Subject – Math (Standard Level)
Topic - Circular trigonometry
Year - Nov 2011 – Nov 2019
Paper -2

Question 1

(a) correct substitution *(A1)*

e.g. $8.5 = \theta(6.8)$, $\theta = \frac{8.5}{6.8}$

$\theta = 1.25$ (accept 71.6°) **A1** **N2**

[2 marks]

(b) **METHOD 1**

correct substitution into area formula (seen anywhere) *(A1)*

e.g. $A = \pi(6.8)^2$, $145.267\ldots$

correct substitution into area formula (seen anywhere) *(A1)*

e.g. $A = \frac{1}{2}(1.25)(6.8^2)$, 28.9

valid approach **M1**

e.g. $\pi(6.8)^2 - \frac{1}{2}(1.25)(6.8^2)$; $145.267\ldots - 28.9$; $\pi r^2 - \frac{1}{2}r^2 \sin\theta$

$A = 116\ (\text{cm}^2)$ **A1** **N2**

[4 marks]

METHOD 2

attempt to find reflex angle *(M1)*

e.g. $2\pi - \theta$, $360 - 1.25$

correct reflex angle *(A1)*

$\hat{AOB} = 2\pi - 1.25\ (= 5.03318\ldots)$

correct substitution into area formula **A1**

e.g. $A = \frac{1}{2}(5.03318...)(6.8^2)$

$A = 116\ (\text{cm}^2)$ **A1** **N2**

[4 marks]

Total [6 marks]

Question 2

Note: accept answers given in degrees, and minutes.

(a) evidence of choosing sine rule *(M1)*

e.g. $\frac{\sin A}{a} = \frac{\sin B}{b}$

correct substitution ***A1***

e.g. $\frac{\sin \theta}{10} = \frac{\sin 30°}{7}$, $\sin \theta = \frac{5}{7}$

$\text{A}\hat{\text{C}}\text{B} = 45.6°$, $\text{A}\hat{\text{C}}\text{B} = 134°$ ***A1A1*** ***N1N1***

Note: If candidates only find the acute angle in part (a), award no marks for (b).

[4 marks]

(b) Attempt to substitute their larger value into angle sum of triangle *(M1)*

e.g. $180° - (134.415...° + 30°)$

$\text{A}\hat{\text{B}}\text{C} = 15.6°$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

Question 3

(a) (i) evidence of valid approach *(M1)*

e.g. choosing cosine rule

correct substitution *(A1)*

e.g. $6^2 = (5p)^2 + (4p)^2 - 2 \times (4p) \times (5p)\cos 0.7$

simplification ***A1***

e.g. $36 = 25p^2 + 16p^2 - 40p^2 \cos 0.7$

$p^2(41 - 40\cos 0.7) = 36$ ***AG*** ***N0***

(ii) 1.85995…

$p = 1.86$ ***A1*** ***N1***

> **Note:** Award ***A0*** for $p = \pm 1.86$, *i.e.* not rejecting the negative value.

[4 marks]

(b) $\text{BD} = 6$ ***A1*** ***N1***

[1 mark]

(c) evidence of valid approach *(M1)*

e.g. choosing sine rule

correct substitution ***A1***

e.g. $\frac{\sin A\hat{D}B}{4p} = \frac{\sin 0.7}{6}$

acute $\text{A}\hat{\text{D}}\text{B} = 0.9253166\ldots$ *(A1)*

$\pi - 0.9253166\ldots = 2.216275\ldots$

$\text{A}\hat{\text{D}}\text{B} = 2.22$ ***A1*** ***N3***

[4 marks]

(d)	(i)	evidence of valid approach *e.g.* recognize isosceles triangle, base angles equal	*(M1)*	
		$\pi - 2(0.9253\ldots)$	*A1*	
		$\hat{CBD} = 1.29$	*AG*	*N0*
	(ii)	area of sector BCD *e.g.* $0.5 \times (1.29) \times (6)^2$	*(A1)*	
		area of triangle BCD *e.g.* $0.5 \times (6)^2 \sin 1.29$	*(A1)*	
		evidence of subtraction $5.92496\ldots$	*M1*	
		$5.937459\ldots$ area $= 5.94$	*A1*	*N3* ***[6 marks]***

Total [15 marks]

Question 4

(a)	$R\hat{P}Q = 65^\circ$	*A1*	*N1* ***[1 mark]***
(b)	evidence of choosing sine rule	*(M1)*	
	correct substitution *e.g.* $\frac{PR}{\sin 45^\circ} = \frac{9}{\sin 65^\circ}$	*A1*	
	7.021854078		
	PR=7.02	*A1*	*N2* ***[3 marks]***
(c)	correct substitution *e.g.* area $= \frac{1}{2} \times 9 \times 7.02\ldots \times \sin 70^\circ$	*(A1)*	
	29.69273008		
	area = 29.7	*A1*	*N2* ***[2 marks]***

Total [6 marks]

Question 5

(a) correct substitution into cosine rule *A1*

e.g. $PQ^2 = r^2 + r^2 - 2(r)(r)\cos(2\theta)$, $PQ^2 = 2r^2 - 2r^2\left(\cos(2\theta)\right)$

substituting $1 - 2\sin^2\theta$ for $\cos 2\theta$ (seen anywhere) *A1*

e.g. $PQ^2 = 2r^2 - 2r^2(1 - 2\sin^2\theta)$

working towards answer *(A1)*

e.g. $PQ^2 = 2r^2 - 2r^2 + 4r^2\sin^2\theta$

recognizing $2r^2 - 2r^2 = 0$ (including crossing out) (seen anywhere) *A1*

e.g. $PQ^2 = 4r^2\sin^2\theta$, $PQ = \sqrt{4r^2\sin^2\theta}$

$PQ = 2r\sin\theta$ *AG* *N0*

[4 marks]

