

Subject – Math (Standard Level)  
Topic - Statistics and Probability  
Year - Nov 2011 – Nov 2019  
Paper -1

Question 1

*[Maximum mark: 6]*

A box contains six red marbles and two blue marbles. Anna selects a marble from the box. She replaces the marble and then selects a second marble.

- (a) Write down the probability that the first marble Anna selects is red. *[1 mark]*
- (b) Find the probability that Anna selects two red marbles. *[2 marks]*
- (c) Find the probability that one marble is red and one marble is blue. *[3 marks]*

Question 2

*[Maximum mark: 6]*

The random variable  $X$  has the following probability distribution.

$x$	1	2	3
$P(X = x)$	$s$	0.3	$q$

Given that  $E(X) = 1.7$ , find  $q$ .

### Question 3

[Maximum mark: 7]

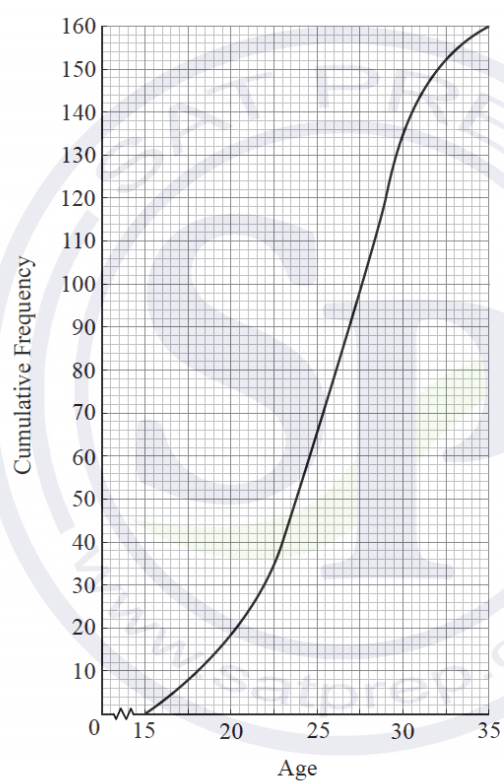
The ages of people attending a music concert are given in the table below.

Age	$15 \leq x < 19$	$19 \leq x < 23$	$23 \leq x < 27$	$27 \leq x < 31$	$31 \leq x < 35$
Frequency	14	26	52	52	16
Cumulative Frequency	14	40	92	$p$	160

(a) Find  $p$ .

[2 marks]

The cumulative frequency diagram is given below.



(b) Use the diagram to estimate

(i) the 80<sup>th</sup> percentile;

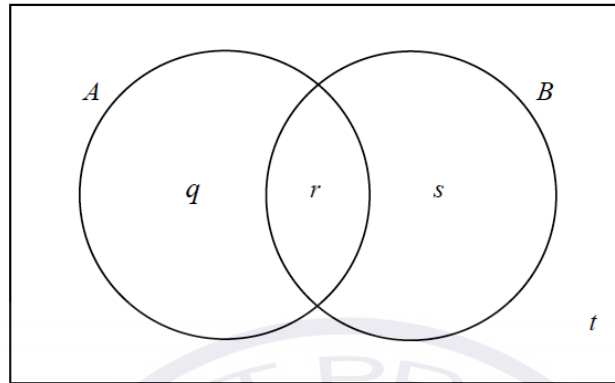
(ii) the interquartile range.

[5 marks]

#### Question 4

[Maximum mark: 7]

Events  $A$  and  $B$  are such that  $P(A) = 0.3$ ,  $P(B) = 0.6$  and  $P(A \cup B) = 0.7$ .



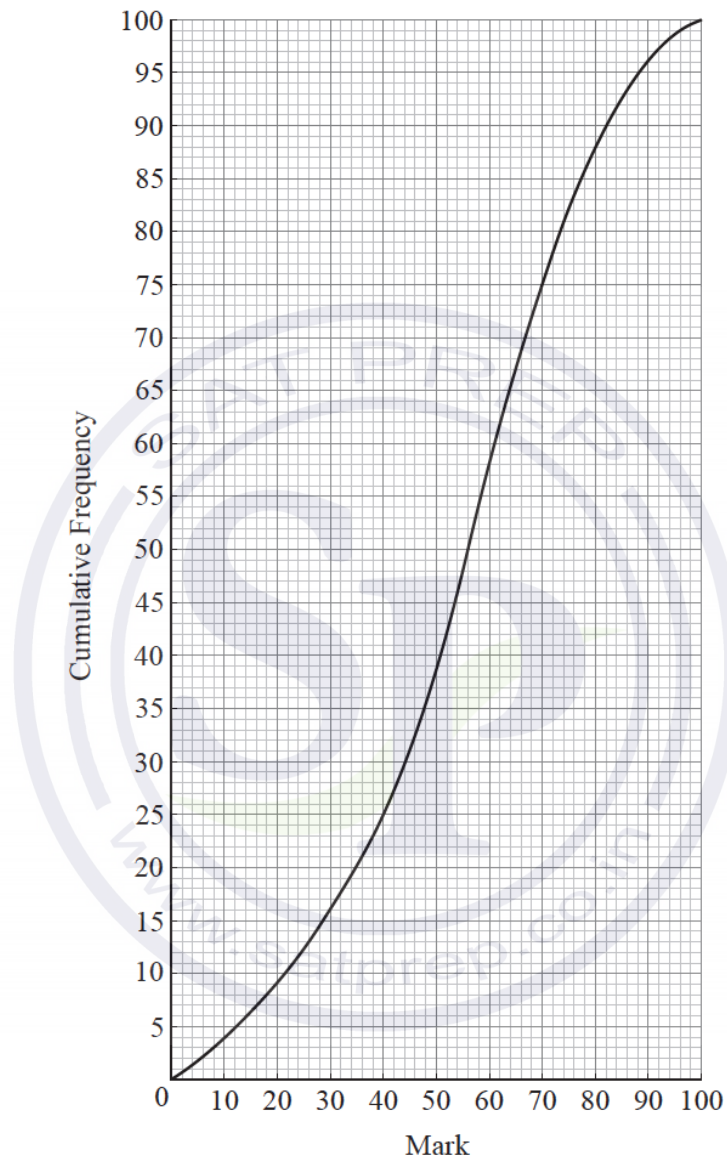
The values  $q$ ,  $r$ ,  $s$  and  $t$  represent probabilities.

- (a) Write down the value of  $t$ . [1 mark]
- (b) (i) Show that  $r = 0.2$ .  
(ii) Write down the value of  $q$  and of  $s$ . [3 marks]
- (c) (i) Write down  $P(B')$ .  
(ii) Find  $P(A|B')$ . [3 marks]

Question 5

[Maximum mark: 5]

The cumulative frequency curve below represents the marks obtained by 100 students.



(a) Find the median mark.

[2 marks]

(b) Find the interquartile range.

[3 marks]

### Question 6

[Maximum mark: 8]

The random variable  $X$  has the following probability distribution, with  $P(X > 1) = 0.5$ .

$x$	0	1	2	3
$P(X = x)$	$p$	$q$	$r$	0.2

(a) Find the value of  $r$ . [2 marks]

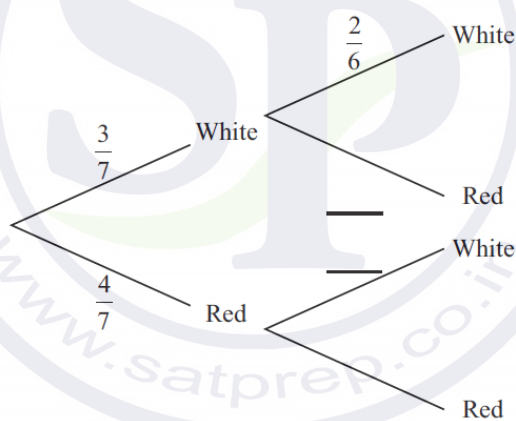
(b) Given that  $E(X) = 1.4$ , find the value of  $p$  and of  $q$ . [6 marks]

### Question 7

[Maximum mark: 14]

Bag A contains three white balls and four red balls. Two balls are chosen at random without replacement.

(a) (i) **Copy** and complete the following tree diagram. (*Do not write on this page.*)



(ii) Find the probability that two white balls are chosen. [5 marks]

Bag B contains four white balls and three red balls. When two balls are chosen at random without replacement from bag B, the probability that they are both white is  $\frac{2}{7}$ .

A standard die is rolled. If 1 or 2 is obtained, two balls are chosen without replacement from bag A, otherwise they are chosen from bag B.

(b) Find the probability that the two balls are white. [5 marks]

(c) Given that both balls are white, find the probability that they were chosen from bag A. [4 marks]

Question 8

*[Maximum mark: 6]*

The following table shows the probability distribution of a discrete random variable  $X$ .

