks]***

(d) (i) $\pm 1, \pm 3$ ***A1***

(ii) $0, \pm 2, \pm 4$ ***A1***

(iii) $0, \pm 2, \pm 4, \pm 6, \pm 8$ ***A1***

***[3 marks]***

(e) (i) $(1, 3), (2, 5), \ldots$ ***(M1)***

$N = 2n + 1$ ***A1***

(ii) Using the formula of the sum of an arithmetic series ***(M1)***

**EITHER**

$$4(1 + 2 + 3 + \ldots + n) = \frac{4}{2}n(n+1)$$

$= 2n(n+1)$ ***A1***

**OR**

$$2(2 + 4 + 6 + \ldots + 2n) = \frac{2}{2}n(2n+2)$$

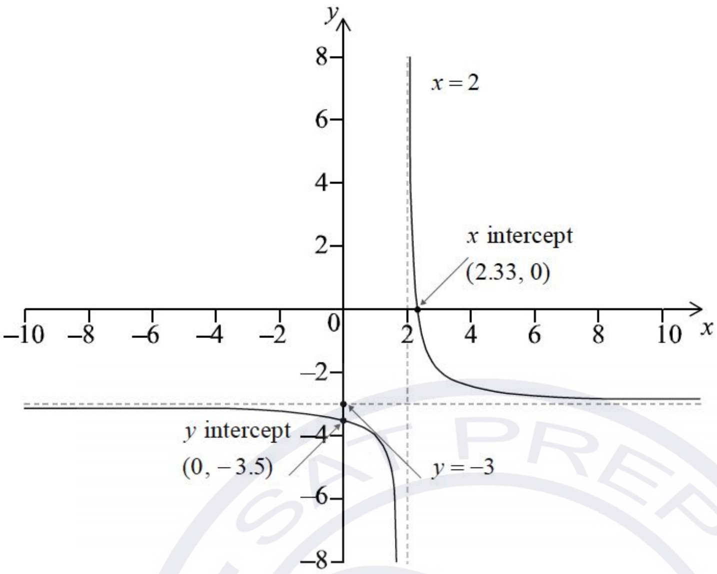
$= 2n(n+1)$ ***A1***

***[4 marks]***

***Total [18 marks]***

Question 9

(a)




***A1A1A1***

| | |
|---|---|
| **Note:** | Award ***A1*** for correct shape, ***A1*** for $x = 2$ clearly stated and ***A1*** for $y = -3$ clearly stated. |

$x$ intercept $(2.33,\ 0)$ and $y$ intercept $(0,\ -3.5)$ ***A1***

| | |
|---|---|
| **Note:** | Accept –3.5 and 2.33 (7/3) marked on the correct axes. |

***[4 marks]***

(b) $x = -3 + \dfrac{1}{y-2}$ ***M1***

| | |
|---|---|
| **Note:** | Award ***M1*** for interchanging $x$ and $y$ (can be done at a later stage). |

$$x + 3 = \frac{1}{y-2}$$

$$y - 2 = \frac{1}{x+3}$$ ***M1***

| | |
|---|---|
| **Note:** | Award ***M1*** for attempting to make $y$ the subject. |

$$f^{-1}(x) = 2 + \frac{1}{x+3}\left(= \frac{2x+7}{x+3}\right),\ x \neq -3$$ ***A1A1***

| | |
|---|---|
| **Note:** | Award ***A1*** only if $f^{-1}(x)$ is seen. Award ***A1*** for the domain. |

***[4 marks]***

***Total [8 marks]***

Question 10

using $p(a) = -7$ to obtain $3a^3 + a^2 + 5a + 7 = 0$ **M1A1**

$(a+1)(3a^2 - 2a + 7) = 0$ **(M1)(A1)**

| |
|---|
| **Note:** Award ***M1*** for a cubic graph with correct shape and ***A1*** for clearly showing that the above cubic crosses the horizontal axis at $(-1, 0)$ only. |

$a = -1$ **A1**

**EITHER**

showing that $3a^2 - 2a + 7 = 0$ has no real (two complex) solutions for $a$ **R1**

**OR**

showing that $3a^3 + a^2 + 5a + 7 = 0$ has one real (and two complex) solutions for $a$ **R1**

| |
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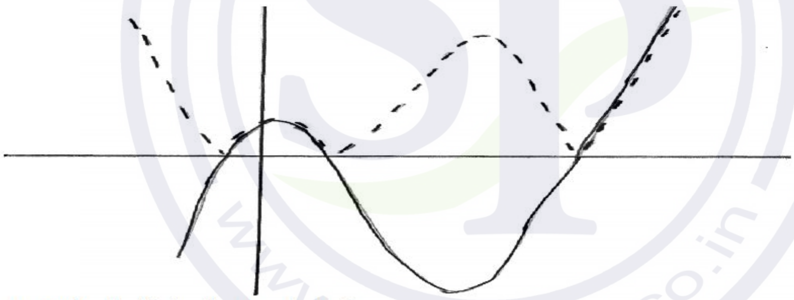
| **Note:** Award ***R1*** for solutions that make specific reference to an appropriate graph. |

