Subject – Math (Standard Level)
Topic - Vector
Year - Nov 2011 – Nov 2017
Paper- 2

Question 1

(a) (i) valid approach *(M1)*

e.g. $\text{OA} + \text{AB}$

$\overrightarrow{\text{OB}} = 4i + 3j$ *A1* *N2*

(ii) valid approach *(M1)*

e.g. $\overrightarrow{\text{OA}} + \overrightarrow{\text{AB}} + \overrightarrow{\text{BF}};\ \overrightarrow{\text{OB}} + \overrightarrow{\text{BF}};\ \overrightarrow{\text{OC}} + \overrightarrow{\text{CG}} + \overrightarrow{\text{GF}}$

$\overrightarrow{\text{OF}} = 4i + 3j + 2k$ *A1* *N2*

(iii) correct approach *A1*

e.g. $\overrightarrow{\text{AO}} + \overrightarrow{\text{OC}} + \overrightarrow{\text{CG}};\ \overrightarrow{\text{AB}} + \overrightarrow{\text{BF}} + \overrightarrow{\text{FG}};\ \overrightarrow{\text{AB}} + \overrightarrow{\text{BC}} + \overrightarrow{\text{CG}}$

$\overrightarrow{\text{AG}} = -4i + 3j + 2k$ *AG* *N0*

[5 marks]

(b) (i) **any** correct equation for (OF) in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ *A2* *N2*

where $\boldsymbol{a}$ is 0 or $4i + 3j + 2k$, and $\boldsymbol{b}$ is a scalar multiple of $4i + 3j + 2k$

e.g. $\boldsymbol{r} = t(4, 3, 2)$, $\boldsymbol{r} = \begin{pmatrix} 4t \\ 3t \\ 2t \end{pmatrix}$, $\boldsymbol{r} = 4i + 3j + 2k + t(4i + 3j + 2k)$

(ii) **any** correct equation for (AG) in the form $\boldsymbol{r} = \boldsymbol{a} + s\boldsymbol{b}$ *A2* *N2*

where $\boldsymbol{a}$ is $4i$ or $3j + 2k$ and $\boldsymbol{b}$ is a scalar multiple of $-4i + 3j + 2k$

e.g. $\boldsymbol{r} = (4, 0, 0) + s(-4, 3, 2)$, $\boldsymbol{r} = \begin{pmatrix} 4 - 4s \\ 3s \\ 2s \end{pmatrix}$, $\boldsymbol{r} = 3j + 2k + s(-4i + 3j + 2k)$

[4 marks]

(c) choosing correct direction vectors, $\vec{OF}$ and $\vec{AG}$ *(A1)(A1)*

scalar product $= -16 + 9 + 4 \ (= -3)$ *(A1)*

magnitudes $\sqrt{4^2 + 3^2 + 2^2}$, $\sqrt{(-4)^2 + 3^2 + 2^2}$ $\left(\sqrt{29}, \sqrt{29}\right)$ *(A1)(A1)*

substitution into formula ***M1***

e.g. $\cos\theta = \dfrac{-16 + 9 + 4}{\left(\sqrt{4^2 + 3^2 + 2^2}\right) \times \sqrt{(-4)^2 + 3^2 + 2^2}} = \left(-\dfrac{3}{29}\right)$

95.93777°, 1.67443 radians

$\theta = 95.9°$ or 1.67 ***A1*** ***N4***

[7 marks]

Total [16 marks]

Question 2

appropriate approach *(M1)*

eg $\begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$, $L_1 = L_2$

any two correct equations ***A1A1***

eg $10 + 2s = 2 + 3t$, $6 - 5s = 1 + 5t$, $-1 - 2s = -3 + 2t$

attempt to solve *(M1)*

eg substituting one equation into another

one correct parameter ***A1***

eg $s = -1$, $t = 2$

correct substitution *(A1)*

eg $2 + 3(2)$, $1 + 5(2)$, $-3 + 2(2)$

$A = (8, 11, 1)$ (accept column vector) ***A1*** ***N4***

[7 marks]

