Subject – Math (Standard Level)
Topic - Circular trigonometry
Year - Nov 2011 – Nov 2019
Paper -2

Question 1

[Maximum mark: 6]

Consider the following circle with centre O and radius 6.8 cm.

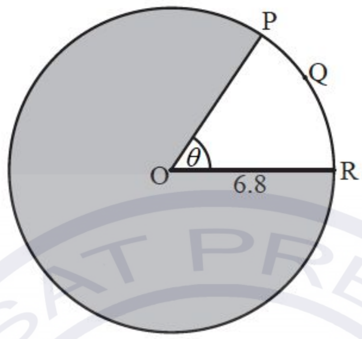


diagram not to scale

The length of the arc PQR is 8.5 cm.

(a) Find the value of θ. *[2 marks]*

(b) Find the area of the shaded region. *[4 marks]*

Question 2

[Maximum mark: 6]

Consider the triangle ABC, where $AB = 10$, $BC = 7$ and $C\hat{A}B = 30^\circ$.

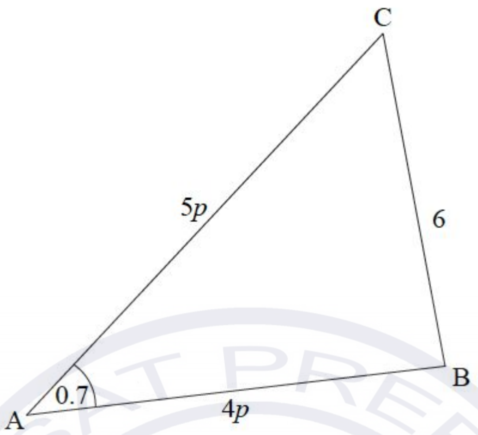
(a) Find the two possible values of $A\hat{C}B$. *[4 marks]*

(b) Hence, find $A\hat{B}C$, given that it is acute. *[2 marks]*

Question 3

[Maximum mark: 15]

The following diagram shows a triangle ABC.

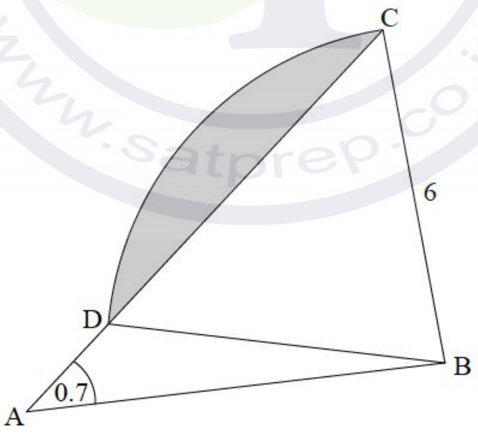


$BC = 6$, $C\hat{A}B = 0.7$ radians, $AB = 4p$, $AC = 5p$, where $p > 0$.

(a) (i) Show that $p^2(41 - 40\cos 0.7) = 36$.

(ii) Find p. *[4 marks]*

Consider the circle with centre B that passes through the point C. The circle cuts the line CA at D, and $A\hat{D}B$ is obtuse. Part of the circle is shown in the following diagram.



(b) Write down the length of BD. *[1 mark]*

(c) Find $A\hat{D}B$. *[4 marks]*

(d) (i) Show that $C\hat{B}D = 1.29$ radians, correct to 2 decimal places.

(ii) Hence, find the area of the shaded region. *[6 marks]*

Question 4

[Maximum mark: 6]

The following diagram shows ΔPQR, where $RQ = 9$ cm, $P\hat{R}Q = 70^\circ$ and $P\hat{Q}R = 45^\circ$.

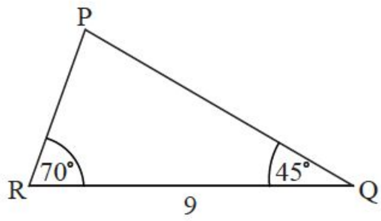


diagram not to scale

(a) Find $R\hat{P}Q$. *[1 mark]*

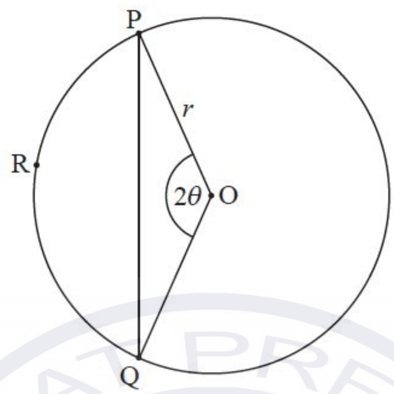
(b) Find PR. *[3 marks]*

(c) Find the area of ΔPQR. *[2 marks]*

Question 5

[Maximum mark: 16]

Consider the following circle with centre O and radius r.



The points P, R and Q are on the circumference, $P\hat{O}Q = 2\theta$, for $0 < \theta < \frac{\pi}{2}$.

(a) Use the cosine rule to show that $PQ = 2r\sin\theta$. *[4 marks]*

Let l be the length of the arc PRQ.

(b) Given that $1.3PQ - l = 0$, find the value of θ. *[5 marks]*

Consider the function $f(\theta) = 2.6\sin\theta - 2\theta$, for $0 < \theta < \frac{\pi}{2}$.

(c) (i) Sketch the graph of f.

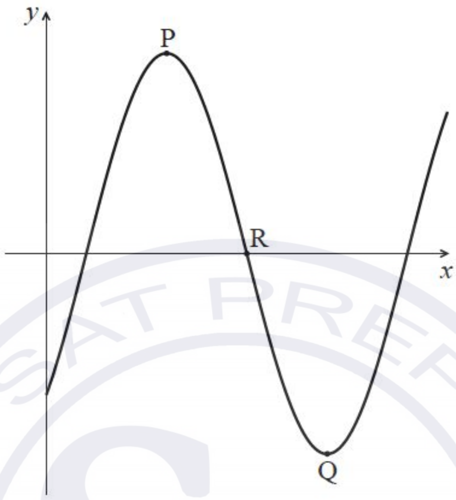
(ii) Write down the root of $f(\theta) = 0$. *[4 marks]*

(d) Use the graph of f to find the values of θ for which $l < 1.3PQ$. *[3 marks]*

Question 6

[Maximum mark: 6]

Let $f(x) = a\cos(b(x-c))$. The diagram below shows part of the graph of f, for $0 \le x \le 10$.



The graph has a local maximum at $P(3, 5)$, a local minimum at $Q(7, -5)$, and crosses the x-axis at R.

(a) Write down the value of

(i) a;

(ii) c. *[2 marks]*

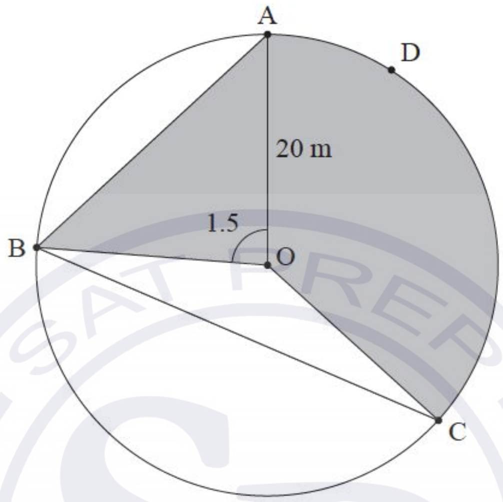
(b) Find the value of b. *[2 marks]*

(c) Find the x-coordinate of R. *[2 marks]*

Question 7

[Maximum mark: 15]

The following diagram shows a circular play area for children.