***Total [6 marks]***

Question 11

**(a)**



as roots of $f(x) = 0$ are $-1, 1, 5$ ***(M1)***

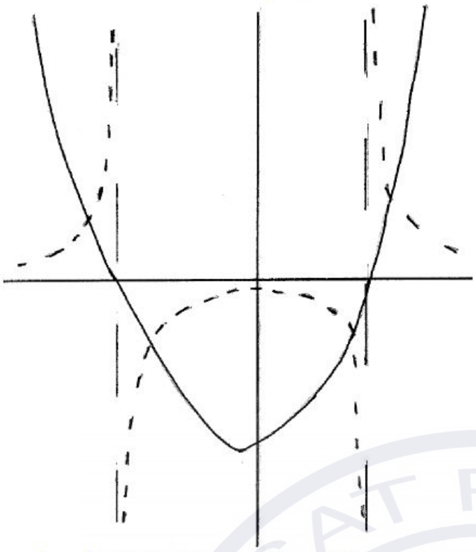
solution is $]-\infty, -1[ \cup ]1, 5[$ ($x < -1$ or $1 < x < 5$) ***A1A1***

| |
|---|
| **:** Award ***A1A0*** for closed intervals. |

***[3 marks]***

**(b) METHOD 1**

(graphs of $g(x)$ and $\frac{1}{g(x)}$)



roots of $g(x) = 0$ are $-3$ and $2$ ***(M1)(A1)***

> **Notes:** Award ***M1*** if quadratic graph is drawn or two roots obtained.
> Roots may be indicated anywhere *eg* asymptotes on graph or in inequalities below.

the intersections of the graphs $g(x)$ and of $1/g(x)$
are $-3.19, -2.79, 1.79, 2.19$ ***(M1)(A1)***

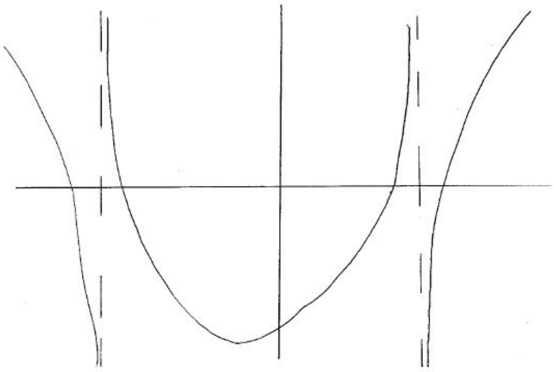
> **Note:** Award ***A1*** for at least one of the values above seen anywhere.

solution is $]-3.19, -3[\cup]-2.79, 1.79[\cup]2, 2.19[$
($-3.19 < x < -3$ or $-2.79 < x < 1.79$ or $2 < x < 2.19$ ) ***A1A1A1***

> **Note:** Award ***A1A1A0*** for closed intervals.

**METHOD 2**

(graph of $g(x) - \frac{1}{g(x)}$)



asymptotes at $x = -3$ and $x = 2$ ***(M1)(A1)***

| |
|---|
| **Note:** May be indicated on the graph. |

roots of graph are $-3.19$, $-2.79$, $1.79$, $2.19$ ***(M1)(A1)***

| |
|---|
| **Note:** Award ***A1*** for at least one of the values above seen anywhere. |

solution is (when graph is negative)
$]-3.19, -3[ \cup ]-2.79, 1.79[ \cup ]2, 2.19[$
($-3.19 < x < -3$ or $-2.79 < x < 1.79$ or $2 < x < 2.19$ ) ***A1A1A1***

| |
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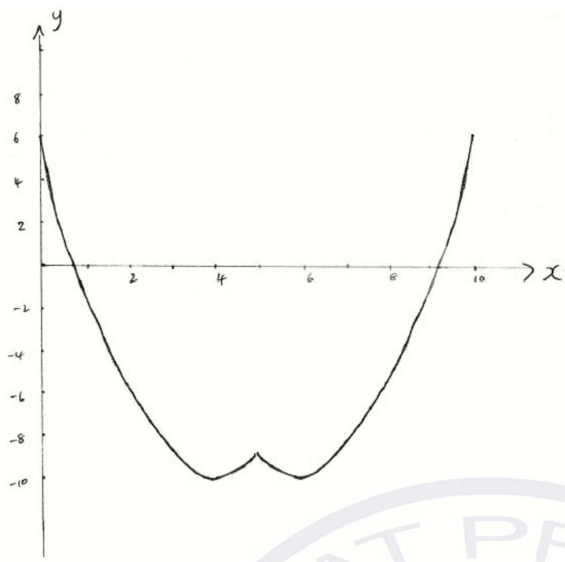
| **Note:** Award ***A1A1A0*** for closed intervals. |

***[7 marks]***

***Total [10 marks]***

## Question 12

(a)




general shape including 2 minimums, cusp **A1A1**

correct domain and symmetrical about the middle $(x = 5)$ **A1**

***[3 marks]***

(b) $x = 9.16$ or $x = 0.838$ **A1A1**

***[2 marks]***

***Total [5 marks]***

## Question 13

(a) (i) $-4 \le y \le -2$ **A1A1**

(ii) $-5 \le y \le -1$ **A1A1**

(iii) $-3 \le 2x - 6 \le 5$ ***(M1)***

> **Note:** Award ***M1*** for $f(2x-6)$.