$x$	0	2	5	9
$P(X = x)$	0.3	$k$	$2k$	0.1

(a) Find the value of  $k$ .

*[3 marks]*

(b) Find  $E(X)$ .

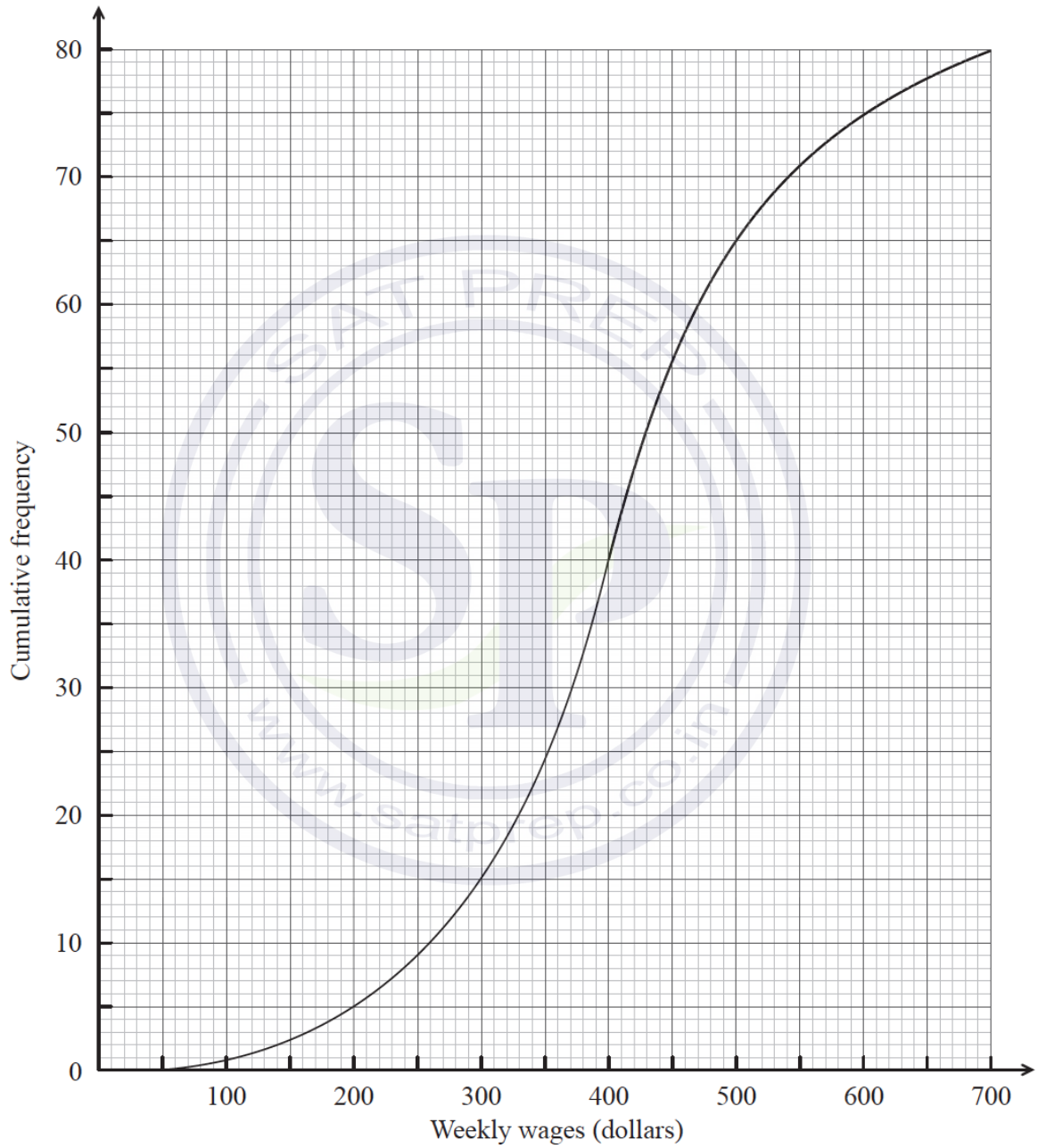
*[3 marks]*



### Question 9

[Maximum mark: 15]

The weekly wages (in dollars) of 80 employees are displayed in the cumulative frequency curve below.

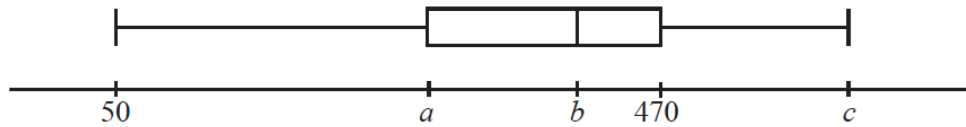


(a) (i) Write down the median weekly wage.

(ii) Find the interquartile range of the weekly wages.

[4 marks]

The box-and-whisker plot below displays the weekly wages of the employees.



(b) Write down the value of

(i)  $a$ ;

(ii)  $b$ ;

(iii)  $c$ .

[3 marks]

Employees are paid \$20 per hour.

(c) Find the median number of **hours** worked per week.

[3 marks]

(d) Find the number of employees who work more than 25 hours per week.

[5 marks]



### Question 10

*[Maximum mark: 16]*

Jar A contains three red marbles and five green marbles. Two marbles are drawn from the jar, one after the other, without replacement.

(a) Find the probability that

(i) none of the marbles are green;

(ii) exactly one marble is green.

*[5 marks]*

(b) Find the expected number of green marbles drawn from the jar.

*[3 marks]*

Jar B contains six red marbles and two green marbles. A fair six-sided die is tossed. If the score is 1 or 2, a marble is drawn from jar A. Otherwise, a marble is drawn from jar B.

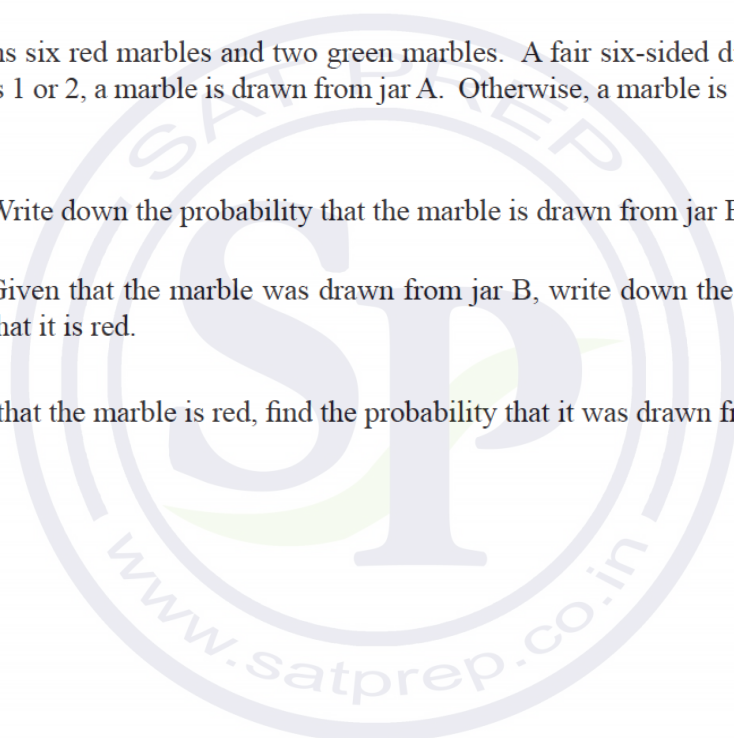
(c) (i) Write down the probability that the marble is drawn from jar B.

(ii) Given that the marble was drawn from jar B, write down the probability that it is red.

*[2 marks]*

(d) Given that the marble is red, find the probability that it was drawn from jar A.

*[6 marks]*



### Question 11

[Maximum mark: 15]

A running club organizes a race to select girls to represent the club in a competition. The times taken by the group of girls to complete the race are shown in the table below.

Time $t$ minutes	$10 \leq t < 12$	$12 \leq t < 14$	$14 \leq t < 20$	$20 \leq t < 26$	$26 \leq t < 28$	$28 \leq t < 30$
Frequency	50	20	$p$	40	20	20
Cumulative Frequency	50	70	120	$q$	180	200

(a) Find the value of  $p$  and of  $q$ . [4 marks]

(b) A girl is chosen at random.

(i) Find the probability that the time she takes is less than 14 minutes.