Question 3

(a)	(i)	appropriate approach	*(M1)*	
		eg $\vec{AO}+\vec{OB}$, $B-A$		
		$\vec{AB}=\begin{pmatrix}1\\3\\2\end{pmatrix}$	*A1*	*N2*
	(ii)	$\vec{AC}=\begin{pmatrix}2\\4\\a\end{pmatrix}$	*A1*	*N1*
				[3 marks]
(b)		valid reasoning (seen anywhere)	*R1*	
		eg scalar product is zero, $\cos\frac{\pi}{2}=\frac{\boldsymbol{u}\cdot\boldsymbol{v}}{\lvert\boldsymbol{u}\rvert\lvert\boldsymbol{v}\rvert}$		
		correct scalar product of **their** $\vec{AB}$ and $\vec{AC}$ (may be seen in part (c))	*(A1)*	
		eg $1(2)+3(4)+2(a)$		
		correct working for **their** $\vec{AB}$ and $\vec{AC}$	*(A1)*	
		eg $2a+14$, $2a=-14$		
		$a=-7$	*A1*	*N3*
				[4 marks]
(c)	(i)	correct magnitudes (may be seen in (b))	*(A1)(A1)*	
		$\sqrt{1^2+3^2+2^2}\left(=\sqrt{14}\right)$, $\sqrt{2^2+4^2+a^2}\left(=\sqrt{20+a^2}\right)$		
		substitution into formula	*(M1)*	
		eg $\cos\theta=\frac{1\times2+3\times4+2\times a}{\sqrt{1^2+3^2+2^2}\sqrt{2^2+4^2+a^2}}$, $\frac{14+2a}{\sqrt{14}\sqrt{4+16+a^2}}$		
		simplification leading to required answer	*A1*	
		eg $\cos\theta=\frac{14+2a}{\sqrt{14}\sqrt{20+a^2}}$		
		$\cos\theta=\frac{2a+14}{\sqrt{14a^2+280}}$	*AG*	*N0*
	(ii)	correct setup	*(A1)*	
		eg $\cos1.2=\frac{2a+14}{\sqrt{14a^2+280}}$		
		valid attempt to solve	*(M1)*	
		eg sketch, $\frac{2a+14}{\sqrt{14a^2+280}}-\cos1.2=0$, attempt to square		
		$a=-3.25$	*A2*	*N3*
				[8 marks]
				Total [15 marks]

Question 4

(a) appropriate approach *(M1)*

eg $\begin{pmatrix}11\\8\\2\end{pmatrix}+s\begin{pmatrix}4\\3\\-1\end{pmatrix}=\begin{pmatrix}1\\1\\-7\end{pmatrix}+t\begin{pmatrix}2\\1\\11\end{pmatrix}$, $L_1 = L_2$

any **two** correct equations ***A1A1***

eg $11+4s=1+2t$, $8+3s=1+t$, $2-s=-7+11t$

attempt to solve system of equations *(M1)*

eg $10+4s=2(7+3s)$, $\begin{cases}4s-2t=-10\\3s-t=-7\end{cases}$

one correct parameter ***A1***

eg $s=-2$, $t=1$

$\mathrm{P}(3,\ 2,\ 4)$ (accept position vector) ***A1*** ***N3***

[6 marks]

(b) choosing correct direction vectors for L_1 and L_2 *(A1)(A1)*

eg $\begin{pmatrix}4\\3\\-1\end{pmatrix}$, $\begin{pmatrix}2\\1\\11\end{pmatrix}$ (or any scalar multiple)

evidence of scalar product (with any vectors) *(M1)*

eg $a \cdot b$, $\begin{pmatrix}4\\3\\-1\end{pmatrix} \cdot \begin{pmatrix}2\\1\\11\end{pmatrix}$

correct substitution ***A1***

eg $4(2)+3(1)+(-1)(11)$, $8+3-11$

calculating $a \cdot b = 0$ ***A1***

Note: Do not award the final ***A1*** without evidence of calculation.

vectors are perpendicular ***AG*** ***N0***

[5 marks]

(c) **Note:** Candidates may take different approaches, which do not necessarily involve vectors. In particular, most of the working could be done on a diagram. Award marks in line with the markscheme.