$3 \le 2x \le 11$

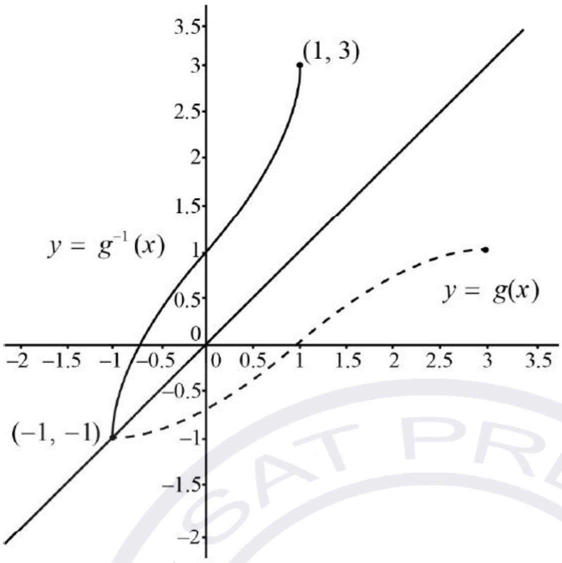
$\frac{3}{2} \le x \le \frac{11}{2}$ **A1A1**

***[7 marks]***

(b) (i) any valid argument *eg* $f$ is not one to one, $f$ is many to one, fails horizontal line test, not injective **R1**

(ii) largest domain for the function $g(x)$ to have an inverse is $[-1, 3]$ **A1A1**

(iii)




y-intercept indicated (coordinates not required) **A1**
correct shape **A1**
coordinates of end points $(1, 3)$ and $(-1, -1)$ **A1**

**Note:** Do not award any of the above marks for a graph that is not one to one.

***[6 marks]***

Question 14

(a) **EITHER**

$y = \ln(x-a) + b = \ln(5x+10)$ ***(M1)***

$y = \ln(x-a) + \ln c = \ln(5x+10)$

$y = \ln(c(x-a)) = \ln(5x+10)$ ***(M1)***

**OR**

$y = \ln(5x+10) = \ln(5(x+2))$ ***(M1)***

$y = \ln(5) + \ln(x+2)$ ***(M1)***

**THEN**

$a = -2$, $b = \ln 5$ ***A1A1***

(c) (i) $y = \dfrac{2x-5}{x+d}$

$(x+d)\,y = 2x-5$ ***M1***

**Note:** Award ***M1*** for attempting to rearrange $x$ and $y$ in a linear expression.

$x(y-2) = -dy-5$ ***(A1)***

$x = \dfrac{-dy-5}{y-2}$ ***(A1)***

**Note:** $x$ and $y$ can be interchanged at any stage

$h^{-1}(x) = \dfrac{-dx-5}{x-2}$ ***A1***

**Note:** Award ***A1*** only if $h^{-1}(x)$ is seen.

(ii) self Inverse $\Rightarrow h(x) = h^{-1}(x)$

$\dfrac{2x-5}{x+d} \equiv \dfrac{-dx-5}{x-2}$ ***(M1)***

$d = -2$ ***A1***

(iii) **METHOD 1**

$\dfrac{2k(x)-5}{k(x)-2} = \dfrac{2x}{x+1}$ ***(M1)***

$k(x) = \dfrac{x+5}{2}$ ***A1***

**METHOD 2**

$h^{-1}\left(\dfrac{2x}{x+1}\right) = \dfrac{2\left(\dfrac{2x}{x+1}\right)-5}{\dfrac{2x}{x+1}-2}$ ***(M1)***

$k(x) = \dfrac{x+5}{2}$ ***A1***

***[8 marks]***

Question 15

(a) $(x+2)^2-6$ ***A1A1***

***[2 marks]***

(b) $(g \circ f)(x) = (x+2)^2-6$ ***(M1)***

$\Rightarrow g(x) = x^2-6$ ***A1***

***[2 marks]***

***Total [4 marks]***

Question 16

(a) $f(-x)=\dfrac{3(-x)^2+10}{(-x)^2-4}$ ***A1***

$=\dfrac{3x^2+10}{x^2-4}=f(x)$

$f(x)=f(-x)$ ***R1***

hence this is an even function ***AG***

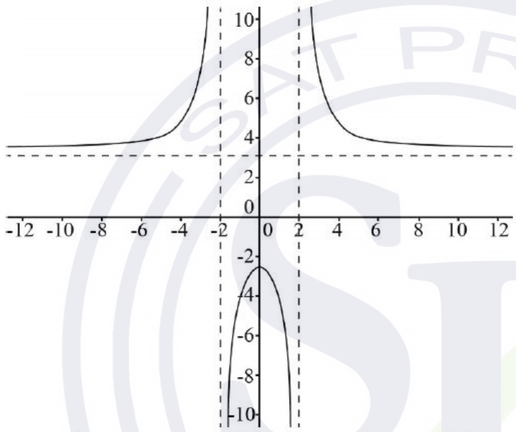
| |
|---|
| **Note:** Award ***A1R1*** for the statement, all the powers are even hence $f(x)=f(-x)$. |

| |
|---|
| **Note:** Just stating all the powers are even is ***A0R0***. |

| |
|---|
| **Note:** Do not accept arguments based on the symmetry of the graph. |