(ii) Find the probability that the time she takes is at least 26 minutes. [3 marks]

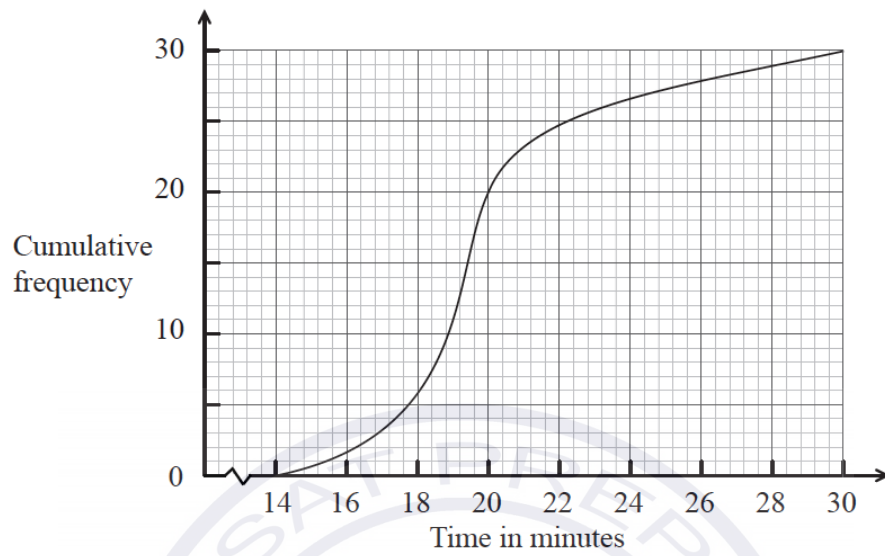
A girl is selected for the competition if she takes less than  $x$  minutes to complete the race.

(c) Given that 40 % of the girls are not selected,

(i) find the number of girls who are not selected;

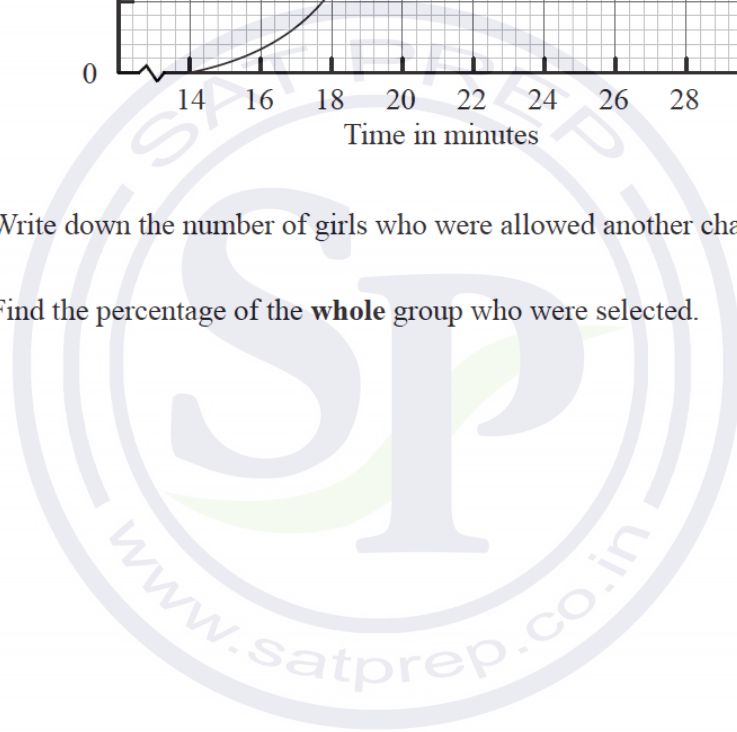
(ii) find  $x$ . [4 marks]

Girls who are not selected, but took less than 25 minutes to complete the race, are allowed another chance to be selected. The new times taken by these girls are shown in the cumulative frequency diagram below.



- (d) (i) Write down the number of girls who were allowed another chance.
- (ii) Find the percentage of the **whole** group who were selected.

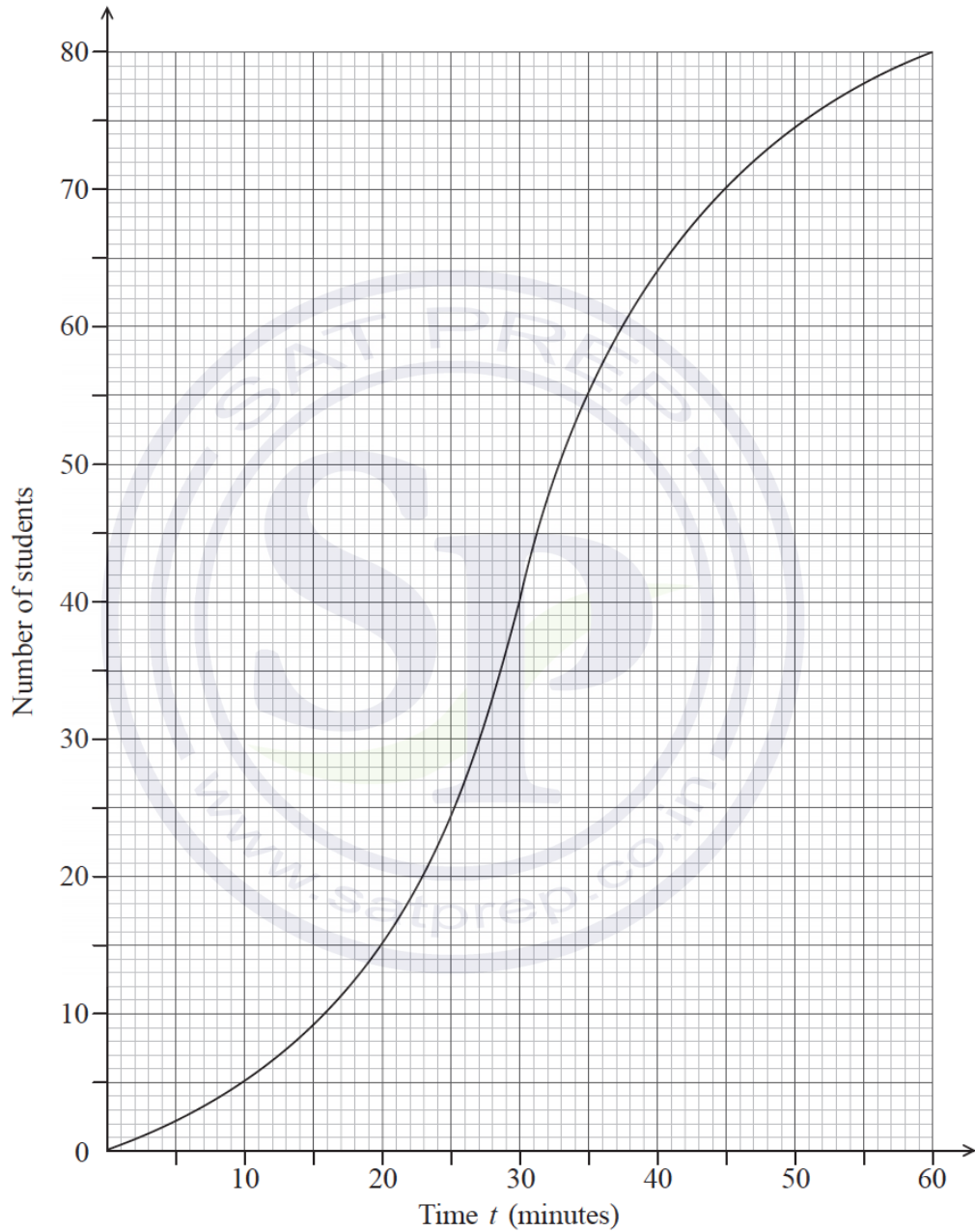
[4 marks]



### Question 12

[Maximum mark: 7]

The following is a cumulative frequency diagram for the time  $t$ , in minutes, taken by 80 students to complete a task.



- (a) Find the number of students who completed the task in less than 45 minutes. [2]
- (b) Find the number of students who took between 35 and 45 minutes to complete the task. [3]
- (c) Given that 50 students take less than  $k$  minutes to complete the task, find the value of  $k$ . [2]

Question 13

[Maximum mark: 6]

Celeste wishes to hire a taxicab from a company which has a large number of taxicabs. The taxicabs are randomly assigned by the company.