(b) $PRQ = r \times 2\theta$ (seen anywhere) *(A1)*

correct set up *A1*

e.g. $1.3 \times 2r\sin\theta - r \times (2\theta) = 0$

attempt to eliminate r *(M1)*

correct equation in terms of the one variable θ *(A1)*

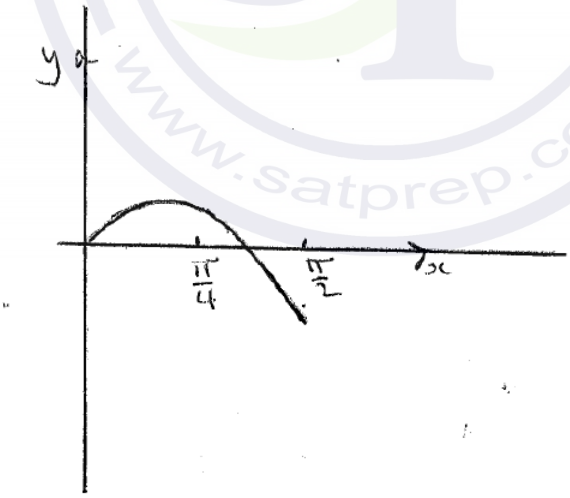
e.g. $1.3 \times 2\sin\theta - 2\theta = 0$

1.221496215

$\theta = 1.22$ (accept 70.0^0 (69.9)) *A1* *N3*

[5 marks]

(c) (i)



A1A1A1 *N3*

Note:	Award ***A1*** for approximately correct shape, ***A1*** for *x*-intercept in approximately correct position, ***A1*** for domain. Do not penalise if sketch starts at origin.

(ii) 1.221496215

$\theta = 1.22$ *A1* *N1*

[4 marks]

(d) evidence of appropriate approach (may be seen earlier) **M2**

e.g. $2\theta < 2.6\sin\theta$, $0 < f(\theta)$, showing positive part of sketch

$0 < \theta < 1.221496215$

$0 < \theta = 1.22$ (accept $\theta < 1.22$) **A1** **N1**

[3 marks]

Total [16 marks]

Question 6

(a) (i) $a = 5$ (accept -5) **A1** **N1**

(ii) $c = 3$ (accept $c = 7$, if $a = -5$) **A1** **N1**

Note: Accept other correct values of c, such as 11, -5, *etc.*

[2 marks]

(b) attempt to find period ***(M1)***

e.g. 8, $b = \frac{2\pi}{\text{period}}$

0.785398…

$b = \frac{2\pi}{8}$ (exact), $\frac{\pi}{4}$, 0.785 [0.785, 0.786] (do not accept 45) **A1** **N2**

[2 marks]

(c) valid approach ***(M1)***

e.g. $f(x) = 0$, symmetry of curve

$x = 5$ (accept $(5, 0)$) **A1** **N2**

[2 marks]

Total [6 marks]

Question 7

(a) **METHOD 1**

choosing cosine rule (must have cos in it) ***(M1)***

e.g. $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution (into rhs) ***A1***

e.g. $20^2 + 20^2 - 2(20)(20)\cos 1.5$, $\text{AB} = \sqrt{800 - 800\cos 1.5}$

$\text{AB} = 27.26555\ldots$

$\text{AB} = 27.3\ [27.2, 27.3]$ ***A1*** ***N2***

[3 marks]

METHOD 2

choosing sine rule ***(M1)***

e.g. $\frac{\sin A}{a} = \frac{\sin B}{b}$, $\frac{\text{AB}}{\sin O} = \frac{\text{AO}}{\sin B}$

correct substitution ***A1***

e.g. $\frac{\text{AB}}{\sin 1.5} = \frac{20}{\sin(0.5(\pi - 1.5))}$

$\text{AB} = 27.26555\ldots$

$\text{AB} = 27.3\ [27.2, 27.3]$ ***A1*** ***N2***

[3 marks]

(b) correct substitution into area formula ***A1***

e.g. $\frac{1}{2}(20)(20)\sin 1.5$, $\frac{1}{2}(20)(27.2655504\ldots)\sin(0.5(\pi - 1.5))$

area $= 199.498997\ldots$ (accept $199.75106 = 200$, from using 27.3)

area $= 199\ [199, 200]$ ***A1*** ***N1***

[2 marks]

(c) appropriate method to find angle AOC *(M1)*

e.g. $2\pi - 1.5 - 2.4$

correct substitution into arc length formula *(A1)*

e.g. $(2\pi - 3.9) \times 20$, $2.3831853\ldots \times 20$

arc length $= 47.6637\ldots$

arc length $= 47.7$ $(47.6, 47.7]$ (*i.e.* do **not** accept 47.6) ***A1*** ***N2***

Notes: Candidates may misread the question and use $A\hat{O}C = 2.4$. If working shown, award ***M0*** then ***A0MRA1*** for the answer 48. Do not then penalize $A\hat{O}C$ in part (d) which, if used, leads to the answer $679.498\ldots$ **However**, if they use the prematurely rounded value of 2.4 for $A\hat{O}C$, penalise 1 mark for premature rounding for the answer 48 in (c). Do not then penalize for this in (d).

[3 marks]

(d) calculating sector area using **their** angle AOC *(A1)*

e.g. $\frac{1}{2}(2.38\ldots)(20^2)$, $200(2.38\ldots)$, $476.6370614\ldots$

shaded area $=$ **their** area of triangle AOB $+$ **their** area of sector *(M1)*

e.g. $199.4989973\ldots + 476.6370614\ldots$, $199 + 476.637$

shaded area $= 676.136\ldots$ (accept $675.637\ldots = 676$ from using 199)

shaded area $= 676$ $[676, 677]$, ***A1*** ***N2***

[3 marks]

(e) dividing to find number of cans *(M1)*

e.g. $\frac{676}{140}$, $4.82857\ldots$

5 cans must be purchased *(A1)*

multiplying to find cost of cans *(M1)*

e.g. $5(32)$, $\frac{676}{140} \times 32$

cost is 160 (dollars) ***A1*** ***N3***

[4 marks]

Total [15 marks]

Question 8

(a) evidence of choosing cosine rule **(M1)**

eg $c^2 = a^2 + b^2 - 2ab\cos C$, $\text{CD}^2 + \text{AD}^2 - 2\times\text{CD}\times\text{AD}\cos D$

correct substitution **A1**

eg $11.5^2 + 8^2 - 2\times 11.5\times 8\cos 104$, $196.25 - 184\cos 104$

$\text{AC} = 15.5\,(\text{m})$ **A1** **N2**

[3 marks]

(b) (i) **METHOD 1**

evidence of choosing sine rule **(M1)**

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$, $\frac{\sin \text{A}\hat{\text{C}}\text{D}}{\text{AD}} = \frac{\sin D}{\text{AC}}$

correct substitution **A1**

eg $\frac{\sin \text{A}\hat{\text{C}}\text{D}}{8} = \frac{\sin 104}{15.516\ldots}$