METHOD 1

attempt to find $\vec{QP}$ or $\vec{PQ}$ ***(M1)***

correct working (may be seen on diagram) ***A1***

eg $\vec{QP} = \begin{pmatrix} -4 \\ -3 \\ 1 \end{pmatrix}$, $\vec{PQ} = \begin{pmatrix} 7 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$

recognizing R is on L_1 (seen anywhere) ***(R1)***

eg on diagram

Q and R are equidistant from P (seen anywhere) ***(R1)***

eg $\vec{QP} = \vec{PR}$, marked on diagram

correct working ***(A1)***

eg $\begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$, $\begin{pmatrix} -4 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$

R $(-1, -1, 5)$ (accept position vector) ***A1*** ***N3***

METHOD 2

recognizing R is on L_1 (seen anywhere) ***(R1)***

eg on diagram

Q and R are equidistant from P (seen anywhere) ***(R1)***

eg P midpoint of QR, marked on diagram

valid approach to find **one** coordinate of mid-point ***(M1)***

eg $x_P = \frac{x_Q + x_R}{2}$, $2y_P = y_Q + y_R$, $\frac{1}{2}(z_Q + z_R)$

one correct substitution ***A1***

eg $x_R = 3 + (3 - 7)$, $2 = \frac{5 + y_R}{2}$, $4 = \frac{1}{2}(z + 3)$

correct working for one coordinate ***(A1)***

eg $x_R = 3 - 4$, $4 - 5 = y_R$, $8 = (z + 3)$

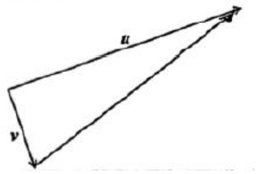
R $(-1, -1, 5)$ (accept position vector) ***A1*** ***N3***

[6 marks]

Total [17 marks]

Question 5

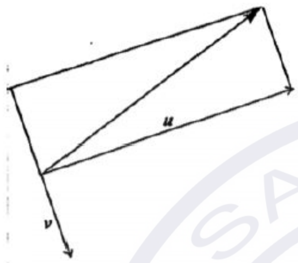
(a) **METHOD 1**



A1A1 *N2*

Note: Award *A1* for segment connecting endpoints and *A1* for direction (must see arrow).

METHOD 2



A1A1 *N2*

Notes: Award *A1* for segment connecting endpoints and *A1* for direction (must see arrow).
Additional lines not required.

[2 marks]

(b) evidence of setting scalar product equal to zero (seen anywhere) *R1*

eg $\boldsymbol{u} \bullet \boldsymbol{v} = 0$, $15 + 2n + 3 = 0$

correct expression for scalar product *(A1)*

eg $3 \times 5 + 2 \times n + 1 \times 3$, $2n + 18 = 0$

attempt to solve equation *(M1)*

eg $2n = -18$

$n = -9$ *A1* *N3*

[4 marks]

Total [6 marks]

Question 6

(a) (i) correct substitution *(A1)*

eg $6\times 2+3\times 2+6\times 1$

$\boldsymbol{u}\bullet\boldsymbol{v}=24$ ***A1*** ***N2***

(ii) correct substitution into magnitude formula for $\boldsymbol{u}$ or $\boldsymbol{v}$ *(A1)*

eg $\sqrt{6^2+3^2+6^2}$, $\sqrt{2^2+2^2+1^2}$, correct value for $|v|$

$|\boldsymbol{u}|=9$ ***A1*** ***N2***

(iii) $|\boldsymbol{v}|=3$ ***A1*** ***N1***

[5 marks]

(b) correct substitution into angle formula *(A1)*

eg $\frac{24}{9\times 3}$, $0.\bar{8}$

0.475882, $27.26604°$ ***A1*** ***N2***

0.476, 27.3°

[2 marks]

Total [7 marks]

Question 7

(a) valid approach (addition or subtraction) *(M1)*

eg $\text{AO} + \text{OB}$, $\text{B} - \text{A}$

$\vec{\text{AB}} = \begin{pmatrix} 9 \\ 6 \\ -3 \end{pmatrix}$ **A1** **N2**

[2 marks]