***[2 marks]***

(b) (i)




correct shape in 3 parts which are asymptotic and symmetrical ***A1***

correct vertical asymptotes clear at 2 and −2 ***A1***

correct horizontal asymptote clear at 3 ***A1***

*continued…*

*Question 5 continued*

(ii) $f(x)>3$ ***A1***

$f(x)\le -2.5$ ***A1***

***[5 marks]***

***Total [7 marks]***

(a) valid method *eg*, sketch of curve or critical values found **(M1)**

$x < -2.24$, $x > 2.24$, **A1**

$-1 < x < 0.8$ **A1**

**Note:** Award ***M1A1A0*** for correct intervals but with inclusive inequalities.

***[3 marks]***

(b) (i) $(1.67, -5.14)$, $(-1.74, -3.71)$ ***A1A1***

**Note:** Award ***A1A0*** for any two correct terms.

(ii) $f'(x) = 4x^3 + 0.6x^2 - 11.6x - 1$

$f''(x) = 12x^2 + 1.2x - 11.6 = 0$ ***(M1)***

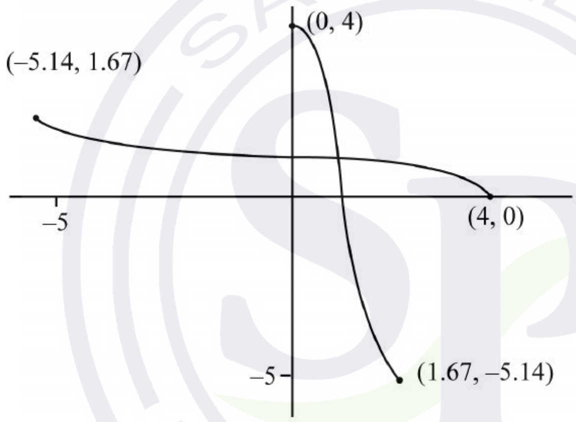
$-1.03$, $0.934$ ***A1A1***

**Note:** ***M1*** should be awarded if graphical method to find zeros of $f''(x)$ or turning points of $f'(x)$ is shown.

***[5 marks]***

(c) (i) $1.67$ ***A1***

(ii)




***M1A1A1***

**Note:** Award ***M1*** for reflection of their $y = f(x)$ in the line $y = x$ provided their $f$ is one-one.

***A1*** for $(0, 4)$, $(4,0)$ (Accept axis intercept values) ***A1*** for the other two sets of coordinates of other end points

(iii) $x = f(1)$ ***M1***

$= -1.6$ ***A1***

(d) (i) $y = 2\sin(x-1) - 3$

$x = 2\sin(y-1) - 3$ ***(M1)***

$\left(g^{-1}(x) =\right) \arcsin\left(\frac{x+3}{2}\right) + 1$ ***A1***

$-5 \le x \le -1$ ***A1A1***

**Note:** Award ***A1*** for $-5$ and $-1$, and ***A1*** for correct inequalities if numbers are reasonable.

(ii) $f^{-1}(g(x)) < 1$

$g(x) > -1.6$ ***(M1)***

$x > g^{-1}(-1.6) = 1.78$ ***(A1)***

**Note:** Accept = in the above.

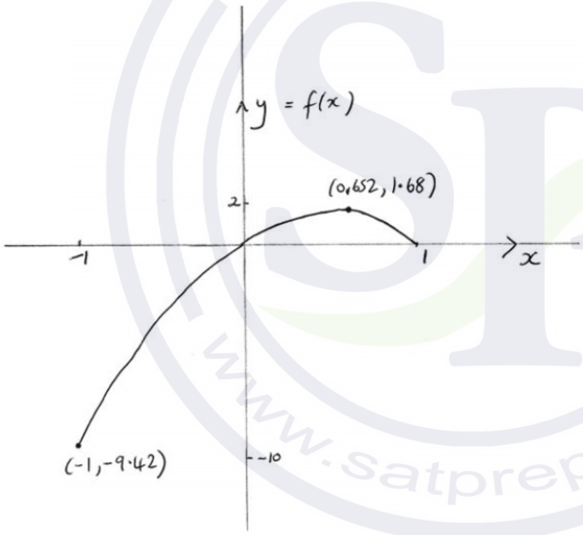
$1.78 < x \le \frac{\pi}{2} + 1$ ***A1A1***

**Note:** ***A1*** for $x > 1.78$ (allow $\ge$) and ***A1*** for $x \le \frac{\pi}{2} + 1$.

***[8 marks]***

## Question 18

(a)