$\text{A}\hat{\text{C}}\text{D} = 30.0^\circ$ **A1** **N2**

METHOD 2

evidence of choosing cosine rule **(M1)**

eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution **A1**

eg $8^2 = 11.5^2 + 15.516\ldots^2 - 2(11.5)(15.516\ldots)\cos C$

$\text{A}\hat{\text{C}}\text{D} = 30.0^\circ$ **A1** **N2**

(ii) subtracting **their** $\text{A}\hat{\text{C}}\text{D}$ from 73 **(M1)**

eg $73 - \text{A}\hat{\text{C}}\text{D}$, $70 - 30.017\ldots$

$\text{A}\hat{\text{C}}\text{B} = 43.0^\circ$ **A1** **N2**

[5 marks]

(c) correct substitution **(A1)**

eg area $\Delta\,\text{ADC} = \frac{1}{2}(8)(11.5)\sin 104$

$\text{area} = 44.6\ (\text{m}^2)$ **A1** **N2**

[2 marks]

(d) attempt to subtract **(M1)**

eg $\text{circle} - \text{ABCD}$, $\pi r^2 - \Delta\,\text{ADC} - \Delta\,\text{ACB}$

area $\Delta\,\text{ACB} = \frac{1}{2}(15.516\ldots)(14)\sin 42.98\ (= 74.0517\ldots)$ **(A1)**

correct working **A1**

eg $\pi(8)^2 - 44.6336\ldots - \frac{1}{2}(15.516\ldots)(14)\sin 42.98$, $64\pi - 44.6 - 74.1$

shaded area is $82.4\ (\text{m}^2)$ **A1** **N3**

[4 marks]

Total [14 marks]

Question 9

(a) valid approach *(M1)*

eg $13 + \text{diameter},\ 13 + 122$

maximum height $= 135$ (m) ***A1*** ***N2***

[2 marks]

(b) (i) $\text{period} = \dfrac{60}{2.4}$ ***A1***

$\text{period} = 25$ (minutes) ***AG*** ***N0***

(ii) $b = \dfrac{2\pi}{25}$ $(= 0.08\pi)$ ***A1*** ***N1***

[2 marks]

(c) **METHOD 1**

valid approach *(M1)*

eg $\max - 74,\ |a| = \dfrac{135-13}{2},\ 74-13$

$|a| = 61$ (accept $a = 61$) *(A1)*

$a = -61$ ***A1*** ***N2***

[3 marks]

METHOD 2

attempt to substitute valid point into equation for h *(M1)*

eg $135 = 74 + a\cos\left(\dfrac{2\pi \times 12.5}{25}\right)$

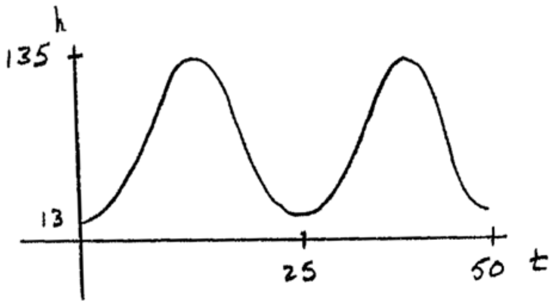
correct equation *(A1)*

eg $135 = 74 + a\cos(\pi),\ 13 = 74 + a$

$a = -61$ ***A1*** ***N2***

[3 marks]

(d)



A1A1A1A1 ***N4***

Note: Award ***A1*** for approximately correct domain, ***A1*** for approximately correct range, ***A1*** for approximately correct sinusoidal shape with 2 cycles.
Only if this last ***A1*** awarded, award ***A1*** for max/min in approximately correct positions.

[4 marks]

(e) setting up inequality (accept equation) ***(M1)***

eg $h > 105$, $105 = 74 + a\cos bt$, sketch of graph with line $y = 105$

any **two** correct values for t (seen anywhere) ***A1A1***

eg $t = 8.371\ldots$, $t = 16.628\ldots$, $t = 33.371\ldots$, $t = 41.628\ldots$,

valid approach ***M1***

eg $\frac{16.628 - 8.371}{25}$, $\frac{t_1 - t_2}{25}$, $\frac{2 \times 8.257}{50}$, $\frac{2(12.5 - 8.371)}{25}$

$p = 0.330$ ***A1*** ***N2***

[5 marks]

Total [16 marks]

Question 10

(a) correct substitution into area formula *(A1)*

eg $\frac{1}{2}(18x)\sin 50$

setting **their** area expression equal to 80 *(M1)*

eg $9x\sin 50 = 80$

$x = 11.6$ ***A1*** ***N2***

[3 marks]

(b) evidence of choosing cosine rule *(M1)*

eg $c^2 = a^2 + b^2 + 2ab\sin C$

correct substitution into right hand side (may be in terms of x) *(A1)*

eg $11.6^2 + 18^2 - 2(11.6)(18)\cos 50$

$\text{BC} = 13.8$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

Question 11

(a) use right triangle trigonometry *A1*

eg $\cos 1.4 = \frac{\text{OC}}{r}$

$\text{OC} = r\cos 1.4$ *AG* *N0*

[1 mark]

(b) correct value for BC

eg $\text{BC} = r\sin 1.4$, $\sqrt{r^2 - (r\cos 1.4)^2}$ *(A1)*

area of $\Delta\text{OBC} = \frac{1}{2}r\sin 1.4 \times r\cos 1.4 \left(= \frac{1}{2}r^2 \sin 1.4 \times \cos 1.4 \right)$ *A1*

area of sector $\text{OAB} = \frac{1}{2}r^2 \times 1.4 \quad (= 0.7r^2)$ *A1*

attempt to subtract in any order *(M1)*

eg sector – triangle, $\frac{1}{2}r^2 \sin 1.4 \times \cos 1.4 - 0.7r^2$

correct equation *A1*

eg $0.7r^2 - \frac{1}{2}r\sin 1.4 \times r\cos 1.4 = 25$

attempt to solve **their** equation *(M1)*

eg sketch, writing as quadratic, $\frac{25}{0.616\ldots}$

$r = 6.37$ *A1* *N4*

[7 marks]

Note: Exception to ***FT*** rule. Award ***A1FT*** for a correct ***FT*** answer from a quadratic equation involving two trigonometric functions.

