# Subject – Math (Standard Level)
# Topic - Vector
# Year - Nov 2011 – Nov 2017
# Paper- 1

## Question 1

*[Maximum mark: 18]*

The line $L_1$ passes through the points $\mathrm{P}(2, 4, 8)$ and $\mathrm{Q}(4, 5, 4)$.

(a) (i) Find $\overrightarrow{\mathrm{PQ}}$.

(ii) Hence write down a vector equation for $L_1$ in the form $\boldsymbol{r} = \boldsymbol{a} + s\boldsymbol{b}$. *[4 marks]*

The line $L_2$ is perpendicular to $L_1$, and parallel to $\begin{pmatrix} 3p \\ 2p \\ 4 \end{pmatrix}$, where $p \in \mathbb{Z}$.

(b) (i) Find the value of $p$.

(ii) Given that $L_2$ passes through $\mathrm{R}(10, 6, -40)$, write down a vector equation for $L_2$. *[7 marks]*

(c) The lines $L_1$ and $L_2$ intersect at the point A. Find the $x$-coordinate of A. *[7 marks]*

## Question 2

*[Maximum mark: 17]*

A line $L_1$ passes though points $\mathrm{P}(-1, 6, -1)$ and $\mathrm{Q}(0, 4, 1)$.

(a) (i) Show that $\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$.

(ii) Hence, write down an equation for $L_1$ in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[3 marks]*

A second line $L_2$ has equation $\boldsymbol{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + s\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$.

(b) Find the cosine of the angle between $\overrightarrow{\mathrm{PQ}}$ and $L_2$. *[7 marks]*

(c) The lines $L_1$ and $L_2$ intersect at the point R. Find the coordinates of R. *[7 marks]*

Question 3

*[Maximum mark: 8]*

The line $L$ passes through the point $(5, -4, 10)$ and is parallel to the vector $\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$.

(a) Write down a vector equation for line $L$. *[2 marks]*

(b) The line $L$ intersects the $x$-axis at the point P. Find the $x$-coordinate of P. *[6 marks]*



Question 4

*[Maximum mark: 14]*

Let A and B be points such that $\overrightarrow{OA} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}$.

(a) Show that $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$. *[1 mark]*

Let C and D be points such that ABCD is a **rectangle**.

(b) Given that $\overrightarrow{AD} = \begin{pmatrix} 4 \\ p \\ 1 \end{pmatrix}$, show that $p = 3$. *[4 marks]*

(c) Find the coordinates of point C. *[4 marks]*

(d) Find the area of rectangle ABCD. *[5 marks]*

Question 5

*[Maximum mark: 6]*

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

(a) Find

(i) $2\boldsymbol{a} + \boldsymbol{b}$;

(ii) $|2\boldsymbol{a} + \boldsymbol{b}|$. *[4 marks]*

Let $2\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = \boldsymbol{0}$, where $\boldsymbol{0}$ is the zero vector.

(b) Find $\boldsymbol{c}$. *[2 marks]*

Question 6

*[Maximum mark: 14]*

Consider points $\mathrm{A}(1, -2, -1)$, $\mathrm{B}(7, -4, 3)$ and $\mathrm{C}(1, -2, 3)$. The line $L_1$ passes through C and is parallel to $\overrightarrow{\mathrm{AB}}$.

(a) (i) Find $\overrightarrow{\mathrm{AB}}$.

(ii) Hence, write down a vector equation for $L_1$. *[4 marks]*

A second line, $L_2$, is given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 2 \\ 15 \end{pmatrix} + s\begin{pmatrix} 3 \\ -3 \\ p \end{pmatrix}$.

(b) Given that $L_1$ is perpendicular to $L_2$, show that $p = -6$. *[3 marks]*

(c) The line $L_1$ intersects the line $L_2$ at point Q. Find the $x$-coordinate of Q. *[7 marks]*

Question 7

*[Maximum mark: 5]*

In the following diagram, $\overrightarrow{\mathrm{OP}} = \boldsymbol{p}$, $\overrightarrow{\mathrm{OQ}} = \boldsymbol{q}$ and $\overrightarrow{\mathrm{PT}} = \frac{1}{2}\overrightarrow{\mathrm{PQ}}$.

