Subject – Math(Higher Level)
Topic - Circular trigonometry
Year - Nov 2011 – Nov 2019
Paper 2

Question 1

(a) $\tan\left(\arctan\frac{1}{2} - \arctan\frac{1}{3}\right) = \tan(\arctan a)$ ***(M1)***

$a = 0.14285\ldots = \frac{1}{7}$ ***(A1)A1***

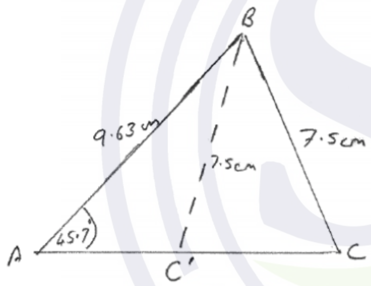
(b) $\arctan\left(\frac{1}{7}\right) = \arcsin(x) \Rightarrow x = \sin\left(\arctan\frac{1}{7}\right) \approx 0.141$ ***(M1)A1***

Note: Accept exact value of $\left(\frac{1}{\sqrt{50}}\right)$.

[5 marks]

Question 2

(a)



A2

Note: Accept 2 separate triangles. The diagram(s) should show that one triangle has an acute angle and the other triangle has an obtuse angle. The values 9.63, 7.5 and 45.7 and/or the letters, A, B C′ and C should be correctly marked on the diagram(s).

[2 marks]

(b) **METHOD 1**

$\frac{\sin 45.7}{7.5} = \frac{\sin C}{9.63}$ ***M1***

$\Rightarrow \hat{C} = 66.77\ldots°, 113.2\ldots°$ ***(A1)(A1)***

$\Rightarrow \hat{B} = 67.52\ldots°, 21.07\ldots°$ ***(A1)***

$\frac{b}{\sin B} = \frac{7.5}{\sin 45.7} \Rightarrow b = 9.68(\text{cm}), b = 3.77(\text{cm})$ ***A1A1***

Note: If only the acute value of $\hat{C}$ is found, award ***M1(A1)(A0)(A0)A1A0***.

METHOD 2

$7.5^2 = 9.63^2 + b^2 - 2\times 9.63 \times b\cos 45.7^\circ$ *M1A1*

$b^2 - 13.45\ldots b + 36.48\ldots = 0$

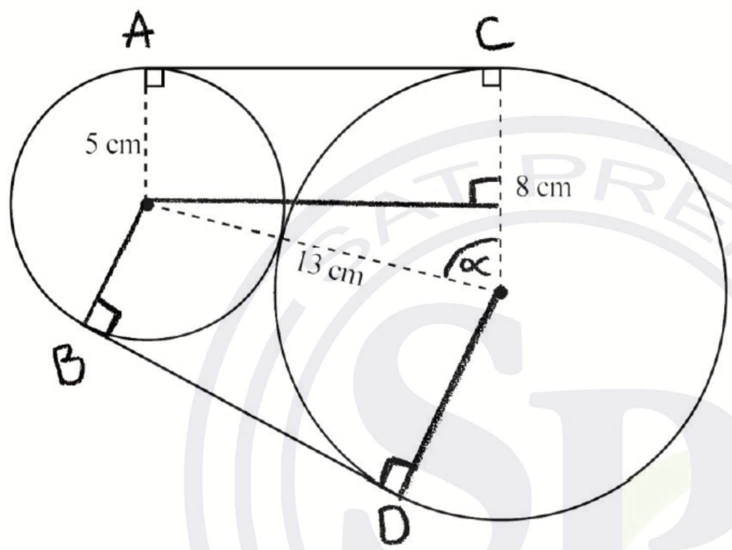
$b = \dfrac{13.45\ldots \pm \sqrt{13.45\ldots^2 - 4\times 36.48\ldots}}{2}$ *(M1)(A1)*

$\text{AC} = 9.68(\text{cm})$, $\text{AC} = 3.77(\text{cm})$ *A1A1*

[6 marks]

Total [8 marks]

Question 3



$\text{AC} = \text{BD} = \sqrt{13^2 - 3^2} = 12.64\ldots$ *(A1)*

$\cos\alpha = \dfrac{3}{13} \Rightarrow \alpha = 1.337\ldots(76.65\ldots^\circ)$ *(M1)(A1)*

attempt to find either arc length AB or arc length CD *(M1)*

arc length $\text{AB} = 5(\pi - 2\times 0.232\ldots)$ $(=13.37\ldots)$ *(A1)*

arc length $\text{CD} = 8(\pi + 2\times 0.232\ldots)$ $(=28.85\ldots)$ *(A1)*

length of string $= 13.37\ldots + 28.85\ldots + 2(12.64\ldots)$ *(M1)*

$= 67.5\,(\text{cm})$ *A1*

[8 marks]

Question 4

$\dfrac{1}{2}r^2 \times 1 = 7$ *M1*

$r = 3.7\ldots\left(=\sqrt{14}\right)$ (or 37… mm) *(A1)*

$\text{height} = 2r\cos\left(\dfrac{\pi-1}{2}\right)$ $\left(\text{or } 2r\sin\dfrac{1}{2}\right)$ *(M1)(A1)*

3.59 or anything that rounds to 3.6 *A1*

so the dimensions are 3.7 by 3.6 (cm or 37 by 36 mm) *A1*

[6 marks]