Total [8 marks]

Question 12

(a) evidence of choosing sine rule *(M1)*

eg $\frac{\sin \hat{A}}{a} = \frac{\sin \hat{B}}{b}$

correct substitution *(A1)*

eg $\frac{\sin \hat{A}}{10.4} = \frac{\sin 1.058}{12.2}$

$\text{B}\hat{\text{A}}\text{C} = 0.837$ ***A1*** ***N2***

[3 marks]

(b) **METHOD 1**

evidence of subtracting angles from π *(M1)*

eg $\text{A}\hat{\text{B}}\text{C} = \pi - A - C$

correct angle (seen anywhere) ***A1***

$\text{A}\hat{\text{B}}\text{C} = \pi - 1.058 - 0.837$, $1.246, 71.4^\circ$

attempt to substitute into cosine or sine rule *(M1)*

correct substitution *(A1)*

eg $12.2^2 + 10.4^2 - 2 \times 12.2 \times 10.4 \cos 71.4$, $\frac{\text{AC}}{\sin 1.246} = \frac{12.2}{\sin 1.058}$

$\text{AC} = 13.3$ (cm) ***A1*** ***N3***

METHOD 2

evidence of choosing cosine rule ***M1***

eg $a^2 = b^2 + c^2 - 2bc\cos A$

correct substitution *(A2)*

eg $12.2^2 = 10.4^2 + b^2 - 2 \times 10.4 b \cos 1.058$

$\text{AC} = 13.3$ (cm) ***A2*** ***N3***

[5 marks]

(c) **METHOD 1**

valid approach *(M1)*

eg $\cos A\hat{O}C = \frac{OA^2 + OC^2 - AC^2}{2 \times OA \times OC}$, $A\hat{O}C = 2 \times A\hat{B}C$

correct working *(A1)*

eg $13.3^2 = 7^2 + 7^2 - 2 \times 7 \times 7 \cos A\hat{O}C$, $O = 2 \times 1.246$

$A\hat{O}C = 2.492 \quad (142.8^\circ)$ *(A1)*

EITHER

correct substitution for arc length (seen anywhere) *A1*

eg $2.492 = \frac{l}{7}$, $l = 17.4$, $14\pi \times \frac{142.8}{360}$

subtracting arc from circumference *(M1)*

eg $2\pi r - l$, $14\pi - 17.4$

OR

attempt to find $A\hat{O}C$ reflex *(M1)*

eg $2\pi - 2.492$, 3.79, $360 - 142.8$

correct substitution for arc length (seen anywhere) *A1*

eg $l = 7 \times 3.79$, $14\pi \times \frac{217.2}{360}$

THEN

arc ABC = 26.5 *A1* *N4*

METHOD 2

valid approach to find $A\hat{O}B$ or $B\hat{O}C$ *(M1)*

eg choosing cos rule, twice angle at circumference

correct working for finding **one** value, $A\hat{O}B$ or $B\hat{O}C$ *(A1)*

eg $\cos A\hat{O}B = \frac{7^2 + 7^2 - 12.2^2}{2 \times 7 \times 7}$, $A\hat{O}B = 2.116$, $B\hat{O}C = 1.6745$

two correct calculations for arc lengths

eg $AB = 7 \times 2 \times 1.058 (= 14.8135)$, $7 \times 1.6745 (= 11.7216)$ *(A1)(A1)*

adding **their** arc lengths (seen anywhere)

eg $rA\hat{O}B + rB\hat{O}C$, $14.8135 + 11.7216$, $7(2.116 + 1.6745)$ *M1*

arc ABC = 26.5 (cm) *A1* *N4*

Note: Candidates may work with other interior triangles using a similar method. Check calculations carefully and award marks in line with markscheme.

[6 marks]

Total [14 marks]

Question 13

(a) evidence of choosing cosine rule *(M1)*

eg $AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos(A\hat{B}C)$

correct substitution into the right-hand side *(A1)*

eg $6^2 + 10^2 - 2(6)(10)\cos 100^\circ$

$AC = 12.5234$

$AC = 12.5$ (cm) ***A1*** ***N2***

[3 marks]

(b) evidence of choosing a valid approach *(M1)*

eg sine rule, cosine rule

correct substitution *(A1)*

eg $\dfrac{\sin(B\hat{C}A)}{6} = \dfrac{\sin 100^\circ}{12.5}$, $\cos(B\hat{C}A) = \dfrac{(AC)^2 + 10^2 - 6^2}{2(AC)(10)}$

$B\hat{C}A = 28.1525$

$B\hat{C}A = 28.2^\circ$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

Question 14

(a) $t = 5$ *(A1)*

correct substitution into formula *(A1)*

eg $210\sin(0.5\times 5 - 2.6) + 990$, $P(5)$

$969.034982\ldots$

969 (deer) (must be an integer) ***A1*** ***N3***

[3 marks]

(b) (i) evidence of considering derivative *(M1)*

eg P'

104.475

104 (deer per month) ***A1*** ***N2***

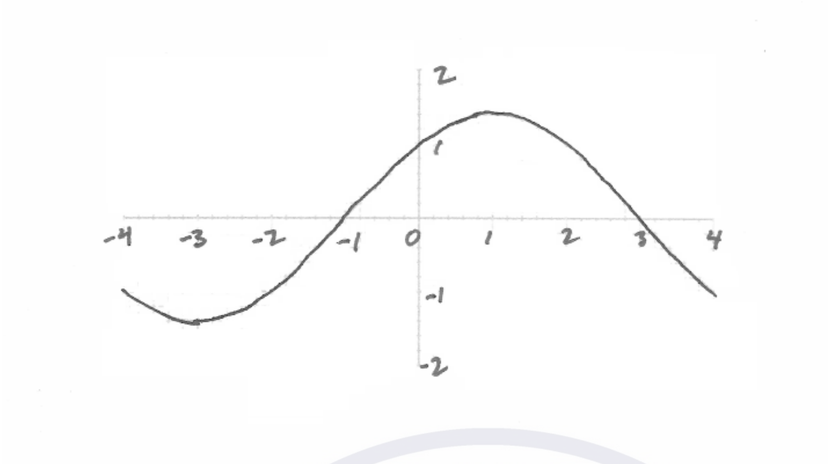
(ii) (the deer population size is) **increasing** ***A1*** ***N1***

[3 marks]

Total [6 marks]

Question 15

(a)



A1A1A1 *N3*

> **Note:** Award *A1* for approximately correct sinusoidal shape.
> **Only** if this *A1* is awarded, award the following:
> *A1* for correct domain,
> *A1* for approximately correct range.