The circle has centre O and a radius of 20 m, and the points A, B, C and D lie on the circle. Angle AOB is 1.5 radians.

(a) Find the length of the chord [AB]. *[3 marks]*

(b) Find the area of triangle AOB. *[2 marks]*

Angle BOC is 2.4 radians.

(c) Find the length of arc ADC. *[3 marks]*

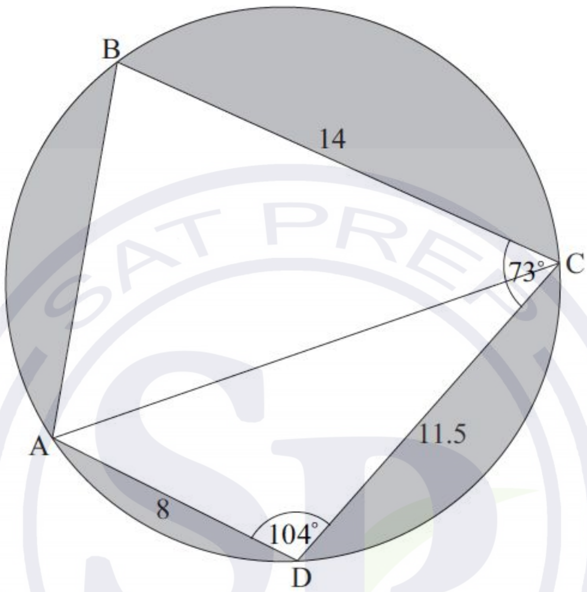
(d) Find the area of the shaded region. *[3 marks]*

(e) The shaded region is to be painted red. Red paint is sold in cans which cost \$32 each. One can covers 140 m^2. How much does it cost to buy the paint? *[4 marks]*

Question 8

[Maximum mark: 14]

The diagram shows a circle of radius 8 metres. The points ABCD lie on the circumference of the circle.



BC = 14 m, CD = 11.5 m, AD = 8 m, $A\hat{D}C = 104^\circ$, and $B\hat{C}D = 73^\circ$

(a) Find AC. *[3 marks]*

(b) (i) Find $A\hat{C}D$.

(ii) Hence, find $A\hat{C}B$. *[5 marks]*

(c) Find the area of triangle ADC. *[2 marks]*

(d) Hence or otherwise, find the total area of the shaded regions. *[4 marks]*

Question 9

[Maximum mark: 16]

A Ferris wheel with diameter 122 metres rotates clockwise at a constant speed. The wheel completes 2.4 rotations every hour. The bottom of the wheel is 13 metres above the ground.

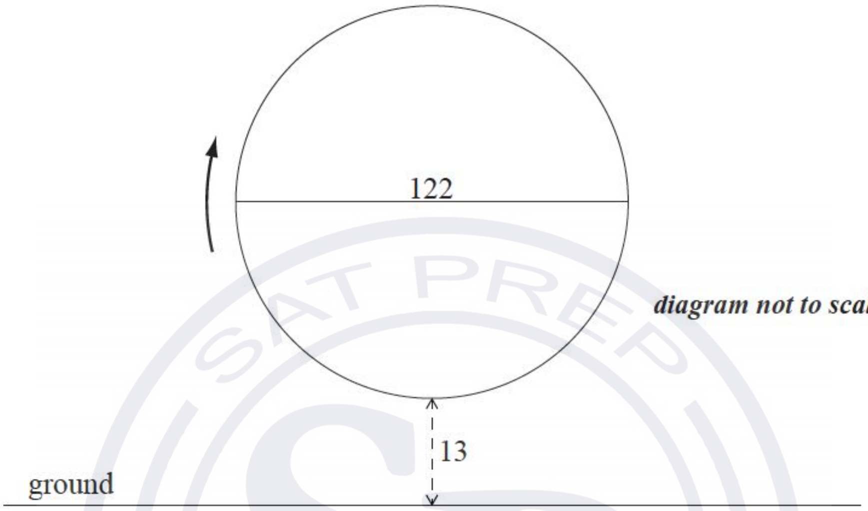


diagram not to scale

A seat starts at the bottom of the wheel.

(a) Find the maximum height above the ground of the seat. *[2 marks]*

After t minutes, the height h metres above the ground of the seat is given by

$$h = 74 + a\cos bt .$$

(b) (i) Show that the period of h is 25 minutes.

(ii) Write down the **exact** value of b. *[2 marks]*

(c) Find the value of a. *[3 marks]*

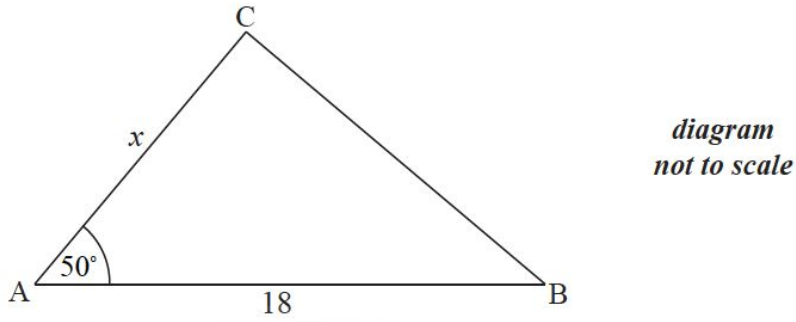
(d) Sketch the graph of h, for $0 \le t \le 50$. *[4 marks]*

(e) In one rotation of the wheel, find the probability that a randomly selected seat is at least 105 metres above the ground. *[5 marks]*

Question 10

[Maximum mark: 6]

The following diagram shows a triangle ABC.



The area of triangle ABC is 80 cm^2, $AB = 18 \text{ cm}$, $AC = x \text{ cm}$ and $B\hat{A}C = 50°$.

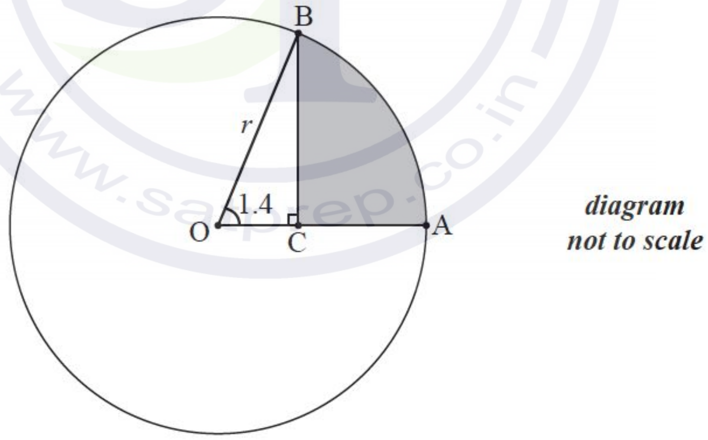
(a) Find x. *[3 marks]*

(b) Find BC. *[3 marks]*

Question 11

[Maximum mark: 8]

The following diagram shows a circle with centre O and radius r cm.



Points A and B are on the circumference of the circle and $A\hat{O}B = 1.4$ radians.
The point C is on [OA] such that $B\hat{C}O = \frac{\pi}{2}$ radians.

