Subject – Math(Standard Level)
Topic - Algebra
Year - Nov 2011 – Nov 2019
Paper -2

Question 1

[Maximum mark: 5]

Consider the expansion of $(3x^2+2)^9$.

(a) Write down the number of terms in the expansion. *[1 mark]*

(b) Find the term in x^4. *[4 marks]*

Question 2

[Maximum mark: 14]

(a) Consider an infinite geometric sequence with $u_1 = 40$ and $r = \frac{1}{2}$.

(i) Find u_4.

(ii) Find the sum of the infinite sequence. *[4 marks]*

Consider an arithmetic sequence with n terms, with first term (-36) and eighth term (-8).

(b) (i) Find the common difference.

(ii) Show that $S_n = 2n^2 - 38n$. *[5 marks]*

(c) The sum of the infinite geometric sequence is equal to twice the sum of the arithmetic sequence. Find n. *[5 marks]*

Question 3

[Maximum mark: 6]

The first three terms of an arithmetic sequence are 36, 40, 44, ….

(a) (i) Write down the value of d.

(ii) Find u_8. *[3 marks]*

(b) (i) Show that $S_n = 2n^2 + 34n$.

(ii) Hence, write down the value of S_{14}. *[3 marks]*

Question 4

[Maximum mark: 6]

Consider the expansion of $\left(2x^3 + \frac{b}{x}\right)^8 = 256x^{24} + 3072x^{20} + \ldots + kx^0 + \ldots$.

(a) Find b. *[3 marks]*

(b) Find k. *[3 marks]*

Question 5

[Maximum mark: 6]

The first term of a geometric sequence is 200 and the sum of the first four terms is 324.8.

(a) Find the common ratio. *[4 marks]*

(b) Find the tenth term. *[2 marks]*

Question 6

[Maximum mark: 6]

The first three terms of an arithmetic sequence are 5, 6.7, 8.4.

(a) Find the common difference. *[2 marks]*

(b) Find the 28^{th} term of the sequence. *[2 marks]*

(c) Find the sum of the first 28 terms. *[2 marks]*

Question 7

[Maximum mark: 7]

The third term in the expansion of $(2x+p)^6$ is $60x^4$. Find the possible values of p.

Question 8

[Maximum mark: 7]

An arithmetic sequence is given by $5, 8, 11, \ldots$.

(a) Write down the value of d. *[1 mark]*

(b) Find

(i) u_{100};

(ii) S_{100}. *[4 marks]*

(c) Given that $u_n = 1502$, find the value of n. *[2 marks]*

Question 9

[Maximum mark: 5]

In the expansion of $(3x-2)^{12}$, the term in x^5 can be expressed as $\binom{12}{r} \times (3x)^p \times (-2)^q$.

(a) Write down the value of p, of q and of r. *[3 marks]*

(b) Find the coefficient of the term in x^5. *[2 marks]*

Question 10

[Maximum mark: 6]

The sum of the first three terms of a geometric sequence is 62.755, and the sum of the infinite sequence is 440. Find the common ratio.

Question 11

[Maximum mark: 7]

The constant term in the expansion of $\left(\frac{x}{a}+\frac{a^2}{x}\right)^6$, where $a \in \mathbb{Z}$, is 1280. Find a.

Question 12

[Maximum mark: 5]

Consider the expansion of $(x+3)^{10}$.

(a) Write down the number of terms in this expansion. *[1]*

(b) Find the term containing x^3. *[4]*

Question 13

[Maximum mark: 7]

Consider the expansion of $x^2\left(3x^2+\frac{k}{x}\right)^8$. The constant term is 16 128.

Find k.

Question 14

[Maximum mark: 6]

Consider the expansion of $\left(\frac{x^3}{2}+\frac{p}{x}\right)^8$. The constant term is 5103. Find the possible values of p.

Question 15

[Maximum mark: 14]

The first two terms of a geometric sequence u_n are $u_1 = 4$ and $u_2 = 4.2$.

(a) (i) Find the common ratio.

(ii) Hence or otherwise, find u_5. *[5]*

Another sequence v_n is defined by $v_n = an^k$, where $a, k \in \mathbb{R}$, and $n \in \mathbb{Z}^+$, such that $v_1 = 0.05$ and $v_2 = 0.25$.

(b) (i) Find the value of a.

(ii) Find the value of k. *[5]*

(c) Find the smallest value of n for which $v_n > u_n$. *[4]*

Question 16

[Maximum mark: 5]

Consider the expansion of $(2x+3)^8$.

(a) Write down the number of terms in this expansion. [1]

(b) Find the term in x^3. [4]

Question 17

[Maximum mark: 6]

In an arithmetic sequence $u_{10} = 8$, $u_{11} = 6.5$.

(a) Write down the value of the common difference. [1]

(b) Find the first term. [3]

(c) Find the sum of the first 50 terms of the sequence. [2]

Question 18

[Maximum mark: 5]

The third term in the expansion of $(x+k)^8$ is $63x^6$. Find the possible values of k.

Question 19

[Maximum mark: 7]

Ramiro walks to work each morning. During the first minute he walks 80 metres. In each subsequent minute he walks 90% of the distance walked during the previous minute. The distance between his house and work is 660 metres. Ramiro leaves his house at $08:00$ and has to be at work by $08:15$.

Explain why he will not be at work on time.

Question 20

[Maximum mark: 7]

The first three terms of a geometric sequence are $u_1 = 0.64$, $u_2 = 1.6$, and $u_3 = 4$.