[3 marks]

(a) recognizes decreasing to the left of minimum or right of maximum,
eg $f'(x) < 0$ ***(R1)***

x-values of minimum and maximum (may be seen on sketch in part (a)) ***(A1)(A1)***
eg $x = -3$, $(1, 1.4)$

two correct intervals *A1A1* *N5*
eg $-4 < x < -3$, $1 \le x \le 4$; $x < -3$, $x \ge 1$

[5 marks]

(c) (i) recognizes that a is found from amplitude of wave ***(R1)***

y-value of minimum or maximum ***(A1)***
eg $(-3, -1.41)$, $(1, 1.41)$

$a = 1.41421$
$a = \sqrt{2}$, (exact), 1.41, *A1* *N3*

Question 16

(a) (i) correct substitution into arc length formula *(A1)*

eg 0.7×5

arc length $= 3.5$ (cm) ***A1*** ***N2***

(ii) valid approach *(M1)*

eg $3.5 + 5 + 5$, $\text{arc} + 2r$

perimeter $= 13.5$ (cm) ***A1*** ***N2***

[4 marks]

(b) correct substitution into area formula *(A1)*

eg $\frac{1}{2}(0.7)(5)^2$

area $= 8.75$ (cm^2) ***A1*** ***N2***

[2 marks]

Total [6 marks]

Question 17

(a) correct substitution into area formula *(A1)*

eg $\frac{1}{2}(6)(8)\sin A = 16$, $\sin A = \frac{16}{24}$

correct working *(A1)*

eg $A = \arcsin\left(\frac{2}{3}\right)$

$A = 0.729727656\ldots, 2.41186499\ldots$; $(41.8103149°, 138.1896851°)$

$A = 0.730$; 2.41 ***A1A1*** ***N3***

(accept degrees *ie* $41.8°$; $138°$)

[4 marks]

(b) evidence of choosing cosine rule *(M1)*

eg $\text{BC}^2 = \text{AB}^2 + \text{AC}^2 - 2(\text{AB})(\text{AC})\cos A$, $a^2 + b^2 - 2ab\cos C$

correct substitution into RHS (angle must be obtuse) *(A1)*

eg $\text{BC}^2 = 6^2 + 8^2 - 2(6)(8)\cos 2.41$, $6^2 + 8^2 - 2(6)(8)\cos 138°$,

$\text{BC} = \sqrt{171.55}$

$\text{BC} = 13.09786$

$\text{BC} = 13.1\text{ cm}$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

Question 18

(a) $r = -4$ *A2* *N2*

Note: Award ***A1*** for $r = 4$.

[2 marks]

(b) (i) evidence of valid approach *(M1)*

eg $\frac{\text{max } y \text{ value} - \text{min } y \text{ value}}{2}$, distance from $y = 10$

$p = 8$ *A1* *N2*

(ii) valid approach *(M1)*

eg period is 24, $\frac{360}{24}$, substitute a point into **their** $f(x)$

$q = \frac{2\pi}{24}$ $\left(\frac{\pi}{12}, \text{exact}\right)$, 0.262 (do not accept degrees) *A1* *N2*

[4 marks]

(c) valid approach *(M1)*

eg line on graph at $y = 7$, $8\cos\left(\frac{2\pi}{24}(x-4)\right)+10 = 7$

$x = 11.46828$

$x = 11.5$ (accept (11.5, 7)) *A1* *N2*

[2 marks]

Note: Do not award the final ***A1*** if additional values are given. If an incorrect value of q leads to multiple solutions, award the final ***A1*** only if **all** solutions within the domain are given.

Total [8 marks]

Question 19

(a) correct substitution into formula *(A1)*

eg $l = 1.2 \times 8$

9.6 (cm) *A1* *N2*

[2 marks]

(b) **METHOD 1**

evidence of choosing cosine rule *(M1)*

eg $2r^2 - 2 \times r^2 \times \cos\left(\mathrm{A\hat{O}B}\right)$

correct substitution into right hand side *(A1)*

eg $8^2 + 8^2 - 2 \times 8 \times 8 \times \cos(1.2)$

9.0342795

$\mathrm{AB} = 9.03$ [9.03, 9.04] (cm) *A1* *N2*

METHOD 2

evidence of choosing sine rule *(M1)*

eg $\dfrac{\mathrm{AB}}{\sin\left(\mathrm{A\hat{O}B}\right)} = \dfrac{\mathrm{OB}}{\sin\left(\mathrm{O\hat{A}B}\right)}$

finding angle OAB or OBA (may be seen in substitution) *(A1)*

eg $\dfrac{\pi - 1.2}{2}$, 0.970796

$\mathrm{AB} = 9.03$ [9.03, 9.04] (cm) *A1* *N2*

[3 marks]

(c) correct working *(A1)*

eg $P = 9.6 + 9.03$

18.6342

18.6 [18.6, 18.7] (cm) *A1* *N2*

[2 marks]

Total [7 marks]

Question 20

(a) valid approach *(M1)*

eg $\frac{2-1}{2}$, $2-1.5$

$p = 0.5$ *A1* *N2*

[2 marks]

(b) valid approach *(M1)*

eg $\frac{1+2}{2}$

$r = 1.5$ *A1* *N2*

[2 marks]

(c) **METHOD 1**

valid approach (seen anywhere) *M1*

eg $q = \frac{2\pi}{\text{period}}$, $\frac{2\pi}{\left(\frac{2\pi}{3}\right)}$

$\text{period} = \frac{2\pi}{3}$ (seen anywhere) *(A1)*

$q = 3$ *A1* *N2*

METHOD 2

attempt to substitute one point and **their** values for p and r into y *M1*

eg $2 = 0.5\sin\left(q\frac{\pi}{6}\right)+1.5$, $\frac{\pi}{2} = 0.5\sin(q1)+1.5$

correct equation in q *(A1)*

eg $q\frac{\pi}{6} = \frac{\pi}{2}$, $q\frac{\pi}{2} = \frac{3\pi}{2}$

$q = 3$ *A1* *N2*

METHOD 3

valid reasoning comparing the graph with that of $\sin x$ *R1*

eg position of max/min, graph goes faster

correct working *(A1)*

eg max at $\frac{\pi}{6}$ not at $\frac{\pi}{2}$, graph goes 3 times as fast

$q = 3$ *A1* *N2*

[3 marks]

Total [7 marks]

Question 21

(a) valid approach *(M1)*

eg speed $= \frac{\text{distance}}{\text{time}}$, 6×1.5

$\text{SL} = 9 \quad (\text{km})$ **A1** **N2**

[2 marks]