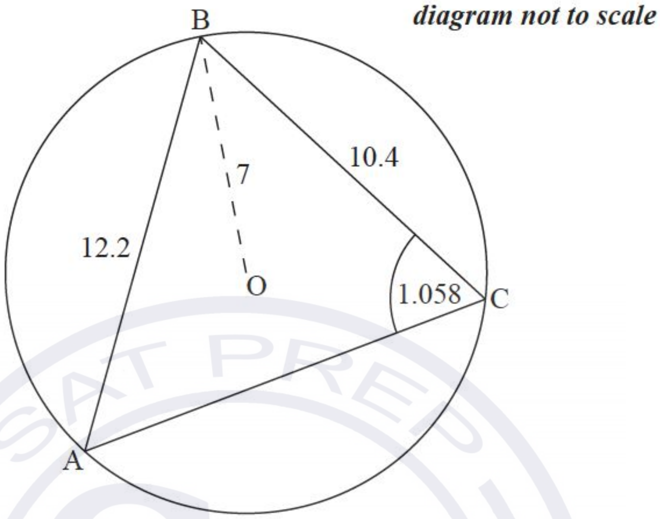
(a) Show that $OC = r\cos 1.4$. *[1 mark]*

(b) The area of the shaded region is 25 cm^2. Find the value of r. *[7 marks]*

Question 12

[Maximum mark: 14]

Consider a circle with centre O and radius 7 cm. Triangle ABC is drawn such that its vertices are on the circumference of the circle.



$AB = 12.2\,\text{cm}$, $BC = 10.4\,\text{cm}$ and $A\hat{C}B = 1.058$ radians.

(a) Find $B\hat{A}C$. *[3]*

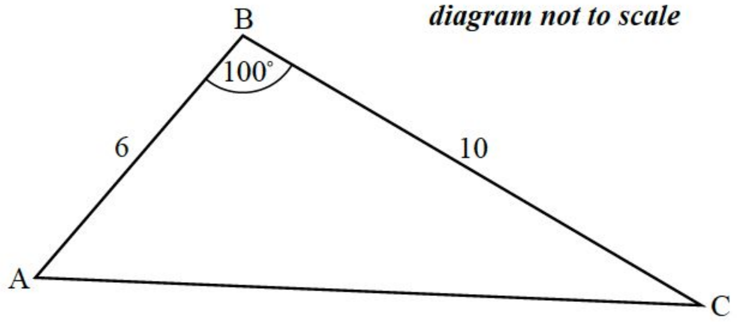
(b) Find AC. *[5]*

(c) Hence or otherwise, find the length of arc ABC. *[6]*

Question 13

[Maximum mark: 6]

The following diagram shows triangle ABC.



$\text{AB} = 6\,\text{cm}$, $\text{BC} = 10\,\text{cm}$, and $\text{A}\hat{\text{B}}\text{C} = 100°$.

(a) Find AC. *[3]*

(b) Find $\text{B}\hat{\text{C}}\text{A}$. *[3]*

Question 14

[Maximum mark: 6]

The population of deer in an enclosed game reserve is modelled by the function $P(t) = 210\sin(0.5t - 2.6) + 990$, where t is in months, and $t = 1$ corresponds to 1 January 2014.

(a) Find the number of deer in the reserve on 1 May 2014. *[3]*

(b) (i) Find the rate of change of the deer population on 1 May 2014.

(ii) Interpret the answer to part (i) with reference to the deer population size on 1 May 2014. *[3]*

Question 15

[Maximum mark: 15]

Let $f(x)=\cos\left(\frac{\pi}{4}x\right)+\sin\left(\frac{\pi}{4}x\right)$, for $-4\le x\le 4$.

(a) Sketch the graph of f. *[3]*

(b) Find the values of x where the function is decreasing. *[5]*

(c) The function f can also be written in the form $f(x)=a\sin\left(\frac{\pi}{4}(x+c)\right)$, where $a\in\mathbb{R}$, and $0\le c\le 2$. Find the value of

(i) a;

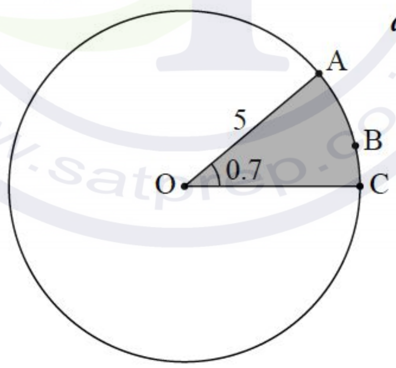
(ii) c. *[7]*

Question 16

[Maximum mark: 6]

The following diagram shows a circle with centre O and radius 5 cm.

diagram not to scale



The points A, B and C lie on the circumference of the circle, and $\hat{AOC}=0.7$ radians.

(a) (i) Find the length of the arc ABC.

(ii) Find the perimeter of the shaded sector. *[4]*

(b) Find the area of the shaded sector. *[2]*

Question 17

[Maximum mark: 7]

In triangle ABC, $AB = 6\text{cm}$ and $AC = 8\text{cm}$. The area of the triangle is 16cm^2.

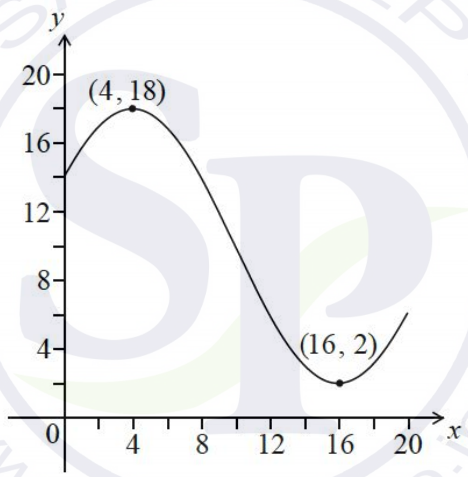
(a) Find the two possible values for $\hat{A}$. *[4]*

(b) Given that $\hat{A}$ is obtuse, find BC. *[3]*

Question 18

[Maximum mark: 8]

Let $f(x) = p\cos(q(x+r))+10$, for $0 \le x \le 20$. The following diagram shows the graph of f.



The graph has a maximum at $(4, 18)$ and a minimum at $(16, 2)$.

(a) Write down the value of r. *[2]*

(b) (i) Find p.

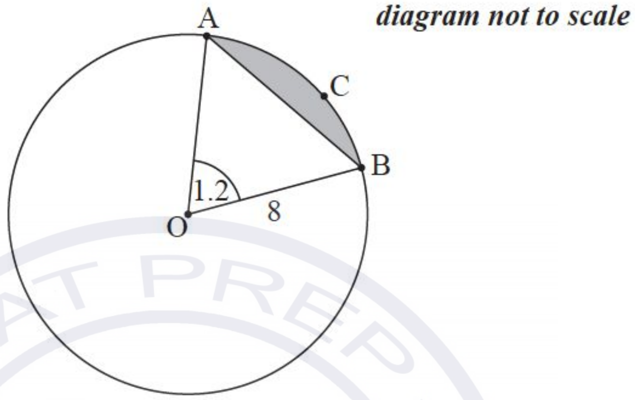
(ii) Find q. *[4]*

(c) Solve $f(x) = 7$. *[2]*

Question 19

[Maximum mark: 7]

The following diagram shows a circle with centre O and radius 8 cm.



The points A, B and C are on the circumference of the circle, and $A\hat{O}B = 1.2$ radians .