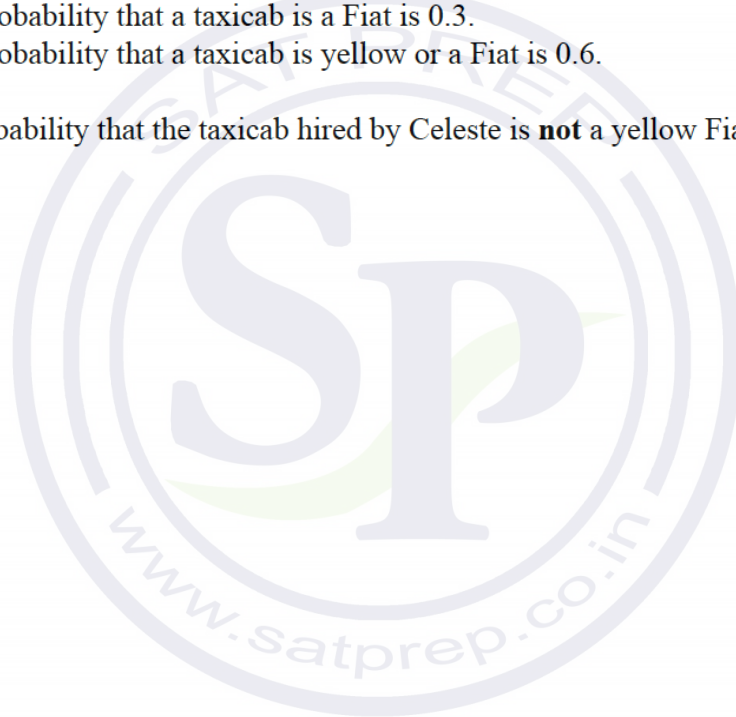
The probability that a taxicab is yellow is 0.4.

The probability that a taxicab is a Fiat is 0.3.

The probability that a taxicab is yellow or a Fiat is 0.6.

Find the probability that the taxicab hired by Celeste is **not** a yellow Fiat.

Question 14



[Maximum mark: 14]

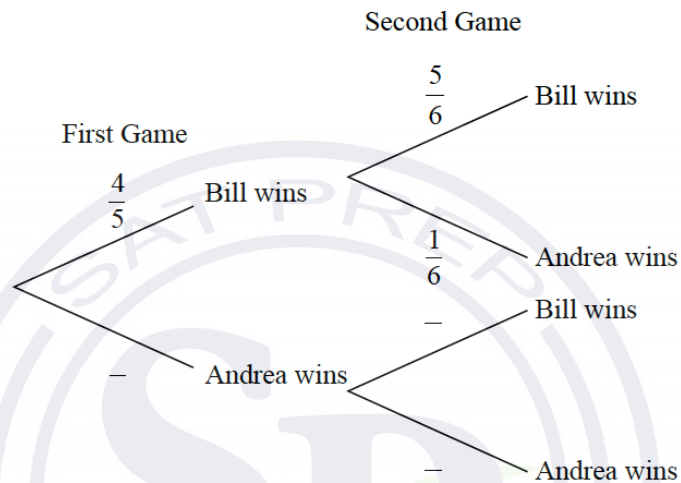
Bill and Andrea play two games of tennis. The probability that Bill wins the first game is  $\frac{4}{5}$ .

If Bill wins the first game, the probability that he wins the second game is  $\frac{5}{6}$ .

If Bill loses the first game, the probability that he wins the second game is  $\frac{2}{3}$ .

(a) **Copy** and complete the following tree diagram. (*Do not write on this page.*)

[3]



(b) Find the probability that Bill wins the first game and Andrea wins the second game. [2]

(c) Find the probability that Bill wins at least one game. [4]

(d) Given that Bill wins at least one game, find the probability that he wins both games. [5]

Question 15

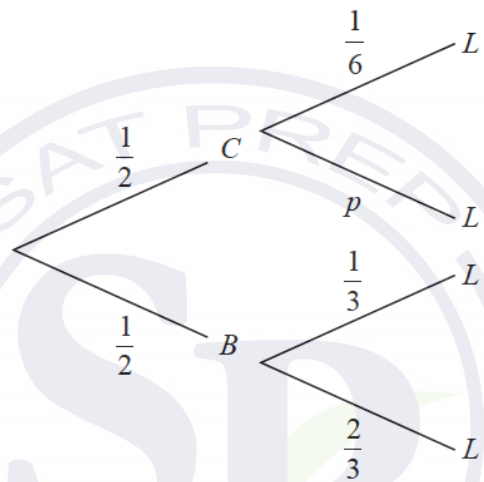
[Maximum mark: 15]

Adam travels to school by car ( $C$ ) or by bicycle ( $B$ ). On any particular day he is equally likely to travel by car or by bicycle.

The probability of being late ( $L$ ) for school is  $\frac{1}{6}$  if he travels by car.

The probability of being late for school is  $\frac{1}{3}$  if he travels by bicycle.

This information is represented by the following tree diagram.



- (a) Find the value of  $p$ . [2]
- (b) Find the probability that Adam will travel by car and be late for school. [2]
- (c) Find the probability that Adam will be late for school. [4]
- (d) Given that Adam is late for school, find the probability that he travelled by car. [3]

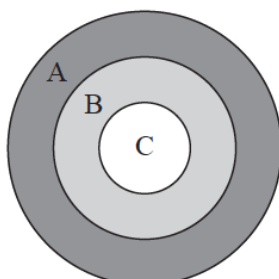
Adam will go to school three times next week.

- (e) Find the probability that Adam will be late exactly once. [4]

### Question 16

[Maximum mark: 7]

The following diagram shows a board which is divided into three regions A, B and C.



A game consists of a contestant throwing one dart at the board. The probability of hitting each region is given in the following table.

Region	A	B	C
Probability	$\frac{5}{20}$	$\frac{4}{20}$	$\frac{1}{20}$

(a) Find the probability that the dart does **not** hit the board.

[3]

The contestant scores points as shown in the following table.

Region	A	B	C	Does not hit the board
Points	0	$q$	10	-3

(b) Given that the game is fair, find the value of  $q$ .

[4]

### Question 17

[Maximum mark: 6]

A discrete random variable  $X$  has the following probability distribution.

$x$	0	1	2	3
$P(X=x)$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{2}{10}$	$p$

(a) Find  $p$ .

[3]

(b) Find  $E(X)$ .

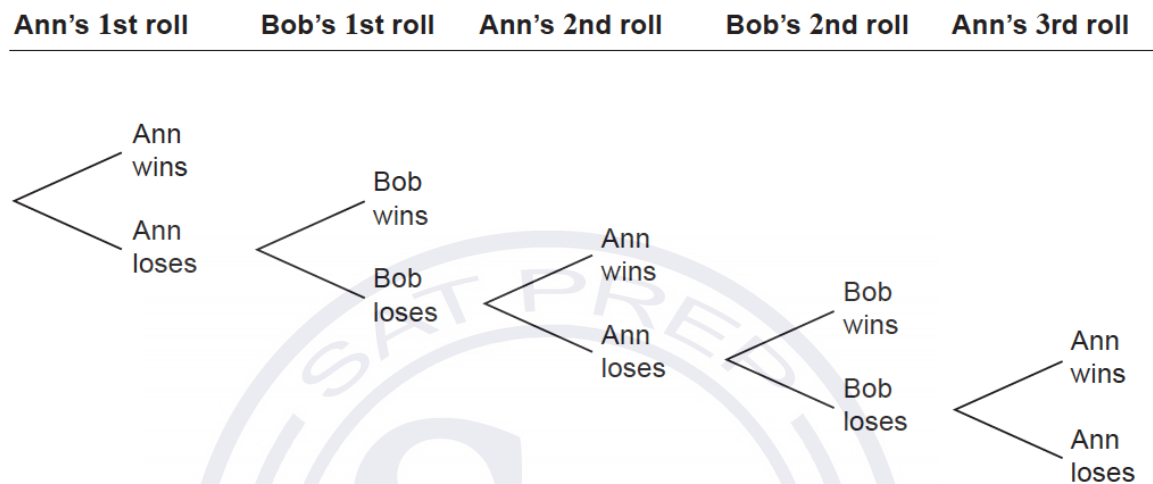
[3]



### Question 18

[Maximum mark: 15]

Ann and Bob play a game where they each have an eight-sided die. Ann's die has three green faces and five red faces; Bob's die has four green faces and four red faces. They take turns rolling their own die and note what colour faces up. The first player to roll green wins. Ann rolls first. Part of a tree diagram of the game is shown below.



(a) Find the probability that Ann wins on her first roll. [2]

(b) (i) The probability that Ann wins on her third roll is  $\frac{5}{8} \times \frac{4}{8} \times p \times q \times \frac{3}{8}$ .

Write down the value of  $p$  and of  $q$ .

(ii) The probability that Ann wins on her tenth roll is  $\frac{3}{8}r^k$  where  $r \in \mathbb{Q}$ ,  $k \in \mathbb{Z}$ .