(b) evidence of choosing sine rule *(M1)*

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$, $\sin\theta = \frac{(\text{SL})\sin 20^\circ}{5}$

correct substitution *(A1)*

eg $\frac{\sin\theta}{9} = \frac{\sin 20^\circ}{5}$

37.9981

$\text{S}\hat{\text{P}}\text{L} = 38.0^\circ$ **A1** **N2**

recognition that second angle is the supplement of first *(M1)*

eg $180 - x$

142.001

$\text{S}\hat{\text{Q}}\text{L} = 142^\circ$ **A1** **N2**

[5 marks]

(c) (i) new store is at Q *A1* *N1*

(ii) **METHOD 1**

attempt to find third angle *(M1)*

eg $\text{S}\hat{\text{L}}\text{P} = 180 - 20 - 38$, $\text{S}\hat{\text{L}}\text{Q} = 180 - 20 - 142$

$\text{S}\hat{\text{L}}\text{Q} = 17.998^\circ$ (seen anywhere) *A1*

evidence of choosing sine rule or cosine rule *(M1)*

correct substitution into sine rule or cosine rule *(A1)*

eg $\frac{x}{\sin 17.998} = \frac{5}{\sin 20}\left(= \frac{9}{\sin 142}\right)$, $9^2 + 5^2 - 2(9)(5)\cos 17.998^\circ$

$4.51708\,\text{km}$

4.52 (km) *A1* *N3*

METHOD 2

evidence of choosing cosine rule *(M1)*

correct substitution into cosine rule *A1*

eg $9^2 = x^2 + 5^2 - 2(x)(5)\cos 142^\circ$

attempt to solve *(M1)*

eg sketch; setting quadratic equation equal to zero;

$0 = x^2 + 7.88x - 56$

one correct value for x *(A1)*

eg $x = -12.3973$, $x = 4.51708$

4.51708

4.52 (km) *A1* *N3*

[6 marks]

Total [13 marks]

Question 22

(a) evidence of choosing sine rule *(M1)*

eg $\frac{AC}{\sin(A\hat{B}C)} = \frac{BC}{\sin(B\hat{A}C)}$

correct substitution *(A1)*

eg $\frac{AC}{\sin 80^\circ} = \frac{10}{\sin 35^\circ}$

$AC = 17.1695$

$AC = 17.2 \quad (cm)$ *A1* *N2*

[3 marks]

(b) $A\hat{C}B = 65^\circ$ (seen anywhere) *(A1)*

correct substitution *(A1)*

eg $\frac{1}{2} \times 10 \times 17.1695 \times \sin 65^\circ$

$\text{area} = 77.8047$

$\text{area} = 77.8 \quad (cm^2)$ *A1* *N2*

[3 marks]

Total [6 marks]

Question 23

(a) correct substitution *(A1)*

eg $l = 1.3 \times 3$

$l = 3.9$ (cm) **A1** **N2**

[2 marks]

(b) **METHOD 1**

valid approach ***(M1)***

eg finding reflex angle, $2\pi - \hat{COA}$

correct angle ***(A1)***

eg $2\pi - 1.3$, 4.98318

correct substitution ***(A1)***

eg $\frac{1}{2}(2\pi - 1.3)3^2$

22.4243

area $= 9\pi - 5.85$ (exact), 22.4 (cm^2) **A1** **N3**

METHOD 2

correct area of small sector ***(A1)***

eg $\frac{1}{2}(1.3)3^2$, 5.85

valid approach ***(M1)***

eg circle – small sector, $\pi r^2 - \frac{1}{2}\theta r^2$

correct substitution ***(A1)***

eg $\pi(3^2) - \frac{1}{2}(1.3)3^2$

22.4243

area $= 9\pi - 5.85$ (exact), 22.4 (cm^2) **A1** **N3**

[4 marks]

Total [6 marks]

Question 24

(a) evidence of choosing sine rule *(M1)*

eg $\frac{\text{AC}}{\sin \text{C}\hat{\text{B}}\text{A}} = \frac{\text{AB}}{\sin \text{A}\hat{\text{C}}\text{B}}$

correct substitution *(A1)*

eg $\frac{\text{AC}}{\sin 44^\circ} = \frac{15}{\sin 83^\circ}$

10.4981

$\text{AC} = 10.5$ (cm) ***A1*** ***N2***

[3 marks]

(b) finding $\text{C}\hat{\text{A}}\text{B}$ (seen anywhere) *(A1)*

eg $180^\circ - 44^\circ - 83^\circ$, $\text{C}\hat{\text{A}}\text{B} = 53^\circ$

correct substitution for area of triangle ABC ***A1***

eg $\frac{1}{2} \times 15 \times 10.4981 \times \sin 53^\circ$

62.8813

area = 62.9 (cm^2) ***A1*** ***N2***

[3 marks]

(c) correct substitution for area of triangle DAC *(A1)*

eg $\frac{1}{2} \times 6 \times 10.4981 \times \sin \theta$

attempt to equate area of triangle ACD to half the area of triangle ABC *(M1)*

eg area $\text{ACD} = \frac{1}{2} \times$ area ABC; $2\text{ACD} = \text{ABC}$

correct equation ***A1***

eg $\frac{1}{2} \times 6 \times 10.4981 \times \sin \theta = \frac{1}{2}(62.9)$, $62.9887 \sin \theta = 62.8813$, $\sin \theta = 0.998294$

86.6531, 93.3468

$\theta = 86.7^\circ$, $\theta = 93.3^\circ$ ***A1A1*** ***N2***

[5 marks]

(d)

Note:	Note: If candidates use an acute angle from part (c) in the cosine rule , award ***M1A0A0*** in part (d).

evidence of choosing cosine rule *(M1)*

eg $\text{CD}^2 = \text{AD}^2 + \text{AC}^2 - 2 \times \text{AD} \times \text{AC} \times \cos \theta$

correct substitution into rhs *(A1)*

eg $\text{CD}^2 = 6^2 + 10.498^2 - 2(6)(10.498)\cos 93.336^\circ$

12.3921

12.4 (cm) ***A1*** ***N2***

[3 marks]

Total [14 marks]

Question 25

(a) valid approach (M1)

eg $70+(180-115),\ 360-(110+115)$

$\text{A}\hat{\text{B}}\text{C}=135°$ *A1* *N2*

[2 marks]

(b) choosing cosine rule (M1)

eg $c^2=a^2+b^2-2ab\cos C$

correct substitution into RHS (A1)

eg $5^2+8^2-2\times5\times8\cos135$

12.0651

12.1 (km) *A1* *N2*

[3 marks]