Question 5

(a) let the distance the cable is laid along the seabed be y

$y^2 = x^2 + 200^2 - 2 \times x \times 200 \cos 60^0$ ***(M1)***

(or equivalent method)

$y^2 = x^2 - 200x + 40000$ ***(A1)***

$\text{cost} = C = 80y + 20x$ ***(M1)***

$C = 80(x^2 - 200x + 40000)^{\frac{1}{2}} + 20x$ ***A1***

[4 marks]

(b) $x = 55.2786\ldots = 55$ (m to the nearest metre) ***(A1)A1***

$\left(x = 100 - \sqrt{2000}\right)$

[2 marks]

Total [6 marks]

Question 6

(a) **EITHER**

$\text{A}\hat{\text{O}}\text{B} = 2\arcsin\left(\frac{3}{4}\right)$ or equivalent (*eg* $\text{A}\hat{\text{O}}\text{B} = 2\arctan\left(\frac{3}{\sqrt{7}}\right)$, $\text{A}\hat{\text{O}}\text{B} = 2\arccos\left(\frac{\sqrt{7}}{4}\right)$)***(M1)***

OR

$\cos \text{A}\hat{\text{O}}\text{B} = \frac{4^2 + 4^2 - 6^2}{2 \times 4 \times 4} \left(= -\frac{1}{8}\right)$ ***(M1)***

THEN

$= 1.696$ (correct to 4sf) ***A1***

[2 marks]

(b) use of area of segment $=$ area of sector $-$ area of triangle ***(M1)***

$= \frac{1}{2} \times 4^2 \times 1.696 - \frac{1}{2} \times 4^2 \times \sin 1.696$ ***(A1)***

$= 5.63\ (\text{cm}^2)$ ***A1***

[3 marks]

Total [5 marks]

Question 7

(a) attempting to solve for $\cos x$ or for u where $u = \cos x$ or for x graphically. ***(M1)***

EITHER

$\cos x = \frac{2}{3}$ (and 2) ***(A1)***

OR

$x = 48.1897\ldots^\circ$ ***(A1)***

THEN

$x = 48^\circ$ ***A1***

Note:	Award ***(M1)(A1)A0*** for $x = 48^\circ, 132^\circ$.

Note:	Award ***(M1)(A1)A0*** for 0.841 radians.

[3 marks]

(b) attempting to solve for $\sec x$ or for v where $v = \sec x$. ***(M1)***

$\sec x = \pm\sqrt{2} \left(\text{and } \pm\sqrt{\frac{2}{3}} \right)$ ***(A1)***

$\sec x = \pm\sqrt{2}$ ***A1***

[3 marks]

Total [6 marks]

Question 8

(a) **EITHER**

$\theta = \pi - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$ (or equivalent) *M1A1*

Note: Accept $\theta = 180° - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$ (or equivalent).

OR

$\theta = \arctan\left(\frac{x}{8}\right) + \arctan\left(\frac{20-x}{13}\right)$ (or equivalent) *M1A1*

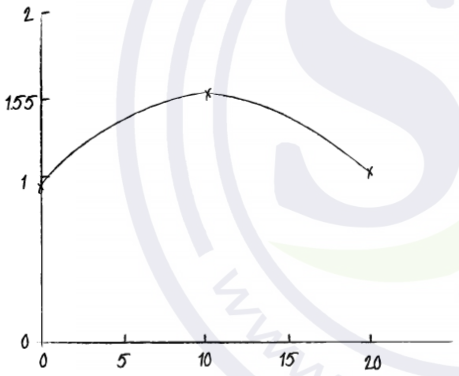
[2 marks]

(b) (i) $\theta = 0.994 \ \left(= \arctan\frac{20}{13}\right)$ *A1*

(ii) $\theta = 1.19 \ \left(= \arctan\frac{5}{2}\right)$ *A1*

[2 marks]

(c) correct shape. *A1*
correct domain indicated. *A1*



[2 marks]

Question 9

(a) **METHOD 1**

$2\arcsin\left(\frac{1.5}{4}\right)$ ***M1***

$\alpha = 0.769^c$ (44.0°) ***A1***

METHOD 2

using the cosine rule:

$3^2 = 4^2 + 4^2 - 2(4)(4)\cos\alpha$ ***M1***

$\alpha = 0.769^c$ (44.0°) ***A1***

[2 marks]

(b) one segment

$A_1 = \frac{1}{2}\times 4^2 \times 0.76879 - \frac{1}{2}\times 4^2 \times \sin(0.76879)$ ***M1A1***

$= 0.58819K$ ***(A1)***

$2A_1 = 1.18$ (cm^2) ***A1***

Note: Award ***M1*** only if both sector and triangle are considered.