(a) Find the length of arc ACB. *[2]*

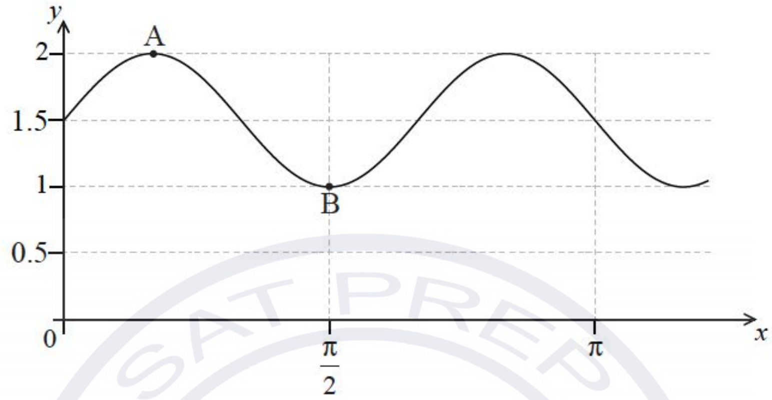
(b) Find AB. *[3]*

(c) Hence, find the perimeter of the shaded segment ABC. *[2]*

Question 20

[Maximum mark: 7]

The following diagram shows part of the graph of $y = p\sin(qx) + r$.



The point $A\left(\frac{\pi}{6}, 2\right)$ is a maximum point and the point $B\left(\frac{\pi}{2}, 1\right)$ is a minimum point. Find the value of

(a) p; *[2]*

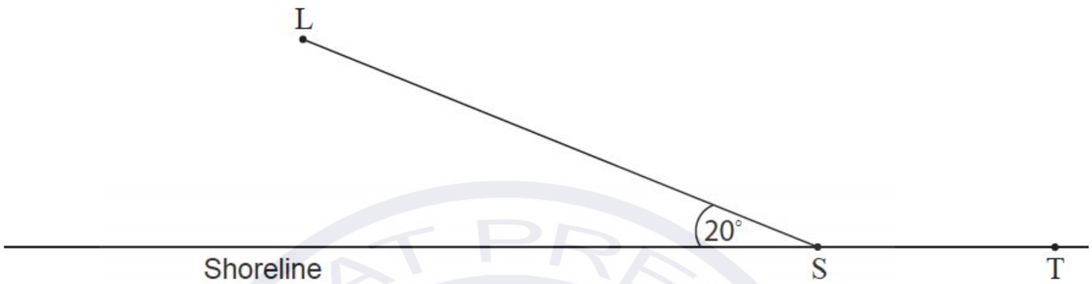
(b) r; *[2]*

(c) q. *[3]*

Question 21

[Maximum mark: 13]

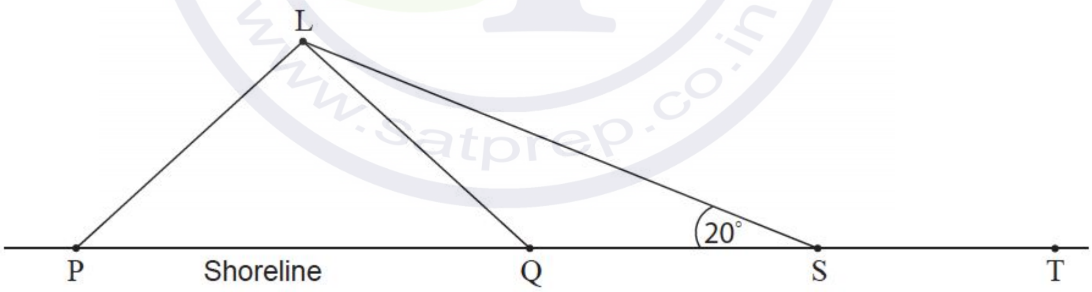
The following diagram shows a straight shoreline, with a supply store at S, a town at T, and an island L.



A boat delivers supplies to the island. The boat leaves S, and sails to the island. Its path makes an angle of 20° with the shoreline.

(a) The boat sails at 6 km per hour, and arrives at L after 1.5 hours. Find the distance from S to L. [2]

It is decided to change the position of the supply store, so that its distance from L is 5 km. The following diagram shows the two possible locations P and Q for the supply store.



(b) Find the size of $\text{S}\hat{\text{P}}\text{L}$ and of $\text{S}\hat{\text{Q}}\text{L}$. [5]

(c) The town wants the new supply store to be as near as possible to the town.

(i) State which of the points P or Q is chosen for the new supply store.

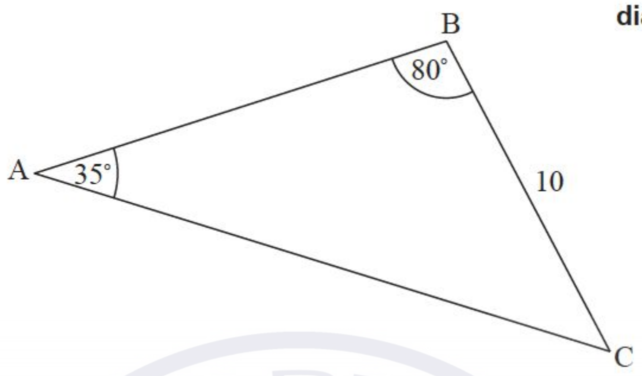
(ii) Hence find the distance between the old supply store and the new one. [6]

Question 22

[Maximum mark: 6]

The following diagram shows triangle ABC.

diagram not to scale



$BC = 10\,\text{cm}$, $A\hat{B}C = 80^\circ$ and $B\hat{A}C = 35^\circ$.

(a) Find AC. [3]

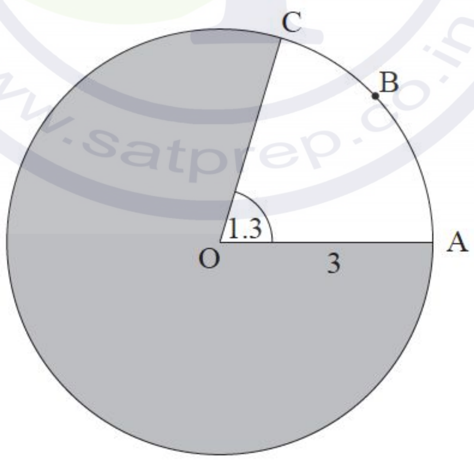
(b) Find the area of triangle ABC. [3]

Question 23

[Maximum mark: 6]

The following diagram shows a circle with centre O and radius $3\,\text{cm}$.

diagram not to scale



Points A, B, and C lie on the circle, and $A\hat{O}C = 1.3$ radians.

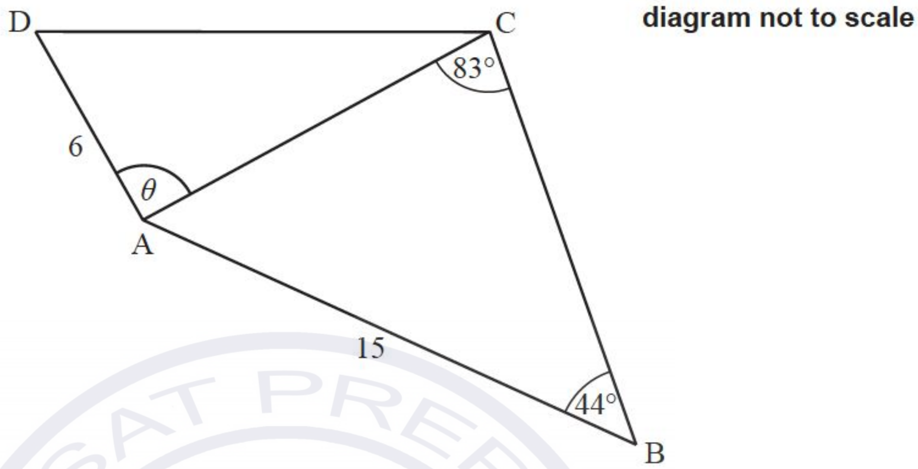
(a) Find the length of arc ABC. [2]

(b) Find the area of the shaded region. [4]

Question 24

[Maximum mark: 14]

The following diagram shows the quadrilateral ABCD.