(c) correct substitution (**must** be into sine rule) *A1*

eg $\frac{\sin \text{A}\hat{\text{C}}\text{B}}{5}=\frac{\sin 135}{\text{AC}}$

17.0398

$\text{A}\hat{\text{C}}\text{B}=17.0$ *A1* *N1*

[2 marks]

Total [7 marks]

Question 26

(a) evidence of choosing sine rule (M1)

eg $\frac{a}{\sin A}=\frac{b}{\sin B}$

correct substitution (A1)

eg $\frac{a}{\sin 1.75}=\frac{7}{\sin 0.82}$

9.42069

$\text{BD}=9.42$ (cm) *A1* *N2*

[3 marks]

(b) evidence of choosing cosine rule (M1)

eg $\cos B=\frac{d^2+c^2-b^2}{2dc},\ a^2=b^2+c^2-2bc\cos B$

correct substitution (A1)

eg $\frac{8^2+9.42069^2-12^2}{2\times8\times9.42069},\ 144=64+\text{BD}^2-16\text{BD}\cos B$

1.51271

$\text{D}\hat{\text{B}}\text{C}=1.51$ (radians) (accept $86.7°$) *A1* *N2*

[3 marks]

Total [6 marks]

Question 27

(a) valid approach **(M1)**

eg $h(0)$, $-15\cos(1.2\times 0)+17$, $-15(1)+17$

$h(0)=2$ (m) **A1** **N2**

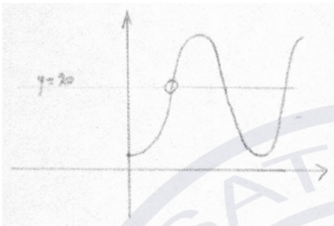
[2 marks]

(b) correct substitution into equation **(A1)**

eg $20=-15\cos 1.2t+17$, $-15\cos 1.2k=3$

valid attempt to solve for k **(M1)**

eg



, $\cos 1.2k=-\dfrac{3}{15}$

1.47679

$k=1.48$ **A1** **N2**

[3 marks]

(c) recognize the need to find the period (seen anywhere) **(M1)**

eg next t value when $h=20$

correct value for period **(A1)**

eg period $=\dfrac{2\pi}{1.2}$, 5.23598, 6.7–1.48

5.2 (min) (must be 1 dp) **A1** **N2**

[3 marks]

Total [8 marks]

Question 28

(a) $\theta = \frac{2\pi}{5}$ **A1** **N1**

[1 mark]

(b) correct expression for area **(A1)**

eg $A = \frac{1}{2}r^2\left(\frac{2\pi}{5}\right), \frac{\pi r^2}{5}$

evidence of equating their expression to 20π **(M1)**

eg $\frac{1}{2}r^2\left(\frac{2\pi}{5}\right) = 20\pi$, $r^2 = 100$, $r = \pm 10$

$r = 10$ **A1** **N2**

[3 marks]

(c) **METHOD 1**

evidence of choosing cosine rule **(M1)**

eg $a^2 = b^2 + c^2 - 2bc\cos A$

correct substitution of **their** r and θ into RHS **(A1)**

eg $10^2 + 10^2 - 2\times 10\times 10\cos\left(\frac{2\pi}{5}\right)$

11.7557

$\text{AB} = 11.8$ (mm) **A1** **N2**

METHOD 2

evidence of choosing sine rule **(M1)**

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$

correct substitution of **their** r and θ **(A1)**

eg $\frac{\sin\frac{2\pi}{5}}{\text{AB}} = \frac{\sin\left(\frac{1}{2}\left(\pi - \frac{2\pi}{5}\right)\right)}{10}$

11.7557

$\text{AB} = 11.8$ (mm) **A1** **N2**

[3 marks]

[Total 7 marks]

Question 29

(a)	(i)	valid approach	*(M1)*	
		eg $\frac{5+17}{2}$		
		$c=11$	***A1***	***N2***
	(ii)	valid approach	*(M1)*	
		eg period is 12, per $=\frac{2\pi}{b}$, $9-3$		
		$b=\frac{2\pi}{12}$	***A1***	
		$b=\frac{\pi}{6}$	***AG***	***N0***
	(iii)	**METHOD 1**		
		valid approach	*(M1)*	
		eg $5=a\sin\left(\frac{\pi}{6}\times 3\right)+11$, substitution of points		
		$a=-6$	***A1***	***N2***
		METHOD 2		
		valid approach	*(M1)*	
		eg $\frac{17-5}{2}$, amplitude is 6		
		$a=-6$	***A1***	***N2***
				[6 marks]
(b)	(i)	$k=2.5$	***A1***	***N1***
	(ii)	$g(x)=-6\sin\left(\frac{\pi}{6}(x-2.5)\right)+11$	***A2***	***N2***
				[3 marks]

(c) (i) **METHOD 1** Using g

recognizing that a point of inflexion is required **_M1_**
eg sketch, recognizing change in concavity

evidence of valid approach **_(M1)_**
eg $g''(x) = 0$, sketch, coordinates of max/min on g'

$w = 8.5$ (exact) **_A1_** **_N2_**

METHOD 2 Using f

recognizing that a point of inflexion is required **_M1_**
eg sketch, recognizing change in concavity

evidence of valid approach involving translation **_(M1)_**
eg $x = w - k$, sketch, $6 + 2.5$

$w = 8.5$ (exact) **_A1_** **_N2_**

(ii) valid approach involving the derivative of g or f (seen anywhere) **_(M1)_**
eg $g'(w)$, $-\pi\cos\left(\frac{\pi}{6}x\right)$, max on derivative, sketch of derivative

attempt to find max value on derivative **_M1_**
eg $-\pi\cos\left(\frac{\pi}{6}(8.5 - 2.5)\right)$, $f'(6)$, dot on max of sketch

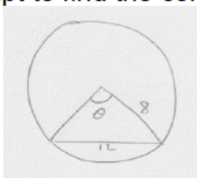
3.14159
max rate of change $= \pi$ (exact), 3.14 **_A1_** **_N2_**

[6 marks]

[Total 15 marks]