[4 marks]

Total [6 marks]

Question 10

(a) each triangle has area $\frac{1}{8}x^2 \sin\frac{2\pi}{n}$ (use of $\frac{1}{2}ab\sin C$) ***(M1)***

there are n triangles so $A = \frac{1}{8}nx^2 \sin\frac{2\pi}{n}$ ***A1***

$C = \dfrac{4\left(\frac{1}{8}nx^2 \sin\frac{2\pi}{n}\right)}{\pi x^2}$ ***A1***

so $C = \frac{n}{2\pi}\sin\frac{2\pi}{n}$ ***AG***

[3 marks]

(b) attempting to find the least value of n such that $\frac{n}{2\pi}\sin\frac{2\pi}{n} > 0.99$ ***(M1)***

$n = 26$ ***A1***

attempting to find the least value of n such that $\dfrac{n\sin\frac{2\pi}{n}}{\pi\left(1+\cos\frac{\pi}{n}\right)} > 0.99$ ***(M1)***

$n = 21$ (and so a regular polygon with 21 sides) ***A1***

> **Note:** Award ***(M0)A0(M1)A1*** if $\frac{n}{2\pi}\sin\frac{2\pi}{n} > 0.99$ is not considered
>
> and $\dfrac{n\sin\frac{2\pi}{n}}{\pi\left(1+\cos\frac{\pi}{n}\right)} > 0.99$ is correctly considered.
>
> Award ***(M1)A1(M0)A0*** for $n = 26$.

[4 marks]

(c) **EITHER**

for even and odd values of n, the value of C seems to increase towards the limiting value of the circle ($C = 1$) *ie* as n increases, the polygonal regions get closer and closer to the enclosing circular region ***R1***

OR

the differences between the odd and even values of n illustrate that this measure of compactness is not a good one. ***R1***

[1 mark]

Total [8 marks]

Question 11

(a) **METHOD 1**

squaring both equations *M1*

$9\sin^2 B + 24\sin B\cos C + 16\cos^2 C = 36$ *(A1)*

$9\cos^2 B + 24\cos B\sin C + 16\sin^2 C = 1$ *(A1)*

adding the equations and using $\cos^2\theta + \sin^2\theta = 1$ to obtain

$9 + 24\sin(B+C) + 16 = 37$ *M1*

$24(\sin B\cos C + \cos B\sin C) = 12$ *A1*

$24\sin(B+C) = 12$ *(A1)*

$\sin(B+C) = \frac{1}{2}$ *AG*

METHOD 2

substituting for $\sin B$ and $\cos B$ to obtain

$\sin(B+C) = \left(\frac{6-4\cos C}{3}\right)\cos C + \left(\frac{1-4\sin C}{3}\right)\sin C$ *M1*

$= \frac{6\cos C + \sin C - 4}{3}$ (or equivalent) *A1*

substituting for $\sin C$ and $\cos C$ to obtain

$\sin(B+C) = \sin B\left(\frac{6-3\sin B}{4}\right) + \cos B\left(\frac{1-3\cos B}{4}\right)$ *M1*

$= \frac{\cos B + 6\sin B - 3}{4}$ (or equivalent) *A1*

Adding the two equations for $\sin(B+C)$:

$2\sin(B+C) = \frac{(18\sin B + 24\cos C) + (4\sin C + 3\cos B) - 25}{12}$ *A1*

$\sin(B+C) = \frac{36+1-25}{24}$ *(A1)*

$\sin(B+C) = \frac{1}{2}$ *AG*

(b) $\sin A = \sin\left(180° - (B+C)\right)$ so $\sin A = \sin(B+C)$ *R1*

$\sin(B+C) = \frac{1}{2} \Rightarrow \sin A = \frac{1}{2}$ *A1*

$\Rightarrow A = 30°$ or $A = 150°$ *A1*

if $A = 150°$, then $B < 30°$ *R1*

for example, $3\sin B + 4\cos C < \frac{3}{2} + 4 < 6$, *ie* a contradiction *R1*

only one possible value ($A = 30°$) *AG*

[5 marks]

Total [11 marks]

Question 12

(a) $A = \frac{1}{2} \times 5 \times 12 \times \sin 100^\circ$ **(M1)**

$= 29.5\ (\text{cm}^2)$ **A1**

[2 marks]

(b) $AC^2 = 5^2 + 12^2 - 2 \times 5 \times 12 \times \cos 100^\circ$ **(M1)**

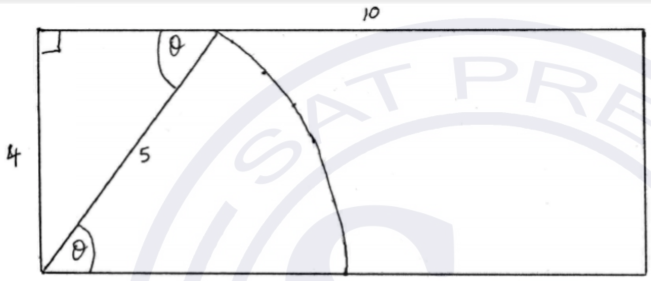
therefore $AC = 13.8\ (\text{cm})$ **A1**

[2 marks]

Total [4 marks]

Question 13

(a)



EITHER

area of triangle $= \frac{1}{2} \times 3 \times 4\ (= 6)$ **A1**

area of sector $= \frac{1}{2} \arcsin\left(\frac{4}{5}\right) \times 5^2\ (= 11.5911\ldots)$ **A1**