correct shape passing through the origin and correct domain ***A1***

**Note:** Endpoint coordinates are not required. The domain can be indicated by $-1$ and $1$ marked on the $x$-axis.

$(0.652, 1.68)$ ***A1***

two correct intercepts (coordinates not required) ***A1***

**Note:** A graph passing through the origin is sufficient for $(0, 0)$.

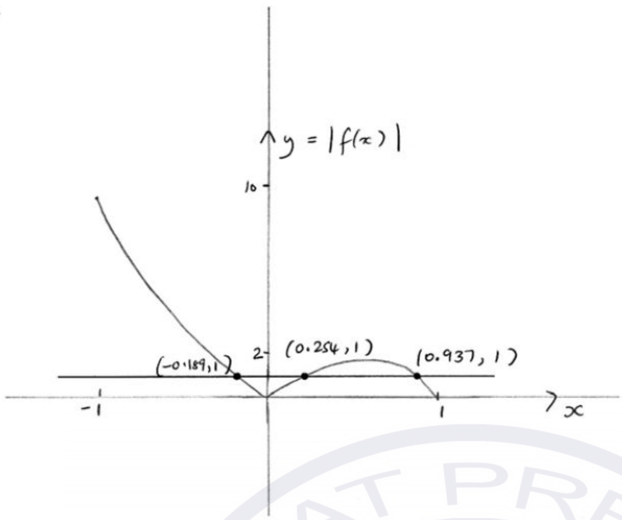
***[3 marks]***

(b) $[-9.42, 1.68]$ (or $[-3\pi, 1.68]$) ***A1A1***

**Note:** Award ***A1A0*** for open or semi-open intervals with correct endpoints. Award ***A1A0*** for closed intervals with one correct endpoint.

***[2 marks]***

(c) attempting to solve either $\left|3x \arccos(x)\right| > 1$ (or equivalent) or $\left|3x \arccos(x)\right| = 1$ (or equivalent) (*eg*. graphically) ***(M1)***




$x = -0.189,\ 0.254,\ 0.937$ ***(A1)***

$-1 \le x < -0.189$ or $0.254 < x < 0.937$ ***A1A1***

**Note:** Award ***A0*** for $x < -0.189$.

***[4 marks]***

***Total [9 marks]***

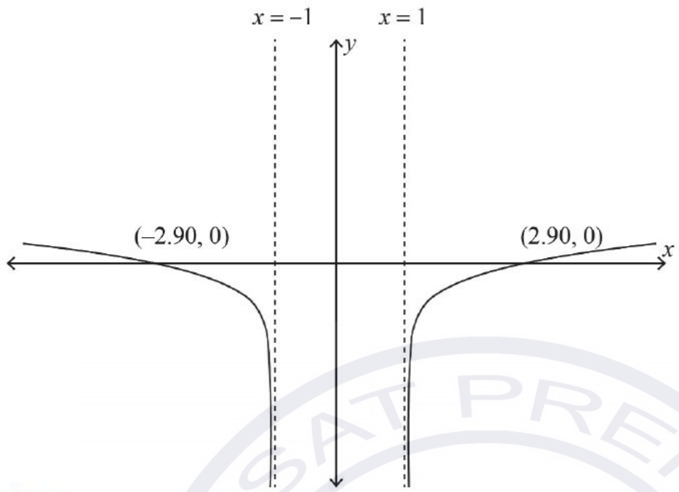
Question 19

(a) $x^2 - 1 > 0$ **(M1)**

$x < -1$ or $x > 1$ **A1**

***[2 marks]***

(b)




shape **A1**

$x = 1$ and $x = -1$ **A1**

$x$-intercepts **A1**

***[3 marks]***

(c) **EITHER**

$f$ is symmetrical about the $y$-axis **R1**

**OR**

$f(-x) = f(x)$ **R1**

***[1 mark]***

(d) **EITHER**

$f$ is not one-to-one function **R1**

**OR**

horizontal line cuts twice **R1**

**Note:** Accept any equivalent correct statement.

***[1 mark]***

(e) $x = -1 + \ln\left(\sqrt{y^2 - 1}\right)$ **M1**

$e^{2x+2} = y^2 - 1$ **M1**

$g^{-1}(x) = \sqrt{e^{2x+2} + 1},\ x \in \mathbb{R}$ **A1A1**

***[4 marks]***

## Question 20

(a) $g(x) = 3x^4 + ax^3 + bx^2 - 7x - 4$

$g(1) = 0 \Rightarrow a + b = 8$ ***M1A1***

$g(-1) = 0 \Rightarrow -a + b = -6$ ***A1***

$\Rightarrow a = 7,\ \mathrm{b} = 1$ ***A1***

***[4 marks]***

(b) $3x^4 + 7x^3 + x^2 - 7x - 4 = \left(x^2 - 1\right)\left(px^2 + qx + r\right)$

attempt to equate coefficients ***(M1)***

$p = 3,\ q = 7,\ r = 4$ ***(A1)***

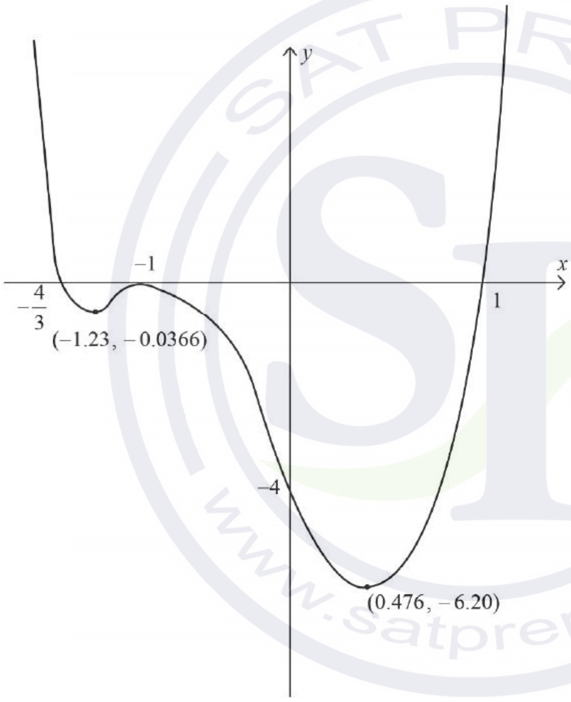
$3x^4 + 7x^3 + x^2 - 7x - 4 = \left(x^2 - 1\right)\left(3x^2 + 7x + 4\right)$