(b) **METHOD 1**

valid approach using $\vec{\text{OC}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ *(M1)*

eg $\vec{\text{AC}} = \begin{pmatrix} x+3 \\ y+2 \\ z-2 \end{pmatrix}$, $\vec{\text{CB}} = \begin{pmatrix} 6-x \\ 4-y \\ -1-z \end{pmatrix}$

correct working **A1**

eg $\begin{pmatrix} x+3 \\ y+2 \\ z-2 \end{pmatrix} = \begin{pmatrix} 12-2x \\ 8-2y \\ -2-2z \end{pmatrix}$

all three equations **A1**

eg $x+3 = 12-2x$, $y+2 = 8-2y$, $z-2 = -2-2z$,

$\vec{\text{OC}} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ **AG** **N0**

METHOD 2

valid approach **(M1)**

eg $\vec{OC} - \vec{OA} = 2\left(\vec{OB} - \vec{OC}\right)$

correct working **A1**

eg $3\vec{OC} = 2\vec{OB} + \vec{OA}$

correct substitution of $\vec{OB}$ and $\vec{OA}$ **A1**

eg $3\vec{OC} = 2\begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}$, $3\vec{OC} = \begin{pmatrix} 9 \\ 6 \\ 0 \end{pmatrix}$

$\vec{OC} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ **AG** **N0**

METHOD 3

valid approach **(M1)**

eg $\vec{AC} = \frac{2}{3}\vec{AB}$, diagram, $\vec{CB} = \frac{1}{3}\vec{AB}$

correct working **A1**

eg $\vec{AC} = \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix}$, $\vec{CB} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$

correct working involving $\vec{OC}$ **A1**

eg $\vec{OC} = \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$

$\vec{OC} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ **AG** **N0**

[3 marks]

(c) finding scalar product and magnitudes *(A1)(A1)(A1)*

scalar product $= (9\times3)+(6\times2)+(-3\times0)$ $(=39)$

magnitudes $\sqrt{81+36+9}$ $(=11.22)$, $\sqrt{9+4}$ $(=3.605)$

substitution into formula ***M1***

eg $\cos\theta = \dfrac{(9\times3)+12}{\sqrt{126}\times\sqrt{13}}$

$\theta = 0.270549$ (accept 15.50135°)

$\theta = 0.271$ (accept 15.5°) ***A1*** ***N4***

[5 marks]

(d) (i) attempt to use a trig ratio ***M1***

eg $\sin\theta = \dfrac{\text{DE}}{\text{CD}}$, $\left|\vec{\text{CE}}\right| = \left|\vec{\text{CD}}\right|\cos\theta$

attempt to express $\vec{\text{CD}}$ in terms of $\vec{\text{OC}}$ ***M1***

eg $\vec{\text{OC}}+\vec{\text{CD}}=\vec{\text{OD}}$, $\text{OC}+\text{CD}=\text{OD}$

correct working ***A1***

eg $\left|k\vec{\text{OC}}-\vec{\text{OC}}\right|\sin\theta$

$\left|\vec{\text{DE}}\right| = (k-1)\left|\vec{\text{OC}}\right|\sin\theta$ ***AG*** ***N0***

(ii) valid approach involving the segment DE ***(M1)***

eg recognizing $\left|\vec{\text{DE}}\right| < 3$, $\text{DE}=3$

correct working (accept equation) ***(A1)***

eg $(k-1)\left(\sqrt{13}\right)\sin 0.271 < 3$, $k-1=3.11324$

$1 < k < 4.11$ (accept $k < 4.11$ but not $k = 4.11$) ***A1*** ***N2***

[6 marks]

Total [16 marks]

Question 8

(a) valid approach **(M1)**

eg $\quad B-A,\ AO+OB,\ \begin{pmatrix}-9\\9\\-6\end{pmatrix}-\begin{pmatrix}1\\5\\-7\end{pmatrix}$

$\vec{AB}=\begin{pmatrix}-10\\4\\1\end{pmatrix}$ **A1** **N2**

[2 marks]

(b) valid approach **(M1)**

eg $\quad OC=OA+AC,\ \begin{pmatrix}1+6\\5-4\\-7+0\end{pmatrix}$

$C(7,1,-7)$ **A1** **N2**

[2 marks]