OR

$\int_0^4 \sqrt{25 - x^2}\,dx$ **M1A1**

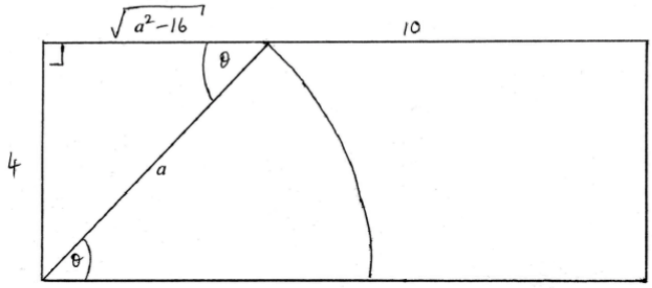
THEN

total area $= 17.5911\ldots\text{m}^2$ **(A1)**

percentage $= \frac{17.5911\ldots}{40} \times 100 = 44\%$ **A1**

[4 marks]

(b) **METHOD 1**



area of triangle $= \frac{1}{2} \times 4 \times \sqrt{a^2-16}$ ***A1***

$\theta = \arcsin\left(\frac{4}{a}\right)$ ***(A1)***

area of sector $= \frac{1}{2} r^2 \theta = \frac{1}{2} a^2 \arcsin\left(\frac{4}{a}\right)$ ***A1***

therefore total area $= 2\sqrt{a^2-16} + \frac{1}{2} a^2 \arcsin\left(\frac{4}{a}\right) = 20$ ***A1***

rearrange to give: $a^2 \arcsin\left(\frac{4}{a}\right) + 4\sqrt{a^2-16} = 40$ ***AG***

Question 14

(a) (i) $A = -3$ ***A1***

(ii) period $= \frac{2\pi}{B}$ ***(M1)***

$B = 2$ ***A1***

Note: Award as above for $A = 3$ and $B = -2$.

(iii) $C = 2$ ***A1***

[4 marks]

(b) $x = 1.74,\ 2.97 \ \left(x = \frac{1}{2}\left(\pi + \arcsin\frac{1}{3}\right), \frac{1}{2}\left(2\pi - \arcsin\frac{1}{3}\right)\right)$ ***(M1)A1***

[2 marks]

Note: Award ***(M1)A0*** if extra correct solutions *eg* $(-1.40,\ -0.170)$ are given outside the domain $0 \le x \le \pi$.

Total [6 marks]

Question 15

$21 = \frac{1}{2} \cdot 6 \cdot 11 \cdot \sin A$ **(M1)**

$\sin A = \frac{7}{11}$ **(A1)**

EITHER

$\hat{A} = 0.6897..., 2.452\ldots \left(\hat{A} = \arcsin\frac{7}{11}, \pi - \arcsin\frac{7}{11} = 39.521...°, 140.478...° \right)$ **(A1)**

OR

$\cos A = \pm\frac{6\sqrt{2}}{11} (= \pm 0.771...)$ **(A1)**

THEN

$BC^2 = 6^2 + 11^2 - 2 \cdot 6 \cdot 11 \cos A$ **(M1)**

$BC = 16.1$ or 7.43 **A1A1**

e: Award ***M1A1A0M1A1A0*** if only one correct solution is given.

[6 marks]

Question 16

attempting to use the area of sector formula (including for a semicircle) ***M1***

semi-circle $\frac{1}{2}\pi \times 5^2 = \frac{25\pi}{2} = 39.26990817\ldots$ ***(A1)***

angle in smaller sector is $\pi - \theta$ ***(A1)***

area of sector $= \frac{1}{2} \times 2^2 \times (\pi - \theta)$ ***(A1)***

attempt to total a sum of areas of regions to 44 ***(M1)***

$2(\pi - \theta) = 44 - 39.26990817\ldots$

$\theta = 0.777 \left(= \frac{29\pi}{4} - 22 \right)$ ***A1***

Note: Award all marks except the final ***A1*** for correct working in degrees.

Note: Attempt to solve with goat inside triangle should lead to nonsense answer and so should only receive a maximum of the two ***M*** marks.

[6 marks]

Question 17

$AC^2 = 7.8^2 + 10.4^2$ ***(M1)***

$AC = 13$ ***(A1)***

use of cosine rule *eg*, $\cos\left(A\hat{B}C\right) = \frac{6.5^2 + 9.1^2 - 13^2}{2(6.5)(9.1)}$ ***M1***

$A\hat{B}C = 111.804\ldots° (= 1.95134\ldots)$ ***(A1)***

$= 112°$ ***A1***

[5 marks]

Question 18

(a) **METHOD 1**

let $\mathrm{AC} = x$

$3^2 = x^2 + 4^2 - 8x\cos\frac{\pi}{9}$ ***M1A1***

attempting to solve for x ***(M1)***

$x = 1.09,\ 6.43$ ***A1A1***

METHOD 2

let $\mathrm{AC} = x$

using the sine rule to find a value of C ***M1***

$4^2 = x^2 + 3^2 - 6x\cos(152.869\ldots°) \Rightarrow x = 1.09$ ***(M1)A1***

$4^2 = x^2 + 3^2 - 6x\cos(27.131\ldots°) \Rightarrow x = 6.43$ ***(M1)A1***

METHOD 3

let $\mathrm{AC} = x$

using the sine rule to find a value of B and a value of C ***M1***

obtaining $B = 132.869\ldots°,\ 7.131\ldots°$ and $C = 27.131\ldots°,\ 152.869\ldots°$ ***A1***