$= (x - 1)(x + 1)^2(3x + 4)$ ***A1***

| **Note:** Accept any equivalent valid method. |
|---|

***[3 marks]***

(c)






***A1*** for correct shape (*ie* with correct number of max/min points)
***A1*** for correct $x$ and $y$ intercepts
***A1*** for correct maximum and minimum points

***[3 marks]***

(d) $c > 0$ ***A1***

$-6.20 < c < -0.0366$ ***A1A1***

| **Note:** Award ***A1*** for correct end points and ***A1*** for correct inequalities. |
|---|

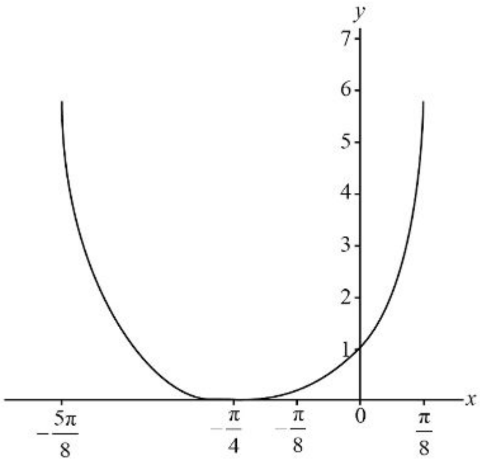
| **Note:** If the candidate has misdrawn the graph and omitted the first minimum point, the maximum mark that may be awarded is ***A1FTA0A0*** for $c > -6.20$ seen. |
|---|

***[3 marks]***

***Total [13 marks]***

Question 21

(a) (i)




***A1A1***

***A1*** for correct concavity, many to one graph, symmetrical about the midpoint of the domain and with two axes intercepts.

**Note:** Axes intercepts and scales not required.

***A1*** for correct domain

(ii) for each value of $x$ there is a unique value of $f(x)$ ***A1***

**Note:** Accept "passes the vertical line test" or equivalent.

(iii) no inverse because the function fails the horizontal line test or equivalent ***R1***

**Note:** No **FT** if the graph is in degrees (one-to-one).

(iv) the expression is not valid at either of $x = \frac{\pi}{4}$ $\left(\text{or } -\frac{3\pi}{4}\right)$ ***R1***

***[5 marks]***

(b) **METHOD 1**

$$f(x) = \frac{\tan\left(x + \frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{4} - x\right)}$$ ***M1***

$$= \frac{\dfrac{\tan x + \tan\frac{\pi}{4}}{1 - \tan x \tan\frac{\pi}{4}}}{\dfrac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}}$$ ***M1A1***

$$= \left(\frac{1+t}{1-t}\right)^2$$ ***AG***

**METHOD 2**

$$f(x) = \tan\left(x + \frac{\pi}{4}\right)\tan\left(\frac{\pi}{2} - \frac{\pi}{4} + x\right)$$ ***(M1)***

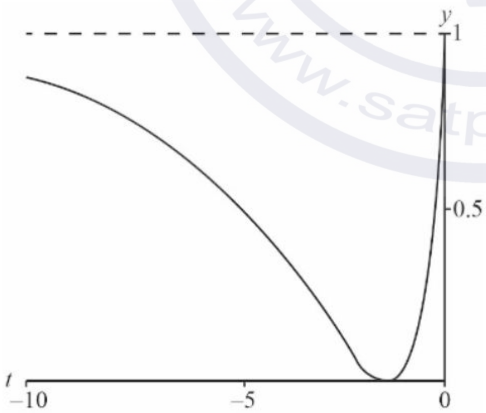
$$= \tan^2\left(x + \frac{\pi}{4}\right)$$ ***A1***

$$g(t) = \left(\frac{\tan x + \tan\frac{\pi}{4}}{1 - \tan x \tan\frac{\pi}{4}}\right)^2$$ ***A1***

$$= \left(\frac{1+t}{1-t}\right)^2$$ ***AG***

***[3 marks]***

(c)




for $t \le 0$, correct concavity with two axes intercepts and with asymptote $y = 1$ ***A1***