(a) Find the value of r. [2]

(b) Find the value of S_6. [2]

(c) Find the least value of n such that $S_n > 75\,000$. [3]

Question 21

[Maximum mark: 6]

(a) Find the term in x^6 in the expansion of $(x+2)^9$. [4]

(b) Hence, find the term in x^7 in the expansion of $5x(x+2)^9$. [2]

Question 22

[Maximum mark: 6]

In a geometric sequence, the fourth term is 8 times the first term. The sum of the first 10 terms is 2557.5. Find the 10th term of this sequence.

Question 23

[Maximum mark: 6]

The first three terms of an arithmetic sequence are $u_1 = 0.3$, $u_2 = 1.5$, $u_3 = 2.7$.

(a) Find the common difference. [2]

(b) Find the 30th term of the sequence. [2]

(c) Find the sum of the first 30 terms. [2]

Question 24

[Maximum mark: 6]

Consider the expansion of $\left(x^2 + \frac{2}{x}\right)^{10}$.

(a) Write down the number of terms of this expansion. [1]

(b) Find the coefficient of x^8. [5]

Question 25

[Maximum mark: 6]

Consider a geometric sequence where the first term is 768 and the second term is 576.

Find the least value of n such that the $n\text{th}$ term of the sequence is less than 7.

Question 26

[Maximum mark: 6]

In the expansion of $ax^3(2 + ax)^{11}$, the coefficient of the term in x^5 is 11880.
Find the value of a.

Question 27

[Maximum mark: 8]

Let $f(x) = \mathrm{e}^{2\sin\left(\frac{\pi x}{2}\right)}$, for $x > 0$.

The kth maximum point on the graph of f has x-coordinate x_k where $k \in \mathbb{Z}^+$.

(a) Given that $x_{k+1} = x_k + a$, find a. [4]

(b) Hence find the value of n such that $\sum_{k=1}^{n} x_k = 861$. [4]

Question 28

[Maximum mark: 13]

The following table shows values of $\ln x$ and $\ln y$.

ln *x*	1.10	2.08	4.30	6.03
ln *y*	5.63	5.22	4.18	3.41

The relationship between $\ln x$ and $\ln y$ can be modelled by the regression equation $\ln y = a \ln x + b$.

(a) Find the value of a and of b. [3]

(b) Use the regression equation to estimate the value of y when $x = 3.57$. [3]

The relationship between x and y can be modelled using the formula $y = kx^n$, where $k \neq 0$, $n \neq 0$, $n \neq 1$.

(c) By expressing $\ln y$ in terms of $\ln x$, find the value of n and of k. [7]

Question 29

[Maximum mark: 7]

The first term of an infinite geometric sequence is 4. The sum of the infinite sequence is 200.

(a) Find the common ratio. [2]

(b) Find the sum of the first 8 terms. [2]

(c) Find the least value of n for which $S_n > 163$. [3]

Question 30

[Maximum mark: 6]

Consider the expansion of $\left(2x+\dfrac{k}{x}\right)^9$, where $k>0$. The coefficient of the term in x^3 is equal to the coefficient of the term in x^5. Find k.

Question 31

[Maximum mark: 6]

The sum of an infinite geometric sequence is 33.25. The second term of the sequence is 7.98. Find the possible values of r.

Question 32

[Maximum mark: 7]

Consider the expansion of $\left(2x^4+\dfrac{x^2}{k}\right)^{12}$, $k\neq 0$. The coefficient of the term in x^{40} is five times the coefficient of the term in x^{38}. Find k.

Question 33

[Maximum mark: 16]

In an arithmetic sequence, $u_1=1.3$, $u_2=1.4$ and $u_k=31.2$.

(a) Find the value of k. [4]

(b) Find the exact value of S_k. [2]

Consider the terms, u_n, of this sequence such that $n\leq k$.

Let F be the sum of the terms for which n is not a multiple of 3.

(c) Show that $F=3240$. [5]

An infinite geometric series is given as $S_\infty = a+\dfrac{a}{\sqrt{2}}+\dfrac{a}{2}+\ldots,\ a\in\mathbb{Z}^+$.

(d) Find the largest value of a such that $S_\infty < F$. [5]

Question 34

[Maximum mark: 7]

In the expansion of the following expression, find the exact value of the constant term.

$$x^3\left(\frac{1}{2x}+x^2\right)^{15}$$

Question 35

[Maximum mark: 7]

The first terms of an infinite geometric sequence, u_n, are $2, 6, 18, 54, \ldots$.
The first terms of a second infinite geometric sequence, v_n, are $2, -6, 18, -54, \ldots$.

The terms of a third sequence, w_n, are defined as $w_n = u_n + v_n$.

(a) Write down the first three **non-zero** terms of w_n. [3]

The finite series, $\sum_{k=1}^{225} w_k$, can also be written in the form $\sum_{k=0}^{m} 4r^k$.

(b) Find the value of

(i) r;

(ii) m. [4]

Question 36

[Maximum mark: 7]

Consider the expansion of $(x^2 + 1.2)^n$ where $n \in \mathbb{Z}$, $n \geq 3$. Given that the coefficient of the term containing x^6 is greater than $200\,000$, find the smallest possible value of n.

Question 37

[Maximum mark: 6]

Consider the graph of the function $f(x) = a(x + 10)^2 + 15$, $x \in \mathbb{R}$.

(a) Write down the coordinates of the vertex. [2]

(b) The graph of f has a y-intercept at -20. Find a. [2]

(c) Point $\mathrm{P}(8, b)$ lies on the graph of f. Find b. [2]

Question 38

[Maximum mark: 7]

The first two terms of a geometric sequence are $u_1 = 2.1$ and $u_2 = 2.226$.

(a) Find the value of r. [2]

(b) Find the value of u_{10}. [2]

(c) Find the least value of n such that $S_n > 5543$. [3]