($B = 2.319\ldots,\ 0.124\ldots$ and $C = 0.473\ldots,\ 2.668\ldots$)

attempting to find a value of x using the cosine rule ***(M1)***

$x = 1.09,\ 6.43$ ***A1A1***

Note: Award ***M1A0(M1)A1A0*** for one correct value of x

[5 marks]

(b) $\frac{1}{2}\times 4\times 6.428\ldots\times\sin\frac{\pi}{9}$ and $\frac{1}{2}\times 4\times 1.088\ldots\times\sin\frac{\pi}{9}$ ***(A1)***

($4.39747\ldots$ and $0.744833\ldots$)

let D be the difference between the two areas

$D = \frac{1}{2}\times 4\times 6.428\ldots\times\sin\frac{\pi}{9} - \frac{1}{2}\times 4\times 1.088\ldots\times\sin\frac{\pi}{9}$ ***(M1)***

$(D = 4.39747\ldots - 0.744833\ldots)$

$= 3.65\left(\mathrm{cm}^2\right)$ ***A1***

[3 marks]

Total [8 marks]

Question 19

(a) $A = 2(\alpha - \sin\alpha)r^2 + \frac{1}{2}(\theta - \sin\theta)r^2$ ***M1A1A1***

Note: Award ***M1A1A1*** for alternative correct expressions *eg*. $A = 4\left(\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)r^2 + \frac{1}{2}\theta r^2$.

[3 marks]

(b) **METHOD 1**

consider for example triangle ADM where M is the midpoint of BD ***M1***

$\sin\frac{\alpha}{4} = \frac{1}{4}$ ***A1***

$\frac{\alpha}{4} = \arcsin\frac{1}{4}$

$\alpha = 4\arcsin\frac{1}{4}$ ***AG***

METHOD 2

attempting to use the cosine rule (to obtain $1 - \cos\frac{\alpha}{2} = \frac{1}{8}$) ***M1***

$\sin\frac{\alpha}{4} = \frac{1}{4}$ (obtained from $\sin\frac{\alpha}{4} = \sqrt{\frac{1-\cos\frac{\alpha}{2}}{2}}$) ***A1***

$\frac{\alpha}{4} = \arcsin\frac{1}{4}$

$\alpha = 4\arcsin\frac{1}{4}$ ***AG***

METHOD 3

$\sin\left(\frac{\pi}{2} - \frac{\alpha}{4}\right) = 2\sin\frac{\alpha}{2}$ where $\frac{\theta}{2} = \frac{\pi}{2} - \frac{\alpha}{4}$

$\cos\frac{\alpha}{4} = 4\sin\frac{\alpha}{4}\cos\frac{\alpha}{4}$ ***M1***

Note: Award ***M1*** either for use of the double angle formula or the conversion from sine to cosine.

$\frac{1}{4} = \sin\frac{\alpha}{4}$ ***A1***

$\frac{\alpha}{4} = \arcsin\frac{1}{4}$

$\alpha = 4\arcsin\frac{1}{4}$ ***AG***

(c) (from triangle ADM), $\theta = \pi - \frac{\alpha}{2}$ $\left(= \pi - 2\arcsin\frac{1}{4} = 2\arccos\frac{1}{4} = 2.6362\ldots\right)$ ***A1***

attempting to solve $2(\alpha - \sin\alpha)r^2 + \frac{1}{2}(\theta - \sin\theta)r^2 = 4$

with $\alpha = 4\arcsin\frac{1}{4}$ and $\theta = \pi - \frac{\alpha}{2}$ $\left(= 2\arccos\frac{1}{4}\right)$ for r ***(M1)***

$r = 1.69$ ***A1***

[3 marks]

Total [8 marks]

Question 20

(a) $x = \frac{\pi}{4}$ **A1**

$x = \frac{5\pi}{4}, x = -\frac{3\pi}{4}$ **A1**

[2 marks]

(b) reflection in the y-axis results in $y = \tan\left(-x + \frac{\pi}{4}\right)\left(= \cot\left(x + \frac{\pi}{4}\right)\right)$ ***(A1)***

vertical stretch gives $y = \frac{1}{2}\tan\left(-x + \frac{\pi}{4}\right)\left(= \frac{1}{2}\cot\left(x + \frac{\pi}{4}\right)\right)$ ***(A1)***

translation

$y = \frac{1}{2}\tan\left[-\left(x - \frac{\pi}{4} - \frac{\pi}{4}\right)\right] - 3$

$= \frac{1}{2}\tan\left(-x + \frac{\pi}{2}\right) - 3\left(= \frac{1}{2}\cot(x) - 3\right)$ ***A1A1***

Notes: Award the ***A1***s independently of each other. Do not penalize the absence of $y =$.