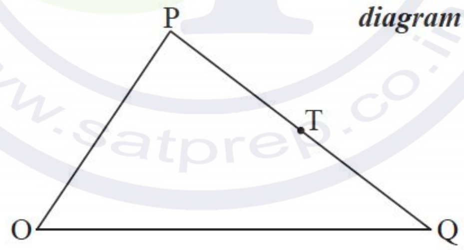



***diagram not to scale***

Express each of the following vectors in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$,

(a) $\overrightarrow{\mathrm{QP}}$; *[2]*

(b) $\overrightarrow{\mathrm{OT}}$. *[3]*

Question 8

*[Maximum mark: 17]*

The line $L_1$ passes through the points $A(2, 1, 4)$ and $B(1, 1, 5)$.

(a) Show that $\vec{AB} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$. *[1]*

(b) Hence, write down

(i) a direction vector for $L_1$;

(ii) a vector equation for $L_1$. *[3]*

Another line $L_2$ has equation $\boldsymbol{r} = \begin{pmatrix} 4 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$. The lines $L_1$ and $L_2$ intersect at the point P.

(c) Find the coordinates of P. *[6]*

(d) (i) Write down a direction vector for $L_2$.

(ii) Hence, find the angle between $L_1$ and $L_2$. *[7]*

Question 9

*[Maximum mark: 7]*

The line $L$ is parallel to the vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

(a) Find the gradient of the line $L$. *[2]*

The line $L$ passes through the point $(9, 4)$.

(b) Find the equation of the line $L$ in the form $y = ax + b$. *[3]*

(c) Write down a vector equation for the line $L$. *[2]*

Question 10

*[Maximum mark: 15]*

*Distances in this question are in metres.*

Ryan and Jack have model airplanes, which take off from level ground. Jack's airplane takes off after Ryan's.

The position of Ryan's airplane $t$ seconds after it takes off is given by $\boldsymbol{r} = \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} + t\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$.

(a) Find the speed of Ryan's airplane. *[3]*

(b) Find the height of Ryan's airplane after two seconds. *[2]*

The position of Jack's airplane $s$ seconds after **it** takes off is given by $\boldsymbol{r} = \begin{pmatrix} -39 \\ 44 \\ 0 \end{pmatrix} + s\begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$.

(c) Show that the paths of the airplanes are perpendicular. *[5]*

The two airplanes collide at the point $(-23, 20, 28)$.

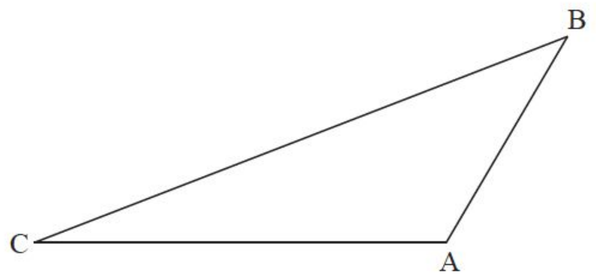
(d) How long after Ryan's airplane takes off does Jack's airplane take off? *[5]*

Question 11

*[Maximum mark: 6]*

The following diagram shows triangle ABC.

***diagram not to scale***




Let $\vec{AB} \bullet \vec{AC} = -5\sqrt{3}$ and $|\vec{AB}||\vec{AC}| = 10$. Find the area of triangle ABC.

Question 12

*[Maximum mark: 17]*

Let $L_x$ be a family of lines with equation given by $\boldsymbol{r} = \begin{pmatrix} x \\ \frac{2}{x} \end{pmatrix} + t\begin{pmatrix} x^2 \\ -2 \end{pmatrix}$, where $x > 0$.

(a) Write down the equation of $L_1$. *[2]*

A line $L_a$ crosses the $y$-axis at a point P.

(b) Show that P has coordinates $\left(0, \frac{4}{a}\right)$. *[6]*

The line $L_a$ crosses the $x$-axis at $\mathrm{Q}(2a, 0)$. Let $d = \mathrm{PQ}^2$.