$AD = 6\text{ cm}$, $AB = 15\text{ cm}$, $A\hat{B}C = 44^\circ$, $A\hat{C}B = 83^\circ$ and $D\hat{A}C = \theta$

(a) Find AC. [3]

(b) Find the area of triangle ABC. [3]

The area of triangle ACD is half the area of triangle ABC.

(c) Find the possible values of θ. [5]

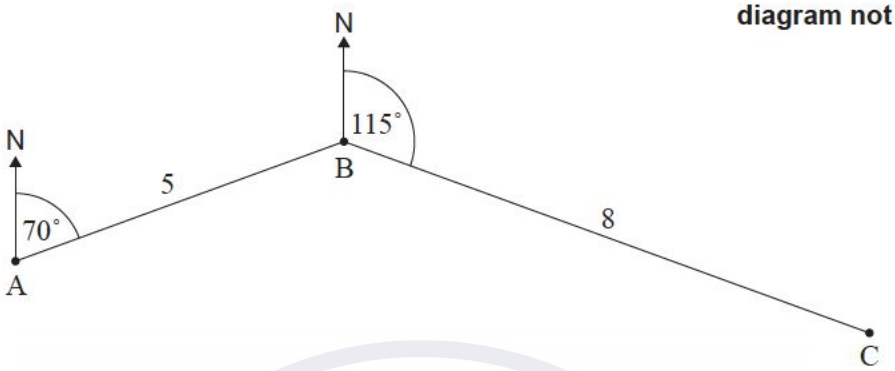
(d) Given that θ is obtuse, find CD. [3]

Question 25

[Maximum mark: 7]

The following diagram shows three towns A, B and C. Town B is 5 km from Town A, on a bearing of 070°. Town C is 8 km from Town B, on a bearing of 115°.

diagram not to scale



(a) Find $A\hat{B}C$. [2]

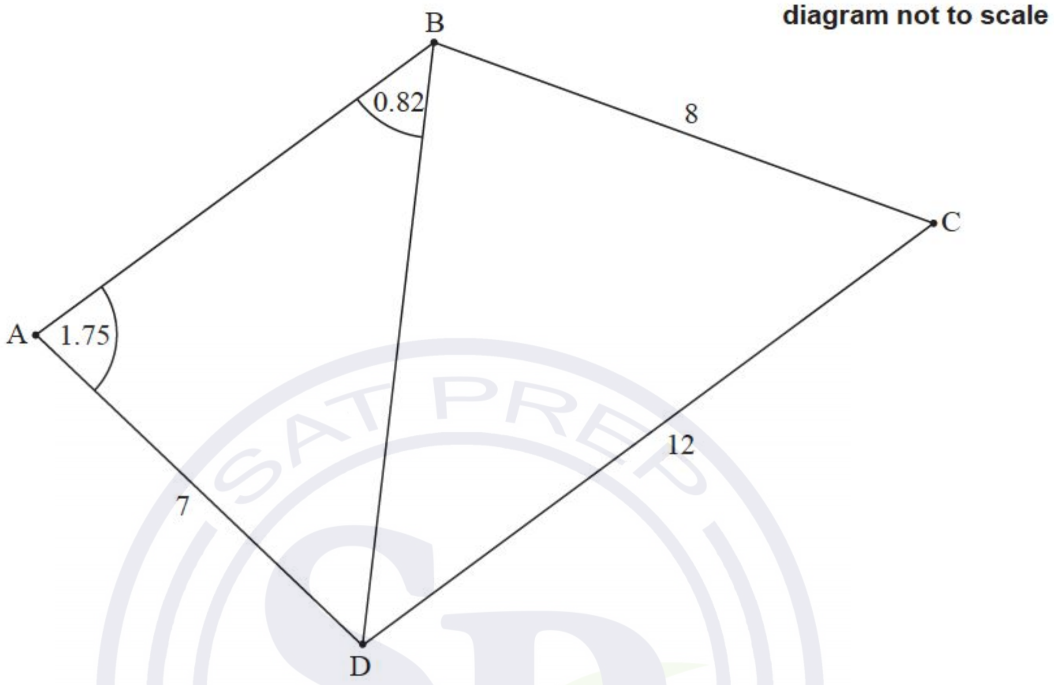
(b) Find the distance from Town A to Town C. [3]

(c) Use the sine rule to find $A\hat{C}B$. [2]

Question 26

[Maximum mark: 6]

The following diagram shows a quadrilateral ABCD.



$AD = 7\text{ cm}$, $BC = 8\text{ cm}$, $CD = 12\text{ cm}$, $D\hat{A}B = 1.75$ radians, $A\hat{B}D = 0.82$ radians.

(a) Find BD. [3]

(b) Find $D\hat{B}C$. [3]

Question 27

[Maximum mark: 8]

The height, h metres, of a seat on a Ferris wheel after t minutes is given by

$$h(t) = -15\cos 1.2t + 17, \text{ for } t \geq 0.$$

(a) Find the height of the seat when $t = 0$. [2]

(b) The seat first reaches a height of 20 m after k minutes. Find k. [3]

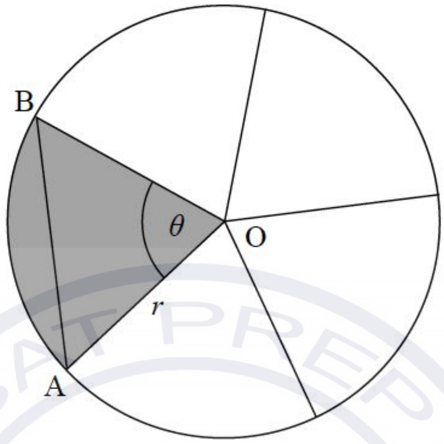
(c) Calculate the time needed for the seat to complete a full rotation, giving your answer correct to one decimal place. [3]

Question 28

[Maximum mark: 7]

The following diagram shows a circle, centre O and radius r mm. The circle is divided into five equal sectors.

diagram not to scale



One sector is OAB, and $\hat{AOB} = \theta$.

(a) Write down the **exact** value of θ in radians. [1]

The area of sector AOB is $20\pi\,\text{mm}^2$.