(c) any correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix}-9\\9\\-6\end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}6\\-4\\0\end{pmatrix}$ **A2** **N2**

eg $\quad \boldsymbol{r}=\begin{pmatrix}-9\\9\\-6\end{pmatrix}+t\begin{pmatrix}6\\-4\\0\end{pmatrix},\ \boldsymbol{r}=-9\boldsymbol{i}+9\boldsymbol{j}-6\boldsymbol{k}+s(6\boldsymbol{i}-4\boldsymbol{j}+0\boldsymbol{k})$

[2 marks]

(d) correct magnitudes **(A1)(A1)**

eg $\quad \sqrt{(-10)^2+(-4)^2+1^2},\ \sqrt{6^2+(-4)^2+(0)^2},\ \sqrt{10^2+4^2+1},\ \sqrt{6^2+4^2}$

$k=\frac{\sqrt{117}}{\sqrt{52}}\ (=1.5)$ (exact) **A1** **N3**

[3 marks]

(e) correct interpretation of relationship between magnitudes *(A1)*

eg $\text{AB} = 1.5\text{AC}$, $\text{BD} = 1.5\text{AC}$, $\sqrt{117} = \sqrt{52t^2}$

recognizing D can have two positions (may be seen in working) **R1**

eg $\vec{\text{BD}} = 1.5\vec{\text{AC}}$ and $\vec{\text{BD}} = -1.5\vec{\text{AC}}$, $t = \pm 1.5$, diagram, two answers

valid approach (seen anywhere) *(M1)*

eg $\vec{\text{OD}} = \vec{\text{OB}} + \vec{\text{BD}}$, $\begin{pmatrix} -9 \\ 9 \\ -6 \end{pmatrix} + t\begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$, $\vec{\text{BD}} = k\begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$

one correct expression for $\vec{\text{OD}}$ *(A1)*

eg $\vec{\text{OD}} = \begin{pmatrix} -9 \\ 9 \\ -6 \end{pmatrix} + 1.5\begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$, $\begin{pmatrix} -9 \\ 9 \\ -6 \end{pmatrix} - 1.5\begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$

$\text{D} = (0, 3, -6)$, $\text{D} = (-18, 15, -6)$ (accept position vectors) **A1A1** **N3**

[6 marks]

Total [15 marks]

Question 9

finding scalar product and magnitudes *(A1)(A1)(A1)*

scalar product $= (-10\times 3) + (2\times -4) + (1\times 0)$ $(= -38)$

magnitudes $= \sqrt{10^2 + 2^2 + 1^2}$, $\sqrt{3^2 + (-4)^2 + (0)^2}$ $\left(\sqrt{105}, \sqrt{25}\right)$

substituting **their** values into formula **M1**

eg $\cos\theta = \dfrac{-30 - 8 + 0}{\left(\sqrt{10^2 + 2^2 + 1^2}\right) \times \left(\sqrt{3^2 + (-4)^2 + (0)^2}\right)}$

2.40637; 137.875°

$\theta = 2.4;\ 137.9°$ **A2** **N4**

[6 marks]

Question 10

(a) correct substitution *(A1)*

eg $\sqrt{4^2+1^2+2^2}$

4.58257

$\left|\vec{AB}\right| = \sqrt{21}$ (exact), 4.58 **A1** **N2**

[2 marks]

(b) finding scalar product and $\left|\vec{AC}\right|$ *(A1)(A1)*

scalar product = $(4\times3)+(1\times0)+(2\times0)\ (=12)$

$\left|\vec{AC}\right| = \sqrt{3^2+0+0}\ (=3)$

substituting **their** values into cosine formula *(M1)*

eg $\cos \hat{BAC} = \dfrac{4\times3+0+0}{\sqrt{3^2}\times\sqrt{21}}, \dfrac{4}{\sqrt{21}}, \cos\theta = 0.873$

0.509739 (29.2059°)

$\hat{BAC} = 0.510$ (29.2°) **A1** **N2**

[4 marks]

Total [6 marks]

Question 11

(a) (i) valid approach *(M1)*

eg $(7, 4, 9)-(3, 2, 5)$, $A-B$

$\overrightarrow{PQ} = 4\boldsymbol{i}+2\boldsymbol{j}+4\boldsymbol{k}\left(=\begin{pmatrix}4\\2\\4\end{pmatrix}\right)$ ***A1*** ***N2***