Find the value of  $r$  and of  $k$ . [6]

(c) Find the probability that Ann wins the game. [7]

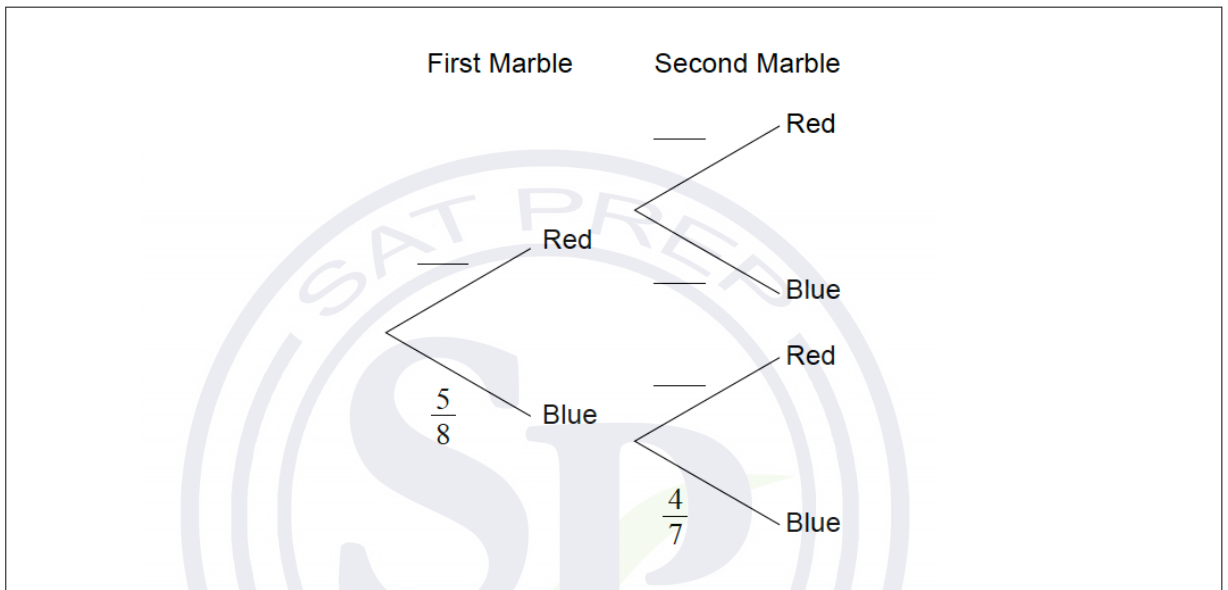
Question 19

[Maximum mark: 6]

A bag contains eight marbles. Three marbles are red and five are blue. Two marbles are drawn from the bag without replacement.

(a) Write down the probability that the first marble drawn is red. [1]

(b) Complete the following tree diagram. [3]

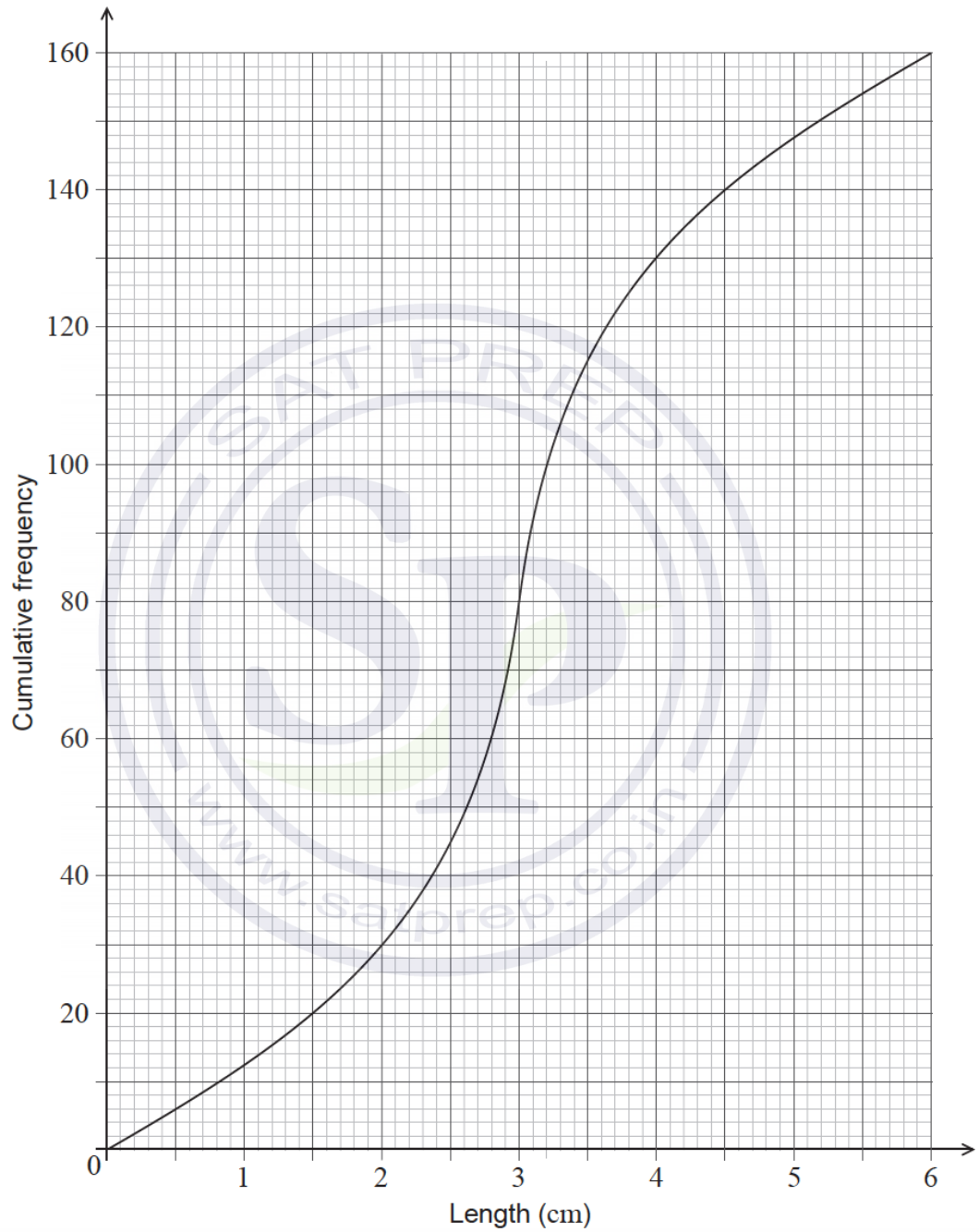


(c) Find the probability that both marbles are blue. [2]

Question 20

[Maximum mark: 6]

The following cumulative frequency diagram shows the lengths of 160 fish, in cm.



(a) Find the median length.

[2]

The following frequency table also gives the lengths of the 160 fish.

<b>Length <math>x</math> cm</b>	$0 \leq x \leq 2$	$2 < x \leq 3$	$3 < x \leq 4.5$	$4.5 < x \leq 6$
<b>Frequency</b>	$p$	50	$q$	20

(b) (i) Write down the value of  $p$ .

(ii) Find the value of  $q$ .

[4]

### Question 21

[Maximum mark: 7]

A bag contains black and white chips. Rose pays \$10 to play a game where she draws a chip from the bag. The following table gives the probability of choosing each colour chip.

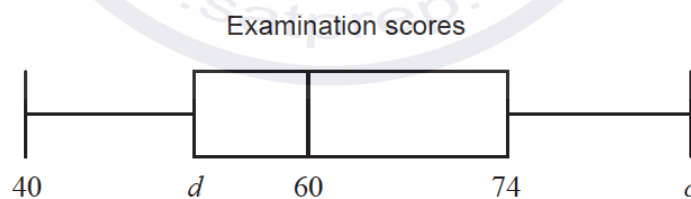
<b>Outcome</b>	black	white
<b>Probability</b>	0.4	0.6

Rose gets no money if she draws a white chip, and gets  $\$k$  if she draws a black chip. The game is fair. Find the value of  $k$ .

### Question 22

[Maximum mark: 5]

The following box-and-whisker plot represents the examination scores of a group of students.



(a) Write down the median score.

[1]

The range of the scores is 47 marks, and the interquartile range is 22 marks.

(b) Find the value of

(i)  $c$ ;

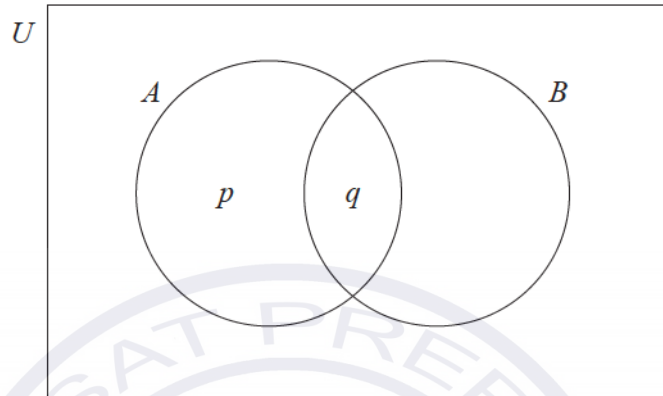
(ii)  $d$ .