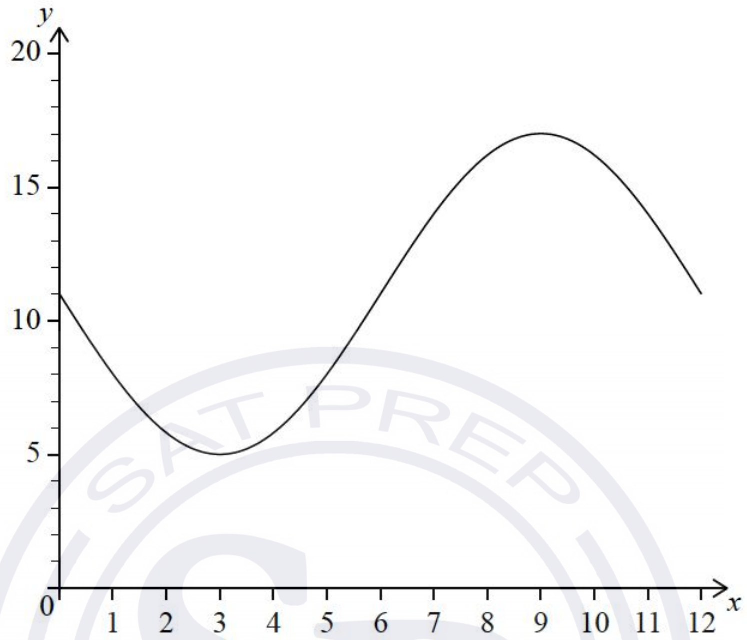
(b) Find the value of r. [3]

(c) Find AB. [3]

Question 29

[Maximum mark: 15]

The following diagram shows the graph of $f(x) = a \sin bx + c$, for $0 \le x \le 12$.



The graph of f has a minimum point at $(3, 5)$ and a maximum point at $(9, 17)$.

(a) (i) Find the value of c.

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of a. [6]

The graph of g is obtained from the graph of f by a translation of $\begin{pmatrix} k \\ 0 \end{pmatrix}$. The maximum point on the graph of g has coordinates $(11.5, 17)$.

(b) (i) Write down the value of k.

(ii) Find $g(x)$. [3]

The graph of g changes from concave-up to concave-down when $x = w$.

(c) (i) Find w.

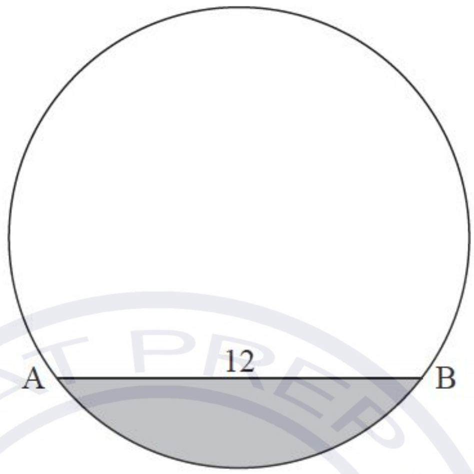
(ii) Hence or otherwise, find the maximum positive rate of change of g. [6]

Question 30

[Maximum mark: 7]

The following diagram shows the chord [AB] in a circle of radius 8 cm, where $AB = 12\text{ cm}$.

diagram not to scale

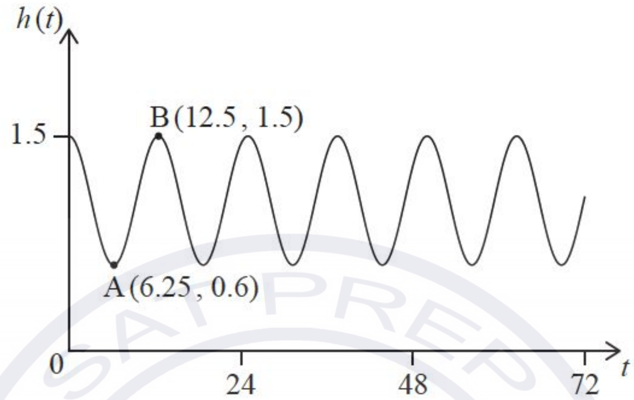


Find the area of the shaded segment.

Question 31

[Maximum mark: 14]

At Grande Anse Beach the height of the water in metres is modelled by the function $h(t) = p\cos(q \times t) + r$, where t is the number of hours after 21:00 hours on 10 December 2017. The following diagram shows the graph of h, for $0 \le t \le 72$.



The point $A(6.25, 0.6)$ represents the first low tide and $B(12.5, 1.5)$ represents the next high tide.

(a) (i) How much time is there between the first low tide and the next high tide?

(ii) Find the difference in height between low tide and high tide. [4]

(b) Find the value of

(i) p;

(ii) q;

(iii) r. [7]

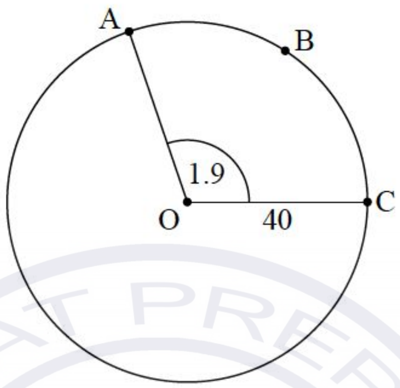
(c) There are two high tides on 12 December 2017. At what time does the second high tide occur? [3]

Question 32

[Maximum mark: 6]

The following diagram shows a circle with centre O and radius 40 cm.

diagram not to scale



The points A, B and C are on the circumference of the circle and $\hat{AOC} = 1.9$ radians .

(a) Find the length of arc ABC. [2]

(b) Find the perimeter of sector OABC. [2]

(c) Find the area of sector OABC. [2]

Question 33

[Maximum mark: 6]

The depth of water in a port is modelled by the function $d(t) = p\cos qt + 7.5$, for $0 \le t \le 12$, where t is the number of hours after high tide.

At high tide, the depth is 9.7 metres.

At low tide, which is 7 hours later, the depth is 5.3 metres.

(a) Find the value of p . [2]

(b) Find the value of q . [2]

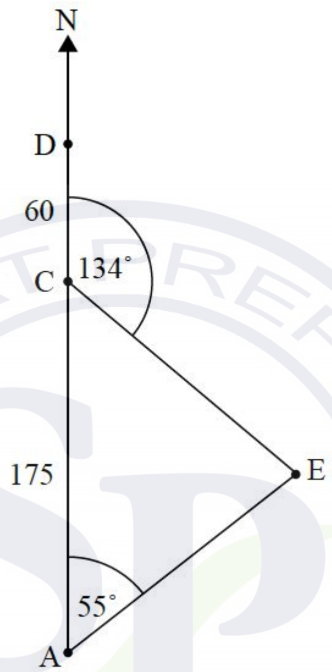
(c) Use the model to find the depth of the water 10 hours after high tide. [2]

Question 34

[Maximum mark: 15]

A ship is sailing north from a point A towards point D. Point C is 175 km north of A. Point D is 60 km north of C. There is an island at E. The bearing of E from A is 055°. The bearing of E from C is 134°. This is shown in the following diagram.

diagram not to scale



(a) Find the bearing of A from E. [2]

(b) Find CE. [5]

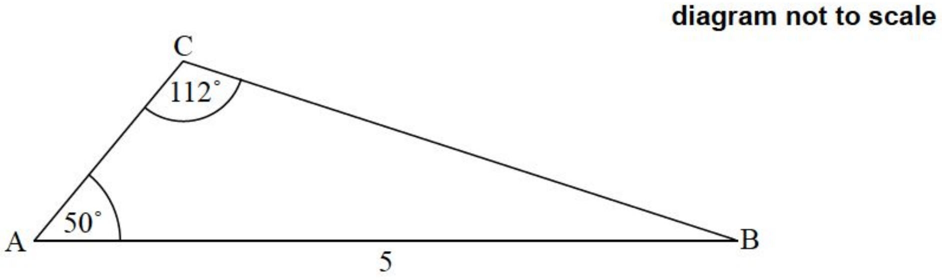
(c) Find DE. [3]

(d) When the ship reaches D, it changes direction and travels directly to the island at 50 km per hour. At the same time as the ship changes direction, a boat starts travelling to the island from a point B. This point B lies on (AC), between A and C, and is the closest point to the island. The ship and the boat arrive at the island at the same time. Find the speed of the boat. [5]

Question 35

[Maximum mark: 6]

The following diagram shows a triangle ABC.



$$\text{AB} = 5\text{ cm},\ \text{C}\hat{\text{A}}\text{B} = 50° \text{ and } \text{A}\hat{\text{C}}\text{B} = 112°$$

(a) Find BC. [3]

(b) Find the area of triangle ABC. [3]

Question 36

[Maximum mark: 17]

Note: In this question, distance is in millimetres.