$t$ intercept at $(-1, 0)$ ***A1***

$y$ intercept at $(0, 1)$ ***A1***

***[3 marks]***

(d) (i) **METHOD 1**

$\alpha, \beta$ satisfy $\frac{(1+t)^2}{(1-t)^2} = k$ ***M1***

$1 + t^2 + 2t = k(1 + t^2 - 2t)$ ***A1***

$(k-1)t^2 - 2(k+1)t + (k-1) = 0$ ***A1***

attempt at using quadratic formula ***M1***

$\alpha, \beta = \frac{k + 1 \pm 2\sqrt{k}}{k-1}$ or equivalent ***A1***

**METHOD 2**

$\alpha, \beta$ satisfy $\frac{1+t}{1-t} = (\pm)\sqrt{k}$ ***M1***

$t + \sqrt{k}t = \sqrt{k} - 1$ ***M1***

$t = \frac{\sqrt{k}-1}{\sqrt{k}+1}$ (or equivalent) ***A1***

$t - \sqrt{k}t = -\left(\sqrt{k}+1\right)$ ***M1***

$t = \frac{\sqrt{k}+1}{\sqrt{k}-1}$ (or equivalent) ***A1***

so for *eg*, $\alpha = \frac{\sqrt{k}-1}{\sqrt{k}+1}$, $\beta = \frac{\sqrt{k}+1}{\sqrt{k}-1}$

(ii) $\alpha + \beta = 2\frac{(k+1)}{(k-1)}\left(= -2\frac{(1+k)}{(1-k)}\right)$ ***A1***

since $1 + k > 1 - k$ ***R1***

$\alpha + \beta < -2$ ***AG***

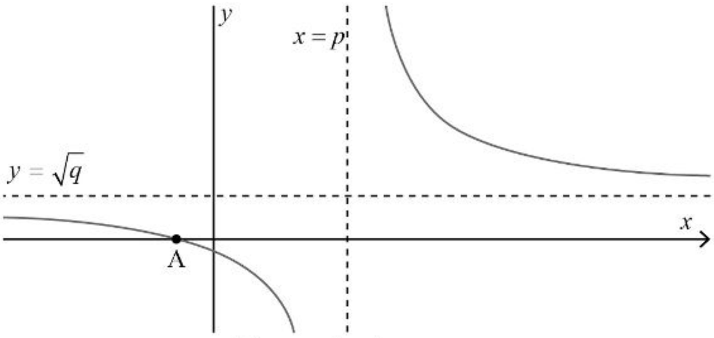
**Note:** Accept a valid graphical reasoning.

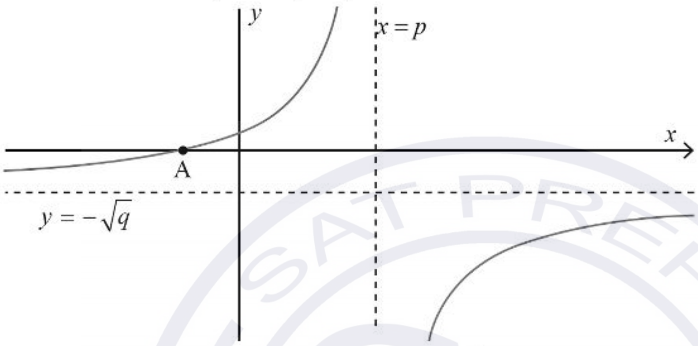
***[7 marks]***

***Total [18 marks]***

Question 22

(a)




either graph passing through (or touching) A **A1**

correct shape and vertical asymptote with correct equation for either graph **A1**

correct horizontal asymptote with correct equation for either graph **A1**

two completely correct sketches **A1**

***[4 marks]***

(b) $a\left(-\frac{1}{2}\right)+1=0 \Rightarrow a=2$ **A1**

from horizontal asymptote, $\left(\frac{a}{b}\right)^2=\frac{4}{9}$ ***(M1)***

$\frac{a}{b}=\pm\frac{2}{3} \Rightarrow b=\pm 3$ **A1**

from vertical asymptote, $b\left(\frac{4}{3}\right)+c=0$

$b=3,\ c=-4$ or $b=-3,\ c=4$ **A1**

***[4 marks]***

***Total [8 marks]***

Question 23

(a) **METHOD 1**

$$f'(x) = \frac{\frac{2(x-3)}{x} - (2\ln x + 1)}{(x-3)^2} \left( = \frac{2(x-3) - x(2\ln x + 1)}{x(x-3)^2} \right) \qquad \textbf{\textit{(M1)A1A1A1}}$$

| **Note:** Award ***M1*** for attempt at quotient rule, ***A1A1*** for numerator and ***A1*** for denominator. |
|---|

**METHOD 2**

$$f(x) = (2\ln x + 1)(x-3)^{-1} \qquad \textbf{\textit{(A1)}}$$

$$f'(x) = \left(\frac{2}{x}\right)(x-3)^{-1} - (2\ln x + 1)(x-3)^{-2} \left( = \frac{2(x-3) - x(2\ln x + 1)}{x(x-3)^2} \right) \qquad \textbf{\textit{(M1)A1A1}}$$

| **Note:** Award ***M1*** for attempt at product rule, ***A1*** for first term, ***A1*** for second term. |
|---|

***[4 marks]***

(b) finding turning point of $y = f'(x)$ or finding root of $y = f''(x)$ ***(M1)***