(ii) correct substitution into magnitude formula *(A1)*

eg $\sqrt{4^2+2^2+4^2}$

$\left|\overrightarrow{PQ}\right| = 6$ ***A1*** ***N2***

[4 marks]

(b) finding scalar product and magnitudes *(A1)(A1)*

scalar product $=(4\times 6)+(2\times(-1))+(4\times 3)\ (=34)$

magnitude of $PR=\sqrt{36+1+9}\ (=6.782)$

correct substitution of **their** values to find cos $Q\hat{P}R$ ***M1***

eg $\cos Q\hat{P}R = \dfrac{24-2+12}{(6)\times\left(\sqrt{46}\right)}$, 0.8355

0.581746

$Q\hat{P}R = 0.582$ radians or $Q\hat{P}R = 33.3^\circ$ ***A1*** ***N3***

[4 marks]

(c) correct substitution *(A1)*

eg $\frac{1}{2}\times\left|\overrightarrow{PQ}\right|\times\left|\overrightarrow{PR}\right|\times\sin P$, $\frac{1}{2}\times 6\times\sqrt{46}\times\sin 0.582$

11.1803

area is 11.2 (sq. units) ***A1*** ***N2***

[2 marks]

(d) recognizing shortest distance is perpendicular distance from R to line through P and Q *(M1)*

eg sketch, height of triangle with base $[PQ]$, $\frac{1}{2}\times 6\times h$, $\sin 33.3^\circ = \dfrac{h}{\sqrt{46}}$

correct working *(A1)*

eg $\frac{1}{2}\times 6\times d = 11.2$, $\left|\overrightarrow{PR}\right|\times\sin P$, $\sqrt{46}\sin 33.3^\circ$

3.72677

distance $=3.73$ (units) ***A1*** ***N2***

[3 marks]

[Total 13 marks]

Question 12

(a) (i) valid approach *(M1)*

eg $\mathrm{B}-\mathrm{A}$, $\mathrm{AO}+\mathrm{OB}$, $\begin{pmatrix}8\\-1\\5\end{pmatrix}-\begin{pmatrix}-3\\4\\2\end{pmatrix}$

$\overrightarrow{\mathrm{AB}}=\begin{pmatrix}11\\-5\\3\end{pmatrix}$ ***A1*** ***N2***

(ii) correct substitution into formula *(A1)*

eg $\sqrt{11^2+(-5)^2+3^2}$

12.4498

$\left|\overrightarrow{\mathrm{AB}}\right|=\sqrt{155}$ (exact), 12.4 ***A1*** ***N2***

[4 marks]

(b) (i) valid approach to find t *(M1)*

eg $\begin{pmatrix}5\\y\\1\end{pmatrix}=\begin{pmatrix}2\\0\\-5\end{pmatrix}+t\begin{pmatrix}1\\-2\\2\end{pmatrix}$, $5=2+t$, $1=-5+2t$

$t=3$ (seen anywhere) *(A1)*

attempt to substitute **their** parameter into the vector equation *(M1)*

eg $\begin{pmatrix}5\\y\\1\end{pmatrix}=\begin{pmatrix}2\\0\\-5\end{pmatrix}+3\begin{pmatrix}1\\-2\\2\end{pmatrix}$, $3\cdot(-2)$

$y=-6$ ***A1*** ***N2***

(ii) correct approach ***A1***

eg $\begin{pmatrix}5\\-6\\1\end{pmatrix}-\begin{pmatrix}-3\\4\\2\end{pmatrix}$, $\mathrm{AO}+\mathrm{OC}$, $c-a$

$\overrightarrow{\mathrm{AC}}=\begin{pmatrix}8\\-10\\-1\end{pmatrix}$ ***AG*** ***N0***

Note: Do not award ***A1*** in part (ii) unless answer in part (i) is correct and does not result from working backwards.