[4]

Question 23

[Maximum mark: 6]

The following Venn diagram shows the events  $A$  and  $B$ , where  $P(A) = 0.4$ ,  $P(A \cup B) = 0.8$  and  $P(A \cap B) = 0.1$ . The values  $p$  and  $q$  are probabilities.



(a) (i) Write down the value of  $q$ .

(ii) Find the value of  $p$ .

[3]

(b) Find  $P(B)$ .

[3]

Question 24

[Maximum mark: 15]

A school collects cans for recycling to raise money. Sam's class has 20 students. The number of cans collected by each student in Sam's class is shown in the following stem and leaf diagram.

Stem	Leaf	<b>Key: 3 1 represents 31 cans</b>
2	0, 1, 4, 9, 9	
3	1, 7, 7, 7, 8, 8	
4	1, 2, 2, 3, 5, 6, 7, 8	
5	0	

- (a) Find the median number of cans collected. [2]

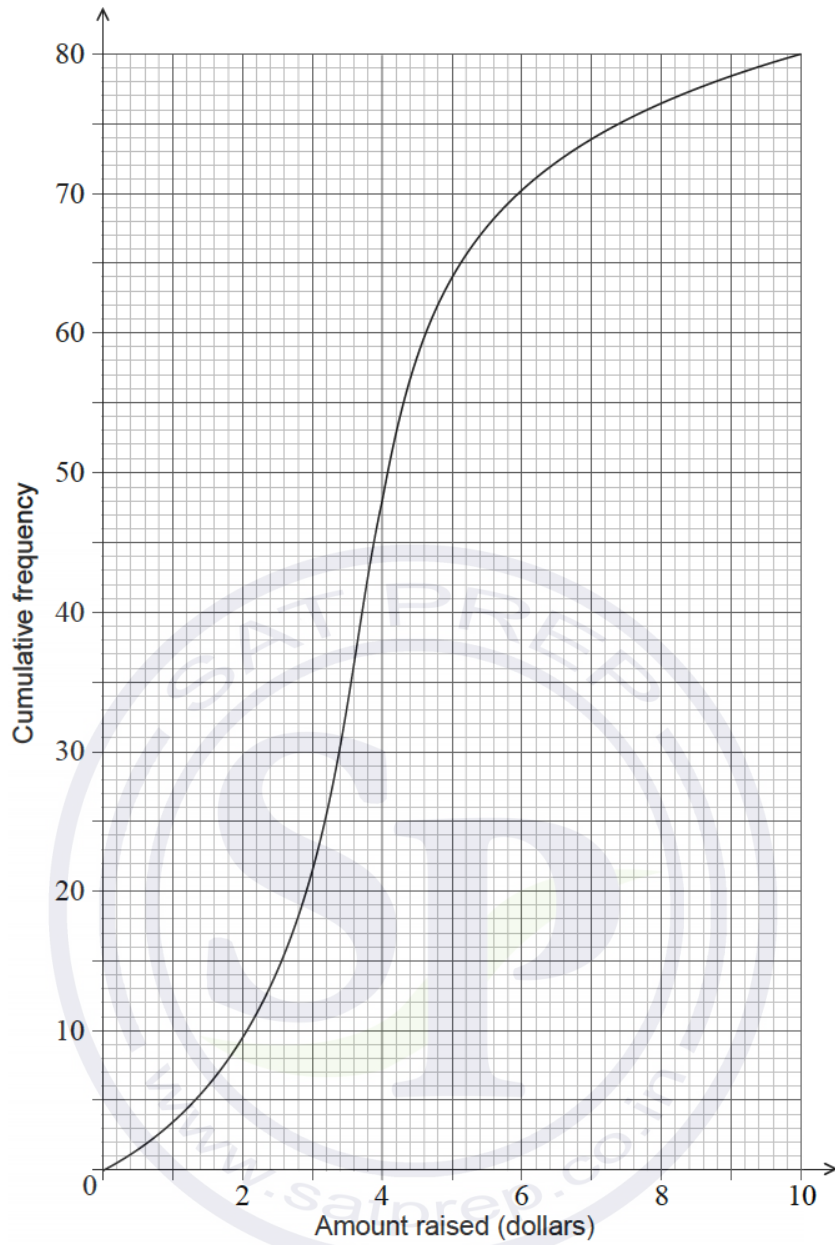
The following box-and-whisker plot also displays the number of cans collected by students in Sam's class.



- (b) (i) Write down the value of  $a$ .
- (ii) The interquartile range is 14. Find the value of  $b$ . [3]
- (c) Sam's class collected 745 cans. They want an average of 40 cans per student. How many more cans need to be collected to achieve this target? [3]

There are 80 students in the school.

- (d) The students raise \$0.10 for each recycled can.
- (i) Find the largest amount raised by a student in Sam's class.
- (ii) The following cumulative frequency curve shows the amounts in dollars raised by all the students in the school. Find the percentage of students in the school who raised more money than anyone in Sam's class. [5]



- (e) The mean number of cans collected is 39.4. The standard deviation is 18.5. Each student then collects 2 more cans.
- (i) Write down the new mean.
  - (ii) Write down the new standard deviation.

[2]

### Question 25

[Maximum mark: 5]

There are 10 items in a data set. The sum of the items is 60.

- (a) Find the mean. [2]

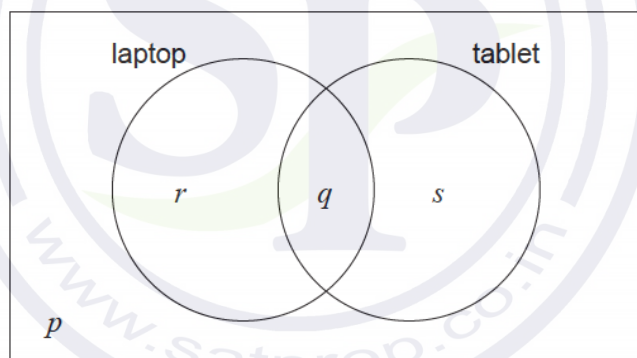
The variance of this data set is 3. Each value in the set is multiplied by 4.

- (b) (i) Write down the value of the new mean.  
(ii) Find the value of the new variance. [3]

### Question 26

[Maximum mark: 13]

In a class of 21 students, 12 own a laptop, 10 own a tablet, and 3 own neither. The following Venn diagram shows the events “own a laptop” and “own a tablet”. The values  $p$ ,  $q$ ,  $r$  and  $s$  represent numbers of students.



- (a) (i) Write down the value of  $p$ .  
(ii) Find the value of  $q$ .  
(iii) Write down the value of  $r$  and of  $s$ . [5]
- (b) A student is selected at random from the class.  
(i) Write down the probability that this student owns a laptop.  
(ii) Find the probability that this student owns a laptop or a tablet but not both. [4]





Question 28

[Maximum mark: 6]

Jim heated a liquid until it boiled. He measured the temperature of the liquid as it cooled. The following table shows its temperature,  $d$  degrees Celsius,  $t$  minutes after it boiled.

$t$ (min)	0	4	8	12	16	20
$d$ ( $^{\circ}\text{C}$ )	105	98.4	85.4	74.8	68.7	62.1

- (a) (i) Write down the independent variable.
- (ii) Write down the boiling temperature of the liquid. [2]

Jim believes that the relationship between  $d$  and  $t$  can be modelled by a linear regression equation.

- (b) Jim describes the correlation as **very strong**. Circle the value below which best represents the correlation coefficient. [2]

0.992	0.251	0	-0.251	-0.992
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- (c) Jim's model is  $d = -2.24t + 105$ , for  $0 \leq t \leq 20$ . Use his model to predict the decrease in temperature for any 2 minute interval. [2]

Question 29

[Maximum mark: 15]

The following table shows the probability distribution of a discrete random variable  $A$ , in terms of an angle  $\theta$ .