(c) Show that $d = 4a^2 + \frac{16}{a^2}$. *[2]*

(d) There is a minimum value for $d$. Find the value of $a$ that gives this minimum value. *[7]*

Question 13

[Maximum mark: 16]

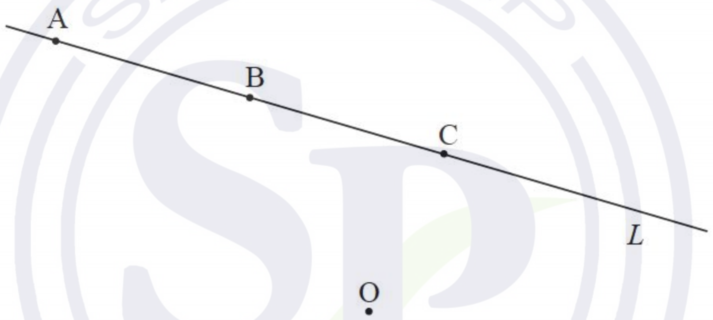
A line $L$ passes through points $A(-2, 4, 3)$ and $B(-1, 3, 1)$.

(a) (i) Show that $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$.

(ii) Find $\left|\overrightarrow{AB}\right|$. [3]

(b) Find a vector equation for $L$. [2]

The following diagram shows the line $L$ and the origin $O$. The point $C$ also lies on $L$.




Point $C$ has position vector $\begin{pmatrix} 0 \\ y \\ -1 \end{pmatrix}$.

(c) Show that $y = 2$. [4]

(d) (i) Find $\overrightarrow{OC} \bullet \overrightarrow{AB}$.

(ii) Hence, write down the size of the angle between OC and $L$. [3]

(e) Hence or otherwise, find the area of triangle OAB. [4]

Question 14

[Maximum mark: 15]

Let P and Q have coordinates $(1, 0, 2)$ and $(-11, 8, m)$ respectively.

(a) Express $\overrightarrow{PQ}$ in terms of $m$. [2]

Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be perpendicular vectors, where $\boldsymbol{a} = \begin{pmatrix} 1 \\ 1 \\ n \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$.

(b) Find $n$. [4]

(c) Given that $\overrightarrow{PQ}$ is parallel to $\boldsymbol{b}$,

(i) express $\overrightarrow{PQ}$ in terms of $\boldsymbol{b}$;

(ii) hence find $m$. [5]

In part (d), distance is in metres, time is in seconds.

(d) A particle moves along a straight line through Q so that its position is given by $\boldsymbol{r} = \boldsymbol{c} + t\boldsymbol{a}$.

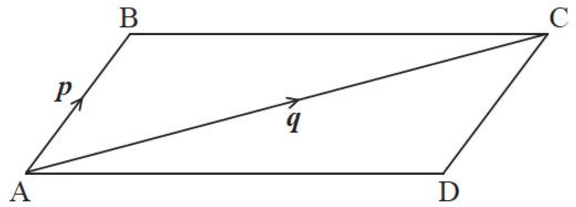
(i) Write down a possible vector $\boldsymbol{c}$.

(ii) Find the speed of the particle. [4]

## Question 15

[Maximum mark: 7]

The following diagram shows the parallelogram ABCD.




Let $\vec{AB} = \boldsymbol{p}$ and $\vec{AC} = \boldsymbol{q}$. Find each of the following vectors in terms of $\boldsymbol{p}$ and/or $\boldsymbol{q}$.

(a) $\vec{CB}$ [2]

(b) $\vec{CD}$ [2]

(c) $\vec{DB}$ [3]

## Question 16

[Maximum mark: 15]

A line $L_1$ passes through the points $A(0, -3, 1)$ and $B(-2, 5, 3)$.

(a) (i) Show that $\vec{AB} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$.

(ii) Write down a vector equation for $L_1$. [3]

A line $L_2$ has equation $\boldsymbol{r} = \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$. The lines $L_1$ and $L_2$ intersect at a point C.

(b) Show that the coordinates of C are $(-1, 1, 2)$. [5]

(c) A point D lies on line $L_2$ so that $\left|\vec{CD}\right| = \sqrt{18}$ and $\vec{CA} \bullet \vec{CD} = -9$. Find $\hat{ACD}$. [7]

Question 17

[Maximum mark: 7]

Let $\boldsymbol{u} = -3\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{v} = m\boldsymbol{j} + n\boldsymbol{k}$, where $m, n \in \mathbb{R}$. Given that $\boldsymbol{v}$ is a unit vector perpendicular to $\boldsymbol{u}$, find the possible values of $m$ and of $n$.

Question 18

[Maximum mark: 7]

The position vectors of points P and Q are $\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ and $7\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$ respectively.