[5 marks]

(c) finding scalar product and magnitude *(A1)(A1)*

scalar product $= 11\times 8 + -5\times -10 + 3\times -1 \quad (=135)$

$\left|\overrightarrow{AC}\right| = \sqrt{8^2+(-10)^2+(-1)^2} \quad \left(=\sqrt{165}, 12.8452\right)$

evidence of substitution into formula *(M1)*

eg $\cos\theta = \dfrac{11\times 8 + -5\times -10 + 3\times -1}{\left|\overrightarrow{AB}\right|\times\sqrt{8^2+(-10)^2+(-1)^2}}$, $\cos\theta = \dfrac{\overrightarrow{AB}\bullet\overrightarrow{AC}}{\sqrt{155}\times\sqrt{8^2+(-10)^2+(-1)^2}}$

correct substitution *(A1)*

eg $\cos\theta = \dfrac{11\times 8 + -5\times -10 + 3\times -1}{\sqrt{155}\times\sqrt{8^2+(-10)^2+(-1)^2}}$, $\cos\theta = \dfrac{135}{159.921...}$,

$\cos\theta = 0.844162...$

$0.565795,\ 32.4177^\circ$

$\hat{A} = 0.566,\ 32.4^\circ$ ***A1*** ***N3***

[5 marks]

(d) correct substitution into area formula *(A1)*

eg $\frac{1}{2}\times\sqrt{155}\times\sqrt{165}\times\sin(0.566...)$, $\frac{1}{2}\times\sqrt{155\times 165}\times\sin(32.4)$

42.8660

area $= 42.9$ ***A1*** ***N2***

[2 marks]

Total [16 marks]

Question 13

METHOD 1 (Distance between the origin and P)

correct position vector for OP *(A1)*

eg $\overrightarrow{OP} = \begin{pmatrix} -1+4t \\ 3+5t \\ 8-t \end{pmatrix}$, $P = \left(-1+4t, 3+5t, 8-t\right)$

correct expression for OP or OP^2 (seen anywhere) ***A1***

eg $\sqrt{(-1+4t)^2+(3+5t)^2+(8-t)^2}$, $(-1+4x)^2+(3+5x)^2+(8-x)^2$

valid attempt to find the minimum of OP *(M1)*

eg $d' = 0$, root on sketch of d', min indicated on sketch of d

$t = -\dfrac{1}{14}, -0.0714285$ *(A1)*

substitute their value of t into L (only award if there is working to find t) ***(M1)***

eg one correct coordinate, $-1+4\left(-\frac{1}{14}\right)$

$(-1.28571, 2.64285, 8.07142)$

$\left(-\frac{9}{7}, \frac{37}{14}, \frac{113}{14}\right) = (-1.29, 2.64, 8.07)$ ***A1*** ***N2***

METHOD 2 (Perpendicular vectors)

recognizing that closest implies perpendicular ***(M1)***

eg $\vec{OP} \perp L$ (may be seen on sketch), $a \bullet b = 0$

valid approach involving $\vec{OP}$ ***(M1)***

eg $\vec{OP} = \begin{pmatrix} -1+4t \\ 3+5t \\ 8-t \end{pmatrix}$, $\begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix} \bullet \vec{OP}$, $\begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix} \perp \vec{OP}$

correct scalar product ***A1***

eg $4(-1+4t)+5(3+5t)-1(8-t)$, $-4+16t+15+25t-8+t=0$, $42t+3$

$t = -\frac{1}{14}, -0.0714285$ ***(A1)***

substitute their value of t into L or $\vec{OP}$ (only award if scalar product used to find t) ***(M1)***

eg one correct coordinate, $-1+4\left(-\frac{1}{14}\right)$

$(-1.28571, 2.64285, 8.07142)$

$\left(-\frac{9}{7}, \frac{37}{14}, \frac{113}{14}\right) = (-1.29, 2.64, 8.07)$ ***A1*** ***N2***

[6 marks]

Question 14

(a) valid approach **(M1)**

eg $L_1 = L_2$, $x = 12$, $y = 1$

$(12, 1)$ (exact) ***A1*** ***N2***

[2 marks]

(b) $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ (or any multiple of $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$) ***A1*** ***N1***

[1 mark]

(c) any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (accept any parameter for t) where $\boldsymbol{a}$ is a position vector for a point on L_1, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ ***A2*** ***N2***

eg $\boldsymbol{r} = \begin{pmatrix} 12 \\ 1 \end{pmatrix} + t\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

Note:	Award ***A1*** for the form $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for the form $L = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for the form $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[2 marks]

Total [5 marks]