$x = 0.899$ ***A1***

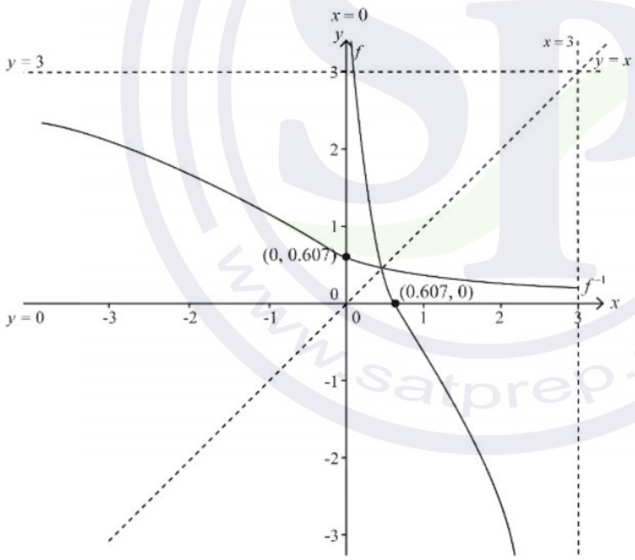
$y = f(0.899048\ldots) = -0.375$ ***(M1)A1***

$(0.899, -0.375)$

| **Note:** Do not accept $x = 0.9$. Accept $y$-coordinates rounding to $-0.37$ or $-0.375$ but not $-0.38$. |
|---|

***[4 marks]***

(c)



(i) smooth curve over the correct domain which does not cross the $y$-axis and is concave down for $x > 1$ **A1**

$x$-intercept at $0.607$ **A1**

equations of asymptotes given as $x = 0$ and $x = 3$ (the latter must be drawn) **A1A1**

***[4 marks]***

(ii) attempt to reflect graph of $f$ in $y = x$ ***(M1)***

smooth curve over the correct domain which does not cross the $x$-axis and is concave down for $y > 1$ **A1**

$y$-intercept at $0.607$ **A1**

equations of asymptotes given as $y = 0$ and $y = 3$ (the latter must be drawn) **A1**

| **Note:** For ***FT*** from (i) to (ii) award max ***M1A0A1A0***. |
|---|

***[4 marks]***

(d) solve $f(x) = f^{-1}(x)$ or $f(x) = x$ to get $x = 0.372$ ***(M1)A1***

$0 < x < 0.372$ **A1**

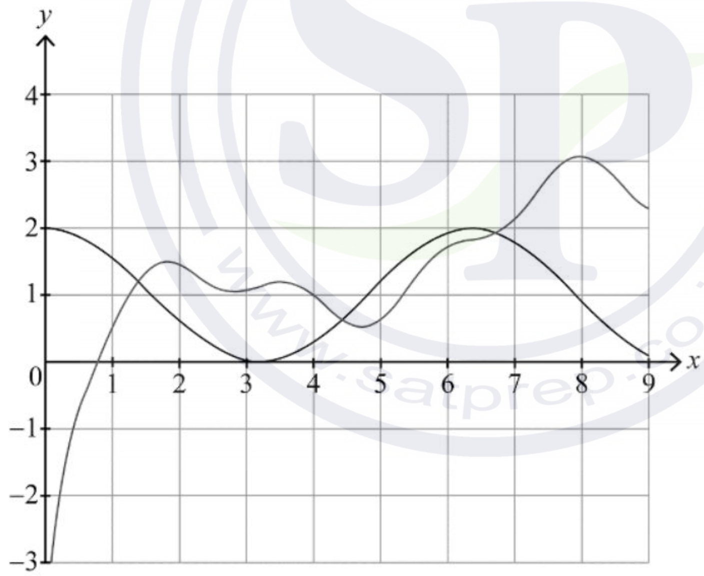
| **Note:** Do not award ***FT*** marks. |
|---|

***[3 marks]***

***Total [19 marks]***

Question 24

(a)




**A1A1**

| **Note:** Award ***A1*** for each correct curve, showing all local max & mins. |
|---|

| **Note:** Award ***A0A0*** for the curves drawn in degrees. |
|---|

***[2 marks]***

(b) $x = 1.35, 4.35, 6.64$ *(M1)*

**Note:** Award ***M1*** for attempt to find points of intersections between two curves.

$0 < x < 1.35$ ***A1***

**Note:** Accept $x < 1.35$.

$4.35 < x < 6.64$ ***A1A1***

**Note:** Award ***A1*** for correct endpoints, ***A1*** for correct inequalities.

**Note:** Award ***M1FTA1FTA0FTA0FT*** for $0 < x < 7.31$.

**Note:** Accept $x < 7.31$.

***[4 marks]***

***Total [6 marks]***

## Question 25

(a) $f(x) \geq 3$ ***A1***

***[1 mark]***

(b) $x = \sec y + 2$ ***(M1)***

**Note:** Exchange of variables can take place at any point.

$\cos y = \dfrac{1}{x-2}$ ***(A1)***

$f^{-1}(x) = \arccos\left(\dfrac{1}{x-2}\right),\ x \geq 3$ ***A1A1***

**Note:** Allow follow through from (a) for last ***A1*** mark which is independent of earlier marks in (b).

***[4 marks]***

***Total [5 marks]***

## Question 26

(a) [illegible]$(1) = 0$ ***(A1)***

[illegible]$(0) = -1$ ***A1***

***[2 marks]***

(b) [illegible] $= f(3)$ ***(M1)***

$\Rightarrow$ [illegible] $= 4$ ***A1***

***[2 marks]***

(c) domain is $-2 \leq x \leq 6$ ***A1***

range is $-6 \leq y \leq 10$ ***A1***

***[2 marks]***

***Total [6 marks]***