(a) Find a vector equation of the line that passes through P and Q. [4]

(b) The line through P and Q is perpendicular to the vector $2\boldsymbol{i} + n\boldsymbol{k}$. Find the value of $n$. [3]

## Question 19

[Maximum mark: 16]

Let $\vec{OA} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$ and $\vec{OB} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$.

(a) (i) Find $\vec{AB}$.

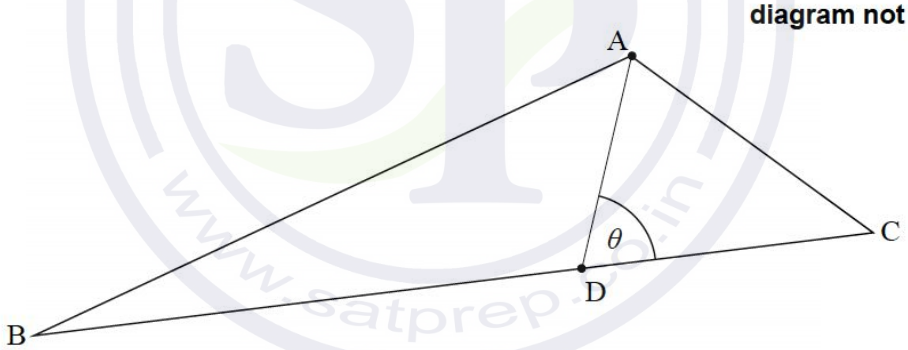
(ii) Find $\left|\vec{AB}\right|$. [4]

The point C is such that $\vec{AC} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$.

(b) Show that the coordinates of C are $(-2, 1, 3)$. [1]

The following diagram shows triangle ABC. Let D be a point on [BC], with acute angle $\text{ADC} = \theta$.

**diagram not to scale**




(c) Write down an expression in terms of $\theta$ for

(i) angle ADB;

(ii) area of triangle ABD. [2]

(d) Given that $\dfrac{\text{area } \Delta\text{ABD}}{\text{area } \Delta\text{ACD}} = 3$, show that $\dfrac{\text{BD}}{\text{BC}} = \dfrac{3}{4}$. [5]

(e) Hence or otherwise, find the coordinates of point D. [4]

## Question 20

[Maximum mark: 17]

A line $L_1$ passes through the points $\mathrm{A}(0, 1, 8)$ and $\mathrm{B}(3, 5, 2)$.

(a) (i) Find $\overrightarrow{\mathrm{AB}}$.

(ii) Hence, write down a vector equation for $L_1$. [4]

(b) A second line $L_2$, has equation $\boldsymbol{r} = \begin{pmatrix} 1 \\ 13 \\ -14 \end{pmatrix} + s\begin{pmatrix} p \\ 0 \\ 1 \end{pmatrix}$.

Given that $L_1$ and $L_2$ are perpendicular, show that $p = 2$. [3]

(c) The lines $L_1$ and $L_2$ intersect at $\mathrm{C}(9, 13, z)$. Find $z$. [5]

(d) (i) Find a unit vector in the direction of $L_2$.

(ii) Hence or otherwise, find one point on $L_2$ which is $\sqrt{5}$ units from C. [5]

## Question 21

[Maximum mark: 7]

The vectors $\boldsymbol{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} k+3 \\ k \end{pmatrix}$ are perpendicular to each other.

(a) Find the value of $k$. [4]

(b) Given that $\boldsymbol{c} = \boldsymbol{a} + 2\boldsymbol{b}$, find $\boldsymbol{c}$. [3]

Question 22

[Maximum mark: 16]

**Note: In this question, distance is in metres and time is in seconds.**

Two particles $P_1$ and $P_2$ start moving from a point A at the same time, along different straight lines.

After $t$ seconds, the position of $P_1$ is given by $\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$.

(a) Find the coordinates of A. [2]

Two seconds after leaving A, $P_1$ is at point B.

(b) Find

(i) $\overrightarrow{AB}$;

(ii) $\left|\overrightarrow{AB}\right|$. [5]

Two seconds after leaving A, $P_2$ is at point C, where $\overrightarrow{AC} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$.

(c) Find $\cos B\hat{A}C$. [5]

(d) Hence or otherwise, find the distance between $P_1$ and $P_2$ two seconds after they leave A. [4]

## Question 23

[Maximum mark: 15]

A line $L$ passes through points $A(-3, 4, 2)$ and $B(-1, 3, 3)$.