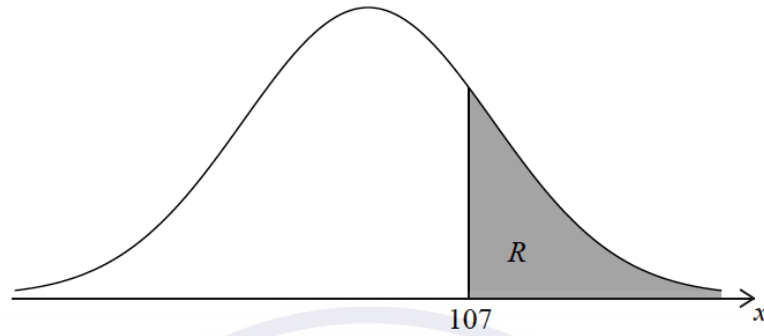
$a$	1	2
$P(A = a)$	$\cos \theta$	$2 \cos 2\theta$

- (a) Show that  $\cos \theta = \frac{3}{4}$ . [6]
- (b) Given that  $\tan \theta > 0$ , find  $\tan \theta$ . [3]
- (c) Let  $y = \frac{1}{\cos x}$ , for  $0 < x < \frac{\pi}{2}$ . The graph of  $y$  between  $x = \theta$  and  $x = \frac{\pi}{4}$  is rotated  $360^{\circ}$  about the  $x$ -axis. Find the volume of the solid formed. [6]

Question 30

[Maximum mark: 6]

The random variable  $X$  is normally distributed with a mean of 100. The following diagram shows the normal curve for  $X$ .



Let  $R$  be the shaded region under the curve, to the right of 107. The area of  $R$  is 0.24.

- (a) Write down  $P(X > 107)$ . [1]
- (b) Find  $P(100 < X < 107)$ . [3]
- (c) Find  $P(93 < X < 107)$ . [2]

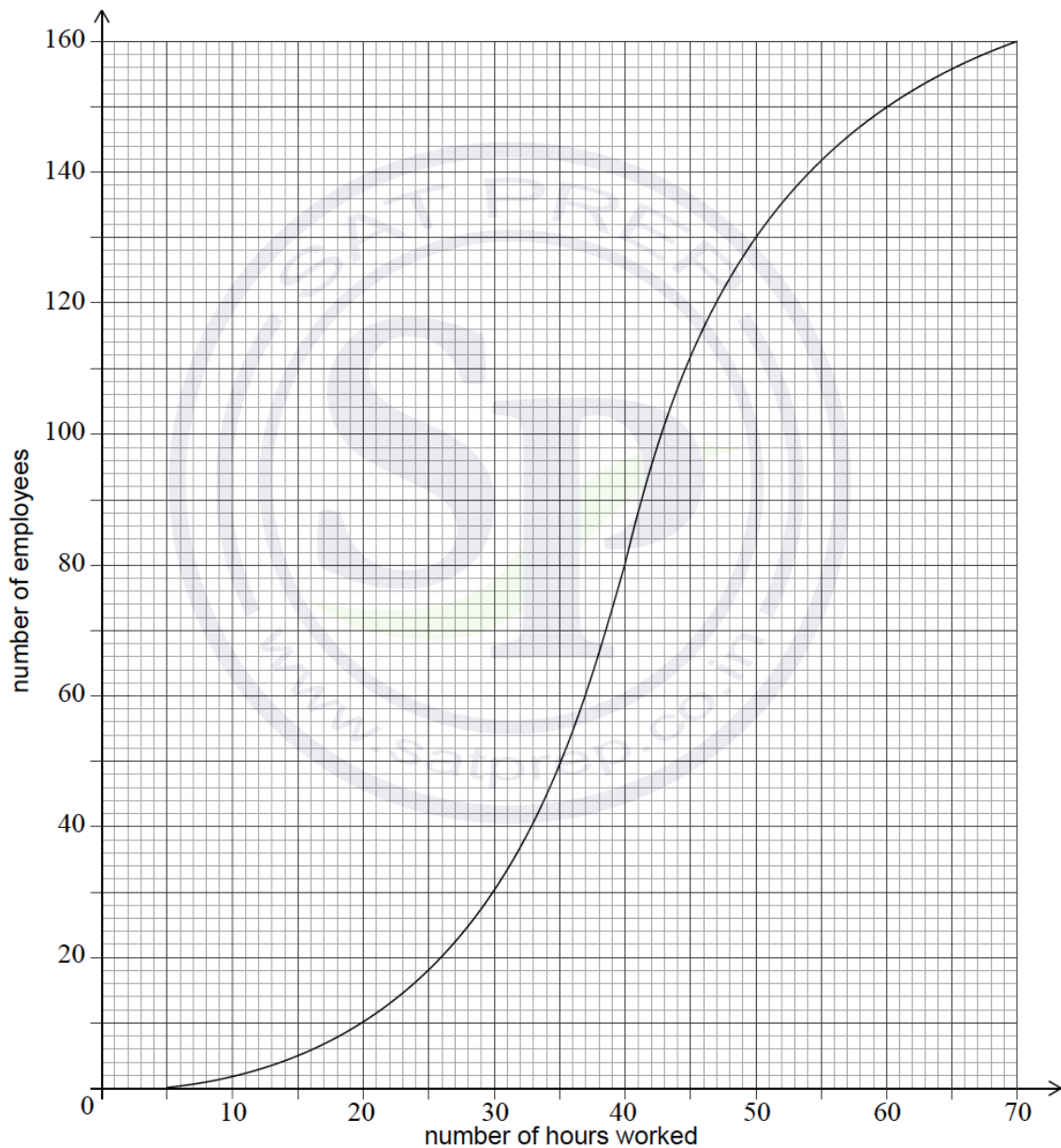
### Question 31

[Maximum mark: 14]

A city hired 160 employees to work at a festival. The following cumulative frequency curve shows the number of hours employees worked during the festival.

- (a) (i) Find the median number of hours worked by the employees.  
(ii) Write down the number of employees who worked 50 hours or less.

[3]



The city paid each of the employees £8 per hour for the first 40 hours worked, and £10 per hour for each hour they worked after the first 40 hours.

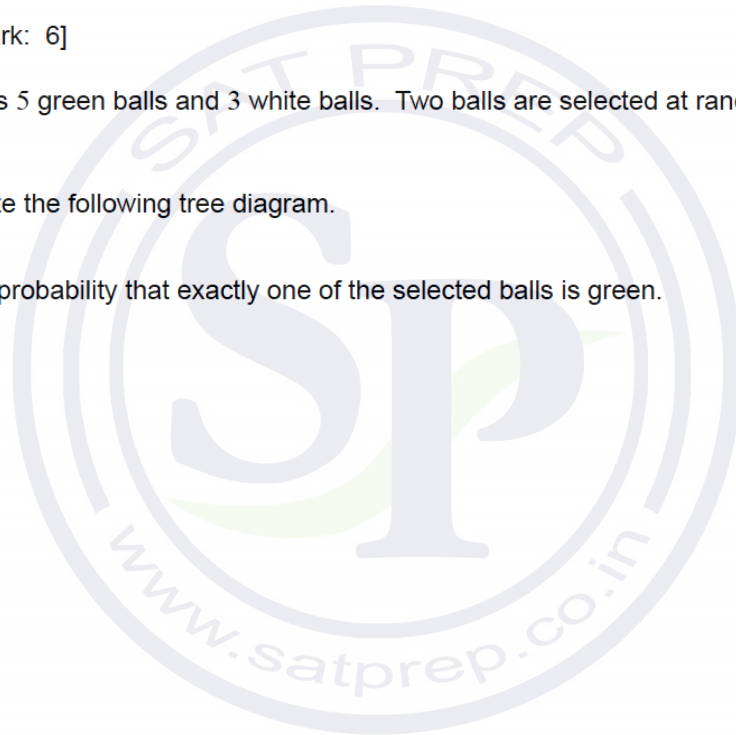
- (b) Find the amount of money an employee earned for working
- (i) 40 hours;
  - (ii) 43 hours. [4]
- (c) Find the number of employees who earned £200 or less. [3]
- (d) Only 10 employees earned more than £ $k$ . Find the value of  $k$ . [4]

### Question 32

[Maximum mark: 6]

A bag contains 5 green balls and 3 white balls. Two balls are selected at random without replacement.

- (a) Complete the following tree diagram. [3]
- (b) Find the probability that exactly one of the selected balls is green. [3]



Question 33

[Maximum mark: 14]

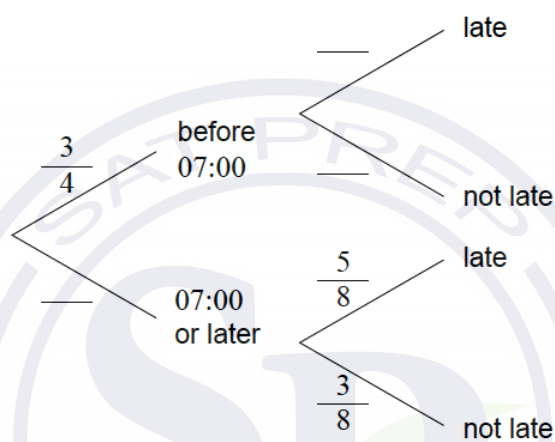
Pablo drives to work. The probability that he leaves home before 07:00 is  $\frac{3}{4}$ .