Let $f(x) = x + a\sin\left(x - \frac{\pi}{2}\right) + a$, for $x \geq 0$.

(a) Show that $f(2\pi) = 2\pi$. [3]

The graph of f passes through the origin. Let P_k be any point on the graph of f with x-coordinate $2k\pi$, where $k \in \mathbb{N}$. A straight line L passes through all the points P_k.

(b) (i) Find the coordinates of P_0 and of P_1.

(ii) Find the equation of L. [6]

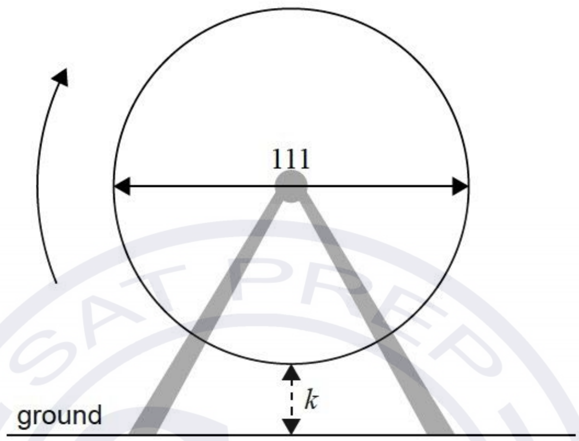
(c) Show that the distance between the x-coordinates of P_k and P_{k+1} is 2π. [2]

Question 37

[Maximum mark: 8]

At an amusement park, a Ferris wheel with diameter 111 metres rotates at a constant speed. The bottom of the wheel is k metres above the ground. A seat starts at the bottom of the wheel.

diagram not to scale



The wheel completes one revolution in 16 minutes.

(a) After 8 minutes, the seat is $117\,\text{m}$ above the ground. Find k. [2]

After t minutes, the height of the seat above ground is given by $h(t) = 61.5 + a\cos\left(\frac{\pi}{8}t\right)$, for $0 \le t \le 32$.

(b) Find the value of a. [3]

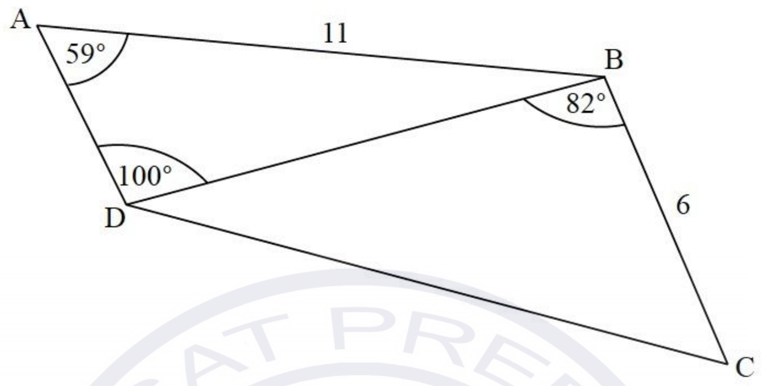
(c) Find when the seat is $30\,\text{m}$ above the ground for the third time. [3]

Question 38

[Maximum mark: 6]

The following diagram shows quadrilateral ABCD.

diagram not to scale



$AB = 11\text{ cm}$, $BC = 6\text{ cm}$, $B\hat{A}D = 59^\circ$, $A\hat{D}B = 100^\circ$, and $C\hat{B}D = 82^\circ$

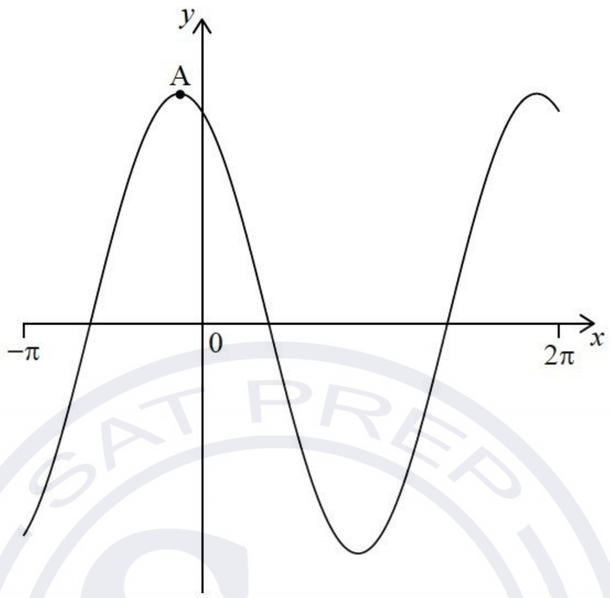
(a) Find DB. [3]

(b) Find DC. [3]

Question 39

[Maximum mark: 15]

Let $f(x) = 12\cos x - 5\sin x$, $-\pi \le x \le 2\pi$, be a periodic function with $f(x) = f(x + 2\pi)$.
The following diagram shows the graph of f.



There is a maximum point at A. The minimum value of f is -13.

(a) Find the coordinates of A. [2]

(b) For the graph of f, write down

(i) the amplitude;

(ii) the period. [2]

(c) Hence, write $f(x)$ in the form $p\cos(x + r)$. [3]

Question 40

[Maximum mark: 7]

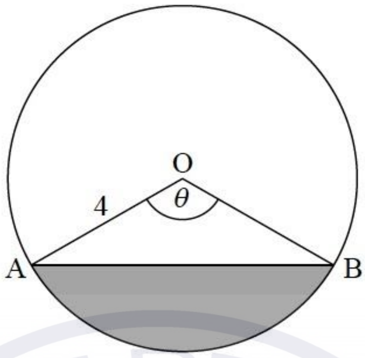
Triangle ABC has $a = 8.1\text{ cm}$, $b = 12.3\text{ cm}$ and area 15 cm^2. Find the largest possible perimeter of triangle ABC.

Question 41

[Maximum mark: 6]

The diagram shows a circle, centre O, with radius 4 cm. Points A and B lie on the circumference of the circle and $\hat{AOB} = \theta$, where $0 \le \theta \le \pi$.

diagram not to scale



(a) Find the area of the shaded region, in terms of θ. [3]

(b) The area of the shaded region is $12\,\text{cm}^2$. Find the value of θ. [3]

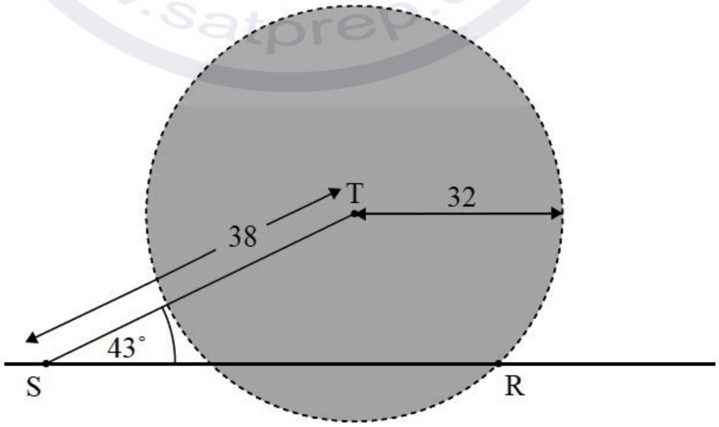
Question 42

[Maximum mark: 6]

A communication tower, T, produces a signal that can reach cellular phones within a radius of 32 km. A straight road passes through the area covered by the tower's signal.