[4 marks]

Total [6 marks]

Question 21

(a) $p^2 = 12^2 + r^2 - 2 \times 12 \times r \times \cos(30^\circ)$ ***M1A1***

$r^2 - 12\sqrt{3}r + 144 - p^2 = 0$ ***AG***

[2 marks]

(b) **EITHER**

$r^2 - 12\sqrt{3}r + 80 = 0$ ***(M1)***

OR

using the sine rule ***(M1)***

THEN

$PQ = 5.10$ (cm) or ***A1***

$PQ = 15.7$ (cm) ***A1***

[3 marks]

(c) area $= \frac{1}{2} \times 12 \times 5.1008\ldots \times \sin(30^\circ)$ ***M1A1***

$= 15.3\,(\text{cm}^2)$ ***A1***

[3 marks]

(d) **METHOD 1**

EITHER

$r^2 - 12\sqrt{3}r + 144 - p^2 = 0$

discriminant $= \left(12\sqrt{3}\right)^2 - 4 \times \left(144 - p^2\right)$ ***M1***

$= 4\left(p^2 - 36\right)$ ***A1***

$\left(p^2 - 36\right) > 0$ ***M1***

$p > 6$ ***A1***

OR

construction of a right angle triangle ***(M1)***

$12 \sin 30° = 6$ ***M1(A1)***

hence for two triangles $p > 6$ ***R1***

THEN

$p < 12$ ***A1***

$144 - p^2 > 0$ to ensure two positive solutions or valid geometric argument ***R1***

$\therefore\ 6 < p < 12$ ***A1***

METHOD 2

diagram showing two triangles ***(M1)***

$12\sin 30° = 6$ ***M1A1***

one right angled triangle when $p = 6$ ***(A1)***

$\therefore p > 6$ for two triangles ***R1***

$p < 12$ for two triangles ***A1***

$6 < p < 12$ ***A1***

[7 marks]

Total [15 marks]

Question 22

METHOD 1

area = (four sector areas radius 9) + (four sector areas radius 3) ***(M1)***

$= 4\left(\frac{1}{2}9^2\frac{\pi}{9}\right) + 4\left(\frac{1}{2}3^2\frac{7\pi}{18}\right)$ ***(A1)(A1)***

$= 18\pi + 7\pi$

$= 25\pi\left(= 78.5\text{cm}^2\right)$ ***A1***

METHOD 2

area =

(area of circle radius 3) + (four sector areas radius 9) – (four sector areas radius 3) ***(M1)***

$\pi 3^2 + 4\left(\frac{1}{2}9^2\frac{\pi}{9}\right) - 4\left(\frac{1}{2}3^2\frac{\pi}{9}\right)$ ***(A1)(A1)***

ote: Award ***A1*** for the second term and ***A1*** for the third term.

$= 9\pi + 18\pi - 2\pi$

$= 25\pi\left(= 78.5\text{cm}^2\right)$ ***A1***

ote: Accept working in degrees.

[4 marks]

Question 23

attempt to use tan, or sine rule, in triangle BXN or BXS ***(M1)***

$\text{NX} = 80\tan 55^\circ \left(= \frac{80}{\tan 35^\circ} = 114.25\right)$ ***(A1)***

$\text{SX} = 80\tan 65^\circ \left(= \frac{80}{\tan 25^\circ} = 171.56\right)$ ***(A1)***

Attempt to use cosine rule ***M1***

$\text{SN}^2 = 171.56^2 + 114.25^2 - 2 \times 171.56 \times 114.25\cos 70^\circ$ ***(A1)***

$\text{SN} = 171(\text{m})$ ***A1***

ote: Award final ***A1*** only if the correct answer has been given to 3 significant figures.

[6 marks]

Question 24

(a) $k^2 - k - 12 < 0$

$(k-4)(k+3) < 0$ *(M1)*

$-3 < k < 4$ **A1**

[2 marks]

(b) $\cos B = \dfrac{2^2 + c^2 - 4^2}{4c}$ $\left(\text{or } 16 = 2^2 + c^2 - 4c\cos B\right)$ **M1**

$\Rightarrow \dfrac{c^2 - 12}{4c} < \dfrac{1}{4}$ **A1**

$\Rightarrow c^2 - c - 12 < 0$

from result in (a)

$0 < \text{AB} < 4$ or $-3 < \text{AB} < 4$ *(A1)*

but AB must be at least 2

$\Rightarrow 2 < AB < 4$ **A1**

Note: Allow $\le AB$ for either of the final two ***A*** marks.

[4 marks]

Total [6 marks]

Question 25

(a) (i) **METHOD 1**

$\text{PC} = \dfrac{\sqrt{3}}{2}$ or 0.8660 *(M1)*

$\text{PM} = \dfrac{1}{2}\text{PC} = \dfrac{\sqrt{3}}{4}$ or 0.4330 *(A1)*

$\text{AM} = \sqrt{\dfrac{1}{4} + \dfrac{3}{16}}$

$= \dfrac{\sqrt{7}}{4}$ or 0.661 (m) **A1**

Note: Award ***M1*** for attempting to solve triangle AMP.

METHOD 2

using the cosine rule

$\text{AM}^2 = 1^2 + \left(\dfrac{\sqrt{3}}{4}\right)^2 - 2 \times \dfrac{\sqrt{3}}{4} \times \cos(30°)$ ***M1A1***

$\text{AM} = \dfrac{\sqrt{7}}{4}$ or 0.661 (m) **A1**

(ii) $\tan\left(\text{A}\hat{\text{M}}\text{P}\right) = \dfrac{2}{\sqrt{3}}$ or equivalent *(M1)*

$= 0.857$ **A1**

[5 marks]

(b) **EITHER**

$\frac{1}{2}\text{AM}^2\left(2\text{A}\hat{\text{M}}\text{P} - \sin(2\text{A}\hat{\text{M}}\text{P})\right)$ ***(M1)A1***

OR

$\frac{1}{2}\text{AM}^2 \times 2\text{A}\hat{\text{M}}\text{P} - \frac{\sqrt{3}}{8}$ ***(M1)A1***

$= 0.158(\text{m}^2)$ ***A1***

Note: Award ***M1*** for attempting to calculate area of a sector minus area of a triangle.