If he leaves home before 07:00 the probability he will be late for work is  $\frac{1}{8}$ .

If he leaves home at 07:00 or later the probability he will be late for work is  $\frac{5}{8}$ .

(a) Copy and complete the following tree diagram.

[3]



(b) Find the probability that Pablo leaves home before 07:00 and is late for work.

[2]

(c) Find the probability that Pablo is late for work.

[3]

(d) Given that Pablo is late for work, find the probability that he left home before 07:00.

[3]

(e) Two days next week Pablo will drive to work. Find the probability that he will be late at least once.

[3]

Question 34

[Maximum mark: 6]

A data set has  $n$  items. The sum of the items is 800 and the mean is 20.

(a) Find  $n$ .

[2]

The standard deviation of this data set is 3. Each value in the set is multiplied by 10.

(b) (i) Write down the value of the new mean.

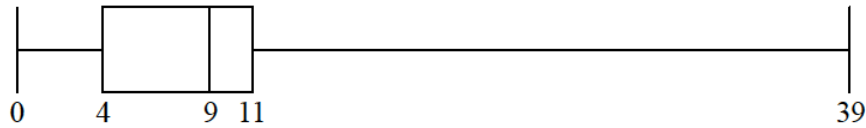
(ii) Find the value of the new variance.

[4]

Question 35

[Maximum mark: 6]

The following box-and-whisker plot shows the number of text messages sent by students in a school on a particular day.



- (a) Find the value of the interquartile range. [2]
- (b) One student sent  $k$  text messages, where  $k > 11$ . Given that  $k$  is an outlier, find the least value of  $k$ . [4]

Question 36

[Maximum mark: 15]

A bag contains  $n$  marbles, two of which are blue. Hayley plays a game in which she randomly draws marbles out of the bag, one after another, without replacement. The game ends when Hayley draws a blue marble.

- (a) Find the probability, in terms of  $n$ , that the game will end on her
- (i) first draw; [4]
- (ii) second draw.
- (b) Let  $n = 5$ . Find the probability that the game will end on her
- (i) third draw; [4]
- (ii) fourth draw.

Hayley plays the game when  $n = 5$ . She pays \$20 to play and can earn money back depending on the number of draws it takes to obtain a blue marble. She earns no money back if she obtains a blue marble on her first draw. Let  $M$  be the amount of money that she earns back playing the game. This information is shown in the following table.

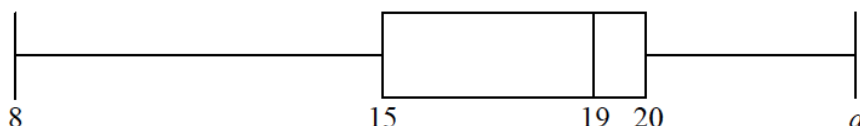
Number of draws	1	2	3	4
Money earned back (\$ $M$ )	0	20	$8k$	$12k$

- (c) Find the value of  $k$  so that this is a fair game. [7]

### Question 37

[Maximum mark: 16]

A group of 10 girls recorded the number of hours they spent watching television during a particular week. Their results are summarized in the box-and-whisker plot below.



(a) The range of the data is 16. Find the value of  $a$ . [2]

(b) Find the value of the interquartile range. [2]

The group of girls watched a total of 180 hours of television.

(c) Find the mean number of hours that the girls in this group spent watching television that week. [2]

A group of 20 boys also recorded the number of hours they spent watching television that same week. Their results are summarized in the table below.

$\bar{x} = 21$	$\sigma = 3$
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(d) (i) Find the total number of hours the group of boys spent watching television that week.  
(ii) Find the mean number of hours that **all 30** girls and boys spent watching television that week. [5]

The following week, the group of boys had exams. During this exam week, the boys spent half as much time watching television compared to the previous week.

(e) For this exam week, find  
(i) the mean number of hours that the group of boys spent watching television;  
(ii) the variance in the number of hours the group of boys spent watching television. [5]



Question 38

[Maximum mark: 6]

The following table shows the probability distribution of a discrete random variable  $X$ .

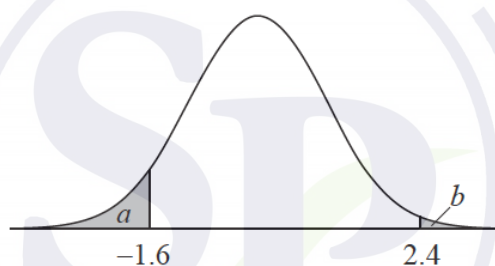
$X$	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>
$P(X=x)$	$\frac{3}{13}$	$\frac{1}{13}$	$\frac{4}{13}$	$k$

- (a) Find the value of  $k$ . [3]
- (b) Find  $E(X)$ . [3]

Question 39

[Maximum mark: 13]

A random variable  $Z$  is normally distributed with mean 0 and standard deviation 1. It is known that  $P(z < -1.6) = a$  and  $P(z > 2.4) = b$ . This is shown in the following diagram.



- (a) Find  $P(-1.6 < z < 2.4)$ . Write your answer in terms of  $a$  and  $b$ . [2]
- (b) Given that  $z > -1.6$ , find the probability that  $z < 2.4$ . Write your answer in terms of  $a$  and  $b$ . [4]

A second random variable  $X$  is normally distributed with mean  $m$  and standard deviation  $s$ .

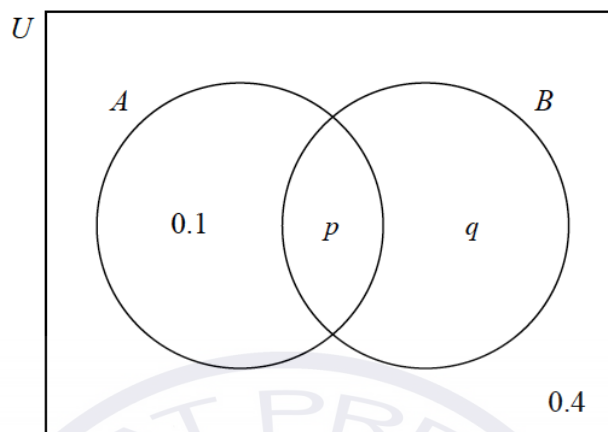
It is known that  $P(x < 1) = a$ .

- (c) Write down the standardized value for  $x = 1$ . [1]
- It is also known that  $P(x > 2) = b$ .
- (d) Find  $s$ . [6]

Question 40

[Maximum mark: 6]

The following Venn diagram shows the events  $A$  and  $B$ , where  $P(A) = 0.3$ . The values shown are probabilities.



- (a) Find the value of  $p$ . [2]
- (b) Find the value of  $q$ . [2]
- (c) Find  $P(A' \cup B)$ . [2]

Question 41

[Maximum mark: 6]

Let  $X$  and  $Y$  be normally distributed with  $X \sim N(14, a^2)$  and  $Y \sim N(22, a^2)$ ,  $a > 0$ .

- (a) Find  $b$  so that  $P(X > b) = P(Y < b)$ . [2]

It is given that  $P(X > 20) = 0.112$ .

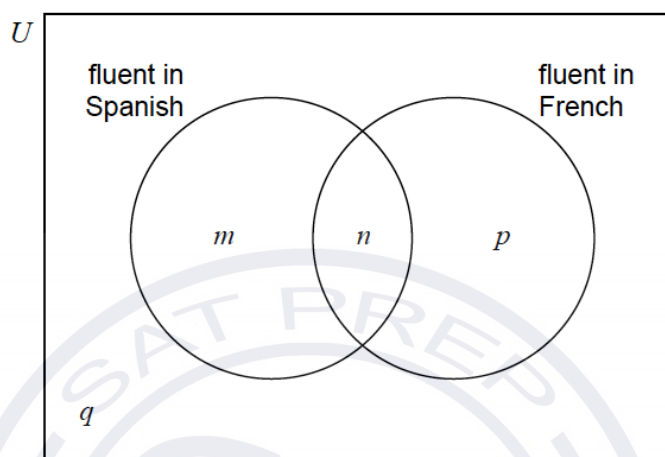
- (b) Find  $P(16 < Y < 28)$ . [4]

Question 42

[Maximum mark: 6]

In a class of 30 students, 18 are fluent in Spanish, 10 are fluent in French, and 5 are not fluent in either of these languages. The following Venn diagram shows the events “fluent in Spanish” and “fluent in French”.

The values  $m$ ,  $n$ ,  $p$  and  $q$  represent numbers of students.



(a) Write down the value of  $q$ .

[1]

(b) Find the value of  $n$ .

[2]

(c) Write down the value of  $m$  and of  $p$ .

[3]