The following diagram shows a line representing the road and a circle representing the area covered by the tower's signal. Point R is on the circumference of the circle and points S and R are on the road. Point S is 38 km from the tower and $R\hat{S}T = 43^\circ$.

diagram not to scale



(a) Let $SR = x$. Use the cosine rule to show that $x^2 - (76\cos 43^\circ)\,x + 420 = 0$. [2]

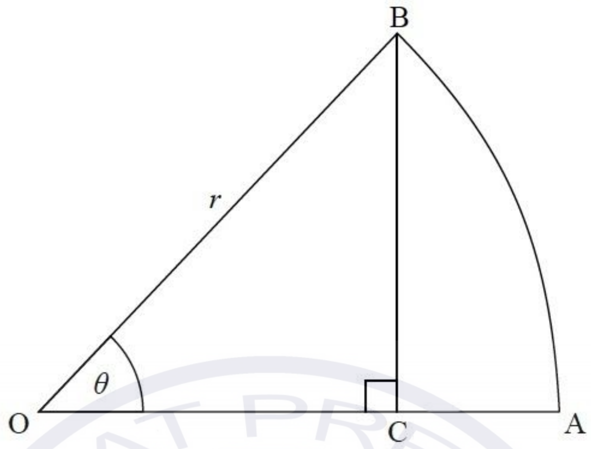
(b) Hence or otherwise, find the total distance along the road where the signal from the tower can reach cellular phones. [4]

Question 43

[Maximum mark: 7]

OAB is a sector of the circle with centre O and radius r, as shown in the following diagram.

diagram not to scale



The angle AOB is θ radians, where $0 < \theta < \frac{\pi}{2}$.

The point C lies on OA and OA is perpendicular to BC.

(a) Show that $\text{OC} = r\cos\theta$. [1]

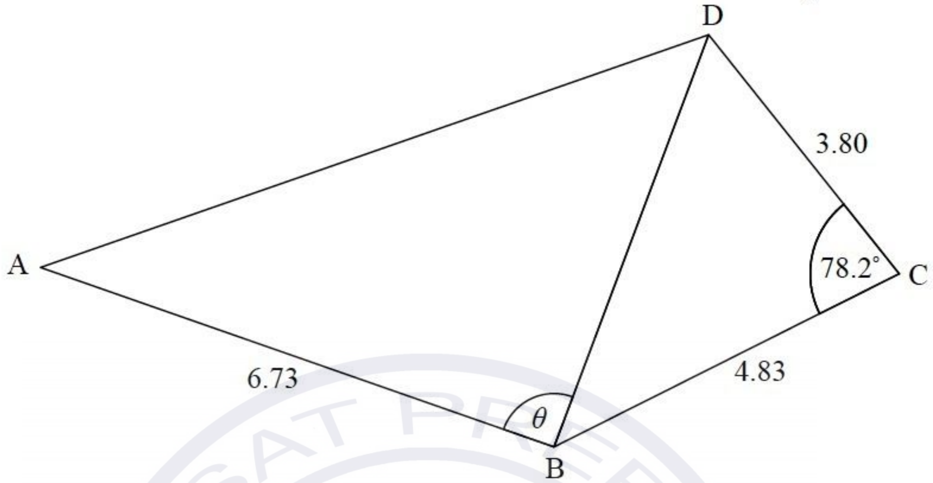
(b) Find the area of triangle OBC in terms of r and θ. [2]

(c) Given that the area of triangle OBC is $\frac{3}{5}$ of the area of sector OAB, find θ. [4]

Question 44

[Maximum mark: 7]

The following diagram shows the quadrilateral ABCD.



$AB = 6.73\,cm$, $BC = 4.83\,cm$, $B\hat{C}D = 78.2^\circ$ and $CD = 3.80\,cm$.

(a) Find BD. [3]

(b) The area of triangle ABD is $18.5\,cm^2$. Find the possible values of θ. [4]

Question 45

[Maximum mark: 13]

Let $f(x) = 2\sin(3x) + 4$ for $x \in \mathbb{R}$.

(a) The range of f is $k \le f(x) \le m$. Find k and m. [3]

Let $g(x) = 5f(2x)$.

(b) Find the range of g. [2]

The function g can be written in the form $g(x) = 10\sin(bx) + c$.

(c) (i) Find the value of b and of c.

(ii) Find the period of g. [5]

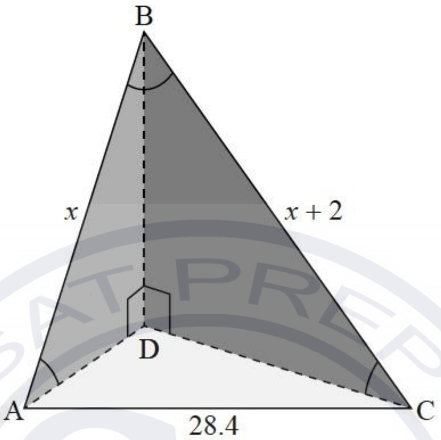
(d) The equation $g(x) = 12$ has two solutions where $\pi \le x \le \frac{4\pi}{3}$. Find both solutions. [3]

Question 46

[Maximum mark: 6]

The diagram below shows a triangular-based pyramid with base ADC.
Edge BD is perpendicular to the edges AD and CD.

diagram not to scale



$AC = 28.4\text{ cm}$, $AB = x\text{ cm}$, $BC = x + 2\text{ cm}$, $A\hat{B}C = 0.667$, $B\hat{A}D = 0.611$

Calculate AD.

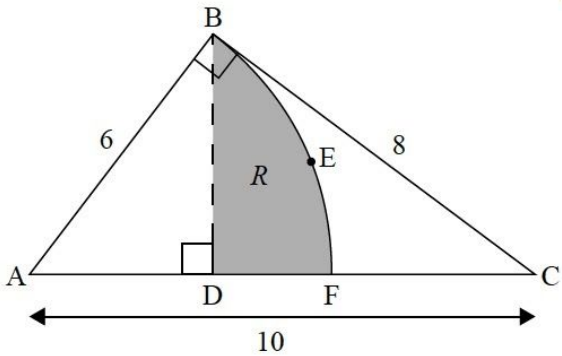
Question 47

[Maximum mark: 7]

The following diagram shows a right-angled triangle, ABC, with $AC = 10\text{ cm}$, $AB = 6\text{ cm}$ and $BC = 8\text{ cm}$.

The points D and F lie on [AC].
[BD] is perpendicular to [AC].
BEF is the arc of a circle, centred at A.
The region R is bounded by [BD], [DF] and arc BEF.

diagram not to scale



(a) Find $B\hat{A}C$. [2]

(b) Find the area of R. [5]