(a) (i) Show that $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

(ii) Find a vector equation for $L$. [3]

The line $L$ also passes through the point $C(3, 1, p)$.

(b) Find the value of $p$. [5]

(c) The point D has coordinates $(q^2, 0, q)$. Given that $\overrightarrow{DC}$ is perpendicular to $L$, find the possible values of $q$. [7]

## Question 24

[Maximum mark: 5]

Let $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$, where O is the origin. $L_1$ is the line that passes through A and B.

(a) Find a vector equation for $L_1$. [2]

(b) The vector $\begin{pmatrix} 2 \\ p \\ 0 \end{pmatrix}$ is perpendicular to $\overrightarrow{AB}$. Find the value of $p$. [3]

## Question 25

[Maximum mark: 16]

Point A has coordinates $(-4, -12, 1)$ and point B has coordinates $(2, -4, -4)$.

(a) Show that $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$. [1]

(b) The line $L$ passes through A and B.

(i) Find a vector equation for $L$.

(ii) Point $C(k, 12, -k)$ is on $L$. Show that $k = 14$. [6]

(c) (i) Find $\overrightarrow{OB} \bullet \overrightarrow{AB}$

(ii) Write down the value of angle OBA. [3]

Point D is also on $L$ and has coordinates $(8, 4, -9)$.

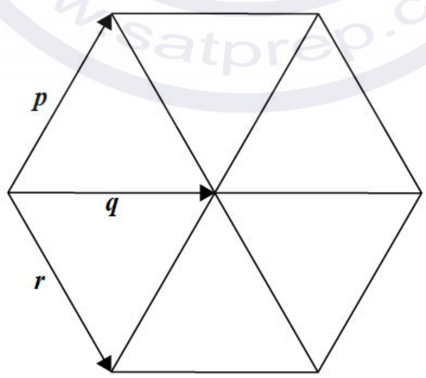
(d) Find the area of triangle OCD. [6]

## Question 26

[Maximum mark: 6]

Six equilateral triangles, each with side length $3\text{ cm}$, are arranged to form a hexagon.
This is shown in the following diagram.

**diagram not to scale**




The vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are shown on the diagram.

Find $\boldsymbol{p} \bullet (\boldsymbol{p} + \boldsymbol{q} + \boldsymbol{r})$.

## Question 27

[Maximum mark: 6]

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 3 \\ 2p \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} p+1 \\ 8 \end{pmatrix}$.

Find the possible values of $p$ for which $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel.

## Question 28

[Maximum mark: 6]

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 0 \\ 3 \\ p \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 0 \\ 6 \\ 18 \end{pmatrix}$.

Find the value of $p$ for which $\boldsymbol{a}$ and $\boldsymbol{b}$ are

(a) parallel; [2]

(b) perpendicular. [4]

## Question 29

[Maximum mark: 7]

The magnitudes of two vectors, $\boldsymbol{u}$ and $\boldsymbol{v}$, are $4$ and $\sqrt{3}$ respectively. The angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is $\frac{\pi}{6}$.

Let $\boldsymbol{w} = \boldsymbol{u} - \boldsymbol{v}$. Find the magnitude of $\boldsymbol{w}$.

## Question 30

[Maximum mark: 6]

A line, $L_1$, has equation $\boldsymbol{r} = \begin{pmatrix} -3 \\ 9 \\ 10 \end{pmatrix} + s\begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}$. Point $\text{P}(15, 9, c)$ lies on $L_1$.

(a) Find $c$. [4]

A second line, $L_2$, is parallel to $L_1$ and passes through $(1, 2, 3)$.

(b) Write down a vector equation for $L_2$. [2]

Question 31

[Maximum mark: 17]

The points A and B have position vectors $\begin{pmatrix} -2 \\ 4 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix}$ respectively.

Point C has position vector $\begin{pmatrix} -1 \\ k \\ 0 \end{pmatrix}$. Let O be the origin.

(a) Find, in terms of $k$,

(i) $\overrightarrow{OA} \cdot \overrightarrow{OC}$;

(ii) $\overrightarrow{OB} \cdot \overrightarrow{OC}$. [3]

(b) Given that $A\hat{O}C = B\hat{O}C$, show that $k = 7$. [8]

(c) Calculate the area of triangle AOC. [6]