[3 marks]

Total [8 marks]

Question 26

$\tan(x+\pi) = \tan x \left(= \frac{\sin x}{\cos x}\right)$ ***(M1)A1***

$\cos\left(x - \frac{\pi}{2}\right) = \sin x$ ***(M1)A1***

e: The two ***M1***'s can be awarded for observation or for expanding.

$\tan(x+\pi)\cos\left(x - \frac{\pi}{2}\right) = \frac{\sin^2 x}{\cos x}$ ***A1***

[5 marks]

Question 27

(a) **METHOD 1**

use of tan ***(M1)***

$\tan\theta_p = \frac{1}{p}$ ***(A1)***

$\theta_p = \arctan\left(\frac{1}{p}\right)$ ***A1***

METHOD 2

$\text{AP} = \sqrt{p^2+1}$ ***(A1)***

use of sin, cos, sine rule or cosine rule using the correct length of AP ***(M1)***

$\theta_p = \arcsin\left(\frac{1}{\sqrt{p^2+1}}\right)$ or $\theta_p = \arccos\left(\frac{p}{\sqrt{p^2+1}}\right)$ ***A1***

[3 marks]

(b) $QR = 1 \Rightarrow r = q + 1$ ***(A1)***

Note: This may be seen anywhere.

$\tan\theta_p = \tan\left(\theta_q + \theta_r\right)$

attempt to use compound angle formula for tan ***M1***

$\tan\theta_p = \dfrac{\tan\theta_q + \tan\theta_r}{1 - \tan\theta_q \tan\theta_r}$ ***(A1)***

$\dfrac{1}{p} = \dfrac{\dfrac{1}{q} + \dfrac{1}{r}}{1 - \left(\dfrac{1}{q}\right)\left(\dfrac{1}{r}\right)}$ ***(M1)***

$\dfrac{1}{p} = \dfrac{\dfrac{1}{q} + \dfrac{1}{q+1}}{1 - \left(\dfrac{1}{q}\right)\left(\dfrac{1}{q+1}\right)}$ or $p = \dfrac{1 - \left(\dfrac{1}{q}\right)\left(\dfrac{1}{q+1}\right)}{\dfrac{1}{q} + \dfrac{1}{q+1}}$ ***A1***

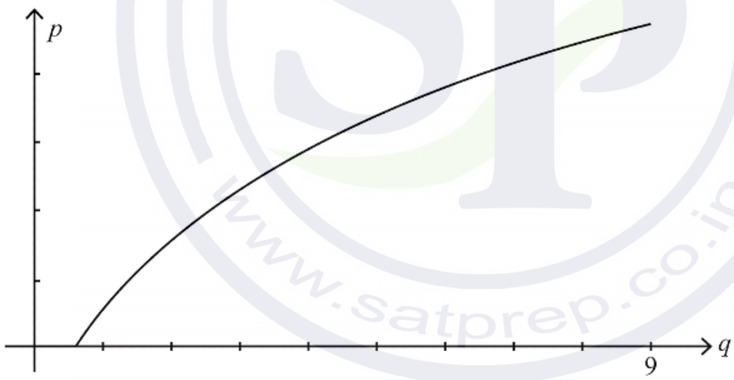
$\dfrac{1}{p} = \dfrac{q + q + 1}{q(q+1) - 1}$ ***M1***

Note: Award ***M1*** for multiplying top and bottom by $q(q+1)$.

$p = \dfrac{q^2 + q - 1}{2q + 1}$ ***AG***

[6 marks]

(c)



increasing function with positive q-intercept ***A1***

Note: Accept curves which extend beyond the domain shown above.

$(0.618 <)\, q < 9$ ***(A1)***

$\Rightarrow$ range is $(0 <)\, p < 4.68$ ***(A1)***

$0 < p < 4.68$ ***A1***

[4 marks]

Total [13 marks]

Question 28

attempt to apply cosine rule **M1**

$\cos A = \dfrac{5^2 + 11^2 - 14^2}{2 \times 5 \times 11} = -0.4545...$

$\Rightarrow A = 117.03569...^\circ$

$\Rightarrow A = 117.0^\circ$ **A1**

attempt to apply sine rule or cosine rule: **M1**

$\dfrac{\sin 117.03569...^\circ}{14} = \dfrac{\sin B}{11}$

$\Rightarrow B = 44.4153...^\circ$

$\Rightarrow B = 44.4^\circ$ **A1**

$C = 180^\circ - A - B$

$C = 18.5^\circ$ **A1**

te: Candidates may attempt to find angles in any order of their choosing.

[5 marks]

Question 29

(a) **METHOD 1**

$\text{LHS} = \dfrac{1+\sin 2x}{\cos 2x} = \dfrac{1+2\sin x\cos x}{\cos^2 x - \sin^2 x}$ **M1**

$= \dfrac{\left(\cos^2 x + \sin^2 x\right) + 2\sin x\cos x}{\cos^2 x - \sin^2 x}$ **M1**

$= \dfrac{(\cos x + \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)}$ **A1**

$= \dfrac{\cos x + \sin x}{\cos x - \sin x}$

$= \dfrac{\dfrac{\cos x}{\cos x} + \dfrac{\sin x}{\cos x}}{\dfrac{\cos x}{\cos x} - \dfrac{\sin x}{\cos x}}$ **A1**

$= \dfrac{1+\tan x}{1-\tan x}$ **AG**

METHOD 2

$\text{LHS} = \dfrac{1+\sin 2x}{\cos 2x} = \dfrac{1+2\sin x\cos x}{\cos^2 x - \sin^2 x}$ **M1**

dividing numerator and denominator by $\cos^2 x$ **M1**

$= \dfrac{\sec^2 x + 2\tan x}{1-\tan^2 x}$

$= \dfrac{1+\tan^2 x + 2\tan x}{1-\tan^2 x}$ **A1**

$= \dfrac{(\tan x + 1)^2}{(1-\tan x)(1+\tan x)}$ **A1**

$= \dfrac{1+\tan x}{1-\tan x}$ **AG**

Note: Candidates may start with RHS; apply MS in reverse.

[4 marks]

(b) valid attempt to solve $\frac{1+\tan x}{1-\tan x}=\sqrt{3}$ ***(M1)***

$\tan x=\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$x=0.262\left(=\frac{\pi}{12}\right),\ x=3.40\left(=\frac{13\pi}{12}\right)$ ***A1***

Note: Award ***M1A0*** if only one correct solution is given.

[2 marks]

Total [6 marks]

Question 30

(a) $3, -3$ **A1A1**

[2 marks]

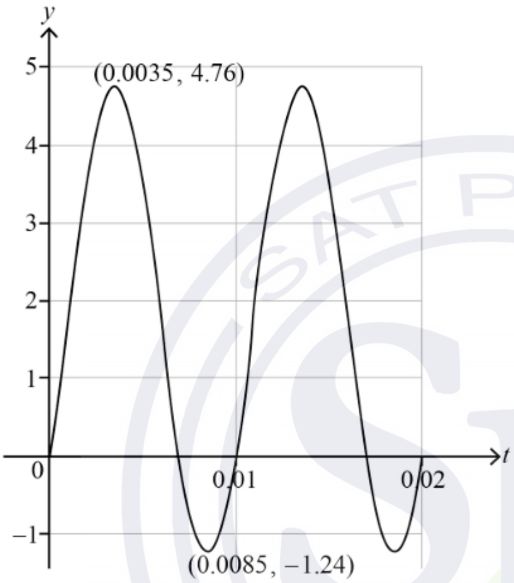
(b) stretch parallel to the y-axis (with x-axis invariant), scale factor $\frac{2}{3}$ **A1**

translation of $\begin{pmatrix} -0.003 \\ 0 \end{pmatrix}$ (shift to the left by 0.003) **A1**

Note: Can be done in either order.

[2 marks]

(c)



correct shape over correct domain with correct endpoints **A1**
first maximum at $(0.0035, 4.76)$ **A1**
first minimum at $(0.0085, -1.24)$ **A1**

[3 marks]

(d) $p \geq 3$ between $t = 0.0016762$ and 0.0053238 and $t = 0.011676$ and 0.015324 ***(M1)(A1)***

Note: Award ***M1A1*** for either interval.

$= 0.00730$ **A1**

[3 marks]

(e) $p_{av} = \frac{1}{0.007}\int_0^{0.007} 6\sin(100\pi t)\sin\left(100\pi(t+0.003)\right)\mathrm{d}t$ ***(M1)***

$= 2.87$ **A1**

[2 marks]

continued…

(f) in each cycle the area under the t axis is smaller than area above the t axis **R1**
the curve begins with the positive part of the cycle **R1**

[2 marks]

(g) $a = \frac{4.76-(-1.24)}{2}$ ***(M1)***

$a = 3.00$ ***A1***

$d = \frac{4.76+(-1.24)}{2}$

$d = 1.76$ ***A1***

$b = \frac{2\pi}{0.01}$

$b = 628 (= 200\pi)$ ***A1***

$c = 0.0035 - \frac{0.01}{4}$ ***(M1)***

$c = 0.00100$ ***A1***

[6 marks]

Total [20 marks]

Question 31

(a) each arc has length $r\theta = 6\times\frac{\pi}{3} = 2\pi\left(= 6.283...\right)$ ***(M1)***

perimeter is therefore $6\pi\ (= 18.8)\ (\text{cm})$ ***A1***

[2 marks]

(b) area of sector, s, is $\frac{1}{2}r^2\theta = 18\times\frac{\pi}{3} = 6\pi\left(= 18.84...\right)$ ***(A1)***

area of triangle, t, is $\frac{1}{2}\times 6\times 3\sqrt{3} = 9\sqrt{3}\left(= 15.58...\right)$ ***(M1)(A1)***

Note: area of segment, k, is 3.261… implies area of triangle

finding $3s - 2t$ or $3k + t$ or similar

area $= 3s - 2t = 18\pi - 18\sqrt{3}\ (= 25.4)\ (\text{cm}^2)$ ***(M1)A1***

[5 marks]

Total [7 marks]