Question 30

attempt to find the central angle or half central angle **(M1)**

eg 

, cosine rule, right triangle

correct working **(A1)**

eg $\cos\theta = \dfrac{8^2 + 8^2 - 12^2}{2 \cdot 8 \cdot 8}$, $\sin^{-1}\left(\dfrac{6}{8}\right)$, 0.722734, $41.4096°$, $\dfrac{\pi}{2} - \sin^{-1}\left(\dfrac{6}{8}\right)$

correct angle $\mathrm{A\hat{O}B}$ (seen anywhere)

eg 1.69612, $97.1807°$, $2 \times \sin^{-1}\left(\dfrac{6}{8}\right)$ **(A1)**

correct sector area

eg $\dfrac{1}{2}(8)(8)(1.70)$, $\dfrac{97.1807}{360}(64\pi)$, 54.2759 **(A1)**

area of triangle (seen anywhere) **(A1)**

eg $\dfrac{1}{2}(8)(8)\sin 1.70$, $\dfrac{1}{2}(8)(12)\sin 0.722$, $\dfrac{1}{2} \times \sqrt{64 - 36} \times 12$, 31.7490

appropriate approach (seen anywhere) **(M1)**

eg $A_{\text{triangle}} - A_{\text{sector}}$, their sector-their triangle

22.5269

area of shaded region $= 22.5\ (\text{cm}^2)$ **A1** **N4**

Question 31

(a) (i) attempt to find the difference of x-values of A and B *(M1)*
eg $6.25–12.5$

6.25 (hours), (6 hours 15 minutes) **A1** **N2**

(ii) attempt to find the difference of y-values of A and B *(M1)*
eg $1.5–0.6$

0.9 (m) **A1** **N2**

[4 marks]

(b) (i) valid approach *(M1)*
eg $\frac{\max-\min}{2}$, $0.9 \div 2$

$p = 0.45$ **A1** **N2**

(ii) **METHOD 1**
period = 12.5 (seen anywhere) *(A1)*

valid approach (seen anywhere) *(M1)*
eg $\text{period} = \frac{2\pi}{b}$, $q = \frac{2\pi}{\text{period}}$, $\frac{2\pi}{12.5}$

0.502654

$q = \frac{4\pi}{25}$, 0.503 $\left(\text{or } -\frac{4\pi}{25}, -0.503\right)$ **A1** **N2**

METHOD 2
attempt to use a coordinate to make an equation *(M1)*
e.g. $p\cos(6.25q) + r = 0.6$, $p\cos(12.5q) + r = 1.5$

correct substitution *(A1)*
eg $0.45\cos(6.25q) + 1.05 = 0.6$, $0.45\cos(12.5q) + 1.05 = 1.5$

0.502654

$q = \frac{4\pi}{25}$, 0.503 $\left(\text{or } -\frac{4\pi}{25}, -0.503\right)$ **A1** **N2**

(iii) valid method to find r *(M1)*
eg $\frac{\max+\min}{2}$, $0.6 + 0.45$

$r = 1.05$ **A1** **N2**

[7 marks]

(c) **METHOD 1**

attempt to find start or end t-values for 12 December **(M1)**

eg $3 + 24,\ t = 27,\ t = 51$

finds t-value for second max **(A1)**

$t = 50$

23:00 (or 11 pm) **A1** **N3**

METHOD 2

valid approach to list either the times of high tides after 21:00 or the t-values of high tides after 21:00, showing at least two times **(M1)**

eg $21{:}00 + 12.5,\ 21{:}00 + 25,\ 12.5 + 12.5,\ 25 + 12.5$

correct time of first high tide on 12 December **(A1)**

eg 10:30 (or 10:30 am)

time of second high tide = 23:00 **A1** **N3**

METHOD 3

attempt to set **their** h equal to 1.5 **(M1)**

eg $h(t) = 1.5,\ 0.45\cos\left(\frac{4\pi}{25}t\right) + 1.05 = 1.5$

correct working to find second max **(A1)**

eg $0.503t = 8\pi,\ t = 50$

23:00 (or 11 pm) **A1** **N3**

[3 marks]

Total [14 marks]

Question 32

(a) correct substitution into arc length formula **(A1)**

eg $(40)(1.9)$

arc length $= 76$ (cm) **A1** **N2**

[2 marks]

(b) valid approach **(M1)**

eg arc $+ 2r$, $76 + 40 + 40$

perimeter $= 156$ (cm) **A1** **N2**

[2 marks]

(c) correct substitution into area formula **(A1)**

eg $\frac{1}{2}(1.9)(40)^2$

area $= 1520$ (cm^2) **A1** **N2**

[2 marks]

[Total 6 marks]

Question 33

(a) valid approach *(M1)*

eg $\frac{\text{max} - \text{min}}{2}$, sketch of graph, $9.7 = p\cos(0) + 7.5$

$p = 2.2$ **A1** **N2**

[2 marks]

(b) valid approach *(M1)*

eg $B = \frac{2\pi}{\text{period}}$, period is 14, $\frac{360}{14}$, $5.3 = 2.2\cos 7q + 7.5$

0.448798

$q = \frac{2\pi}{14}\left(\frac{\pi}{7}\right)$, 0.449 (do not accept degrees) **A1** **N2**

[2 marks]

(c) valid approach *(M1)*

eg $d(10)$, $2.2\cos\left(\frac{20\pi}{14}\right) + 7.5$

7.01045

7.01 (m) **A1** **N2**

[2 marks]

[Total 6 marks]

Question 34

(a) valid method **(M1)**

eg $180+55$, $360-90-35$

235° (accept S55W, W35S) **A1** **N2**

[2 marks]

(b) valid approach to find $\hat{AEC}$ (may be seen in (a)) **(M1)**

eg $\hat{AEC}=180-55-\hat{ACE}$, $134=E+55$

correct working to find $\hat{AEC}$ (may be seen in (a)) **(A1)**

eg $180-55-46$, $134-55$, $\hat{AEC}=79^{\circ}$

evidence of choosing sine rule (seen anywhere) **(M1)**

eg $\frac{a}{\sin A}=\frac{b}{\sin B}$

correct substitution into sine rule **(A1)**

eg $\frac{CE}{\sin 55^{\circ}}=\frac{175}{\sin \hat{AEC}}$

146.034

$CE=146$ (km) **A1** **N2**

[5 marks]

(c) evidence of choosing cosine rule **(M1)**

eg $DE^2=DC^2+CE^2-2\times DC\times CE\times\cos\theta$

correct substitution into right-hand side **(A1)**

eg $60^2+146.034^2-2\times60\times146.034\cos134$

192.612

$DE=193$ (km) **A1** **N2**

[3 marks]

(d) valid approach for locating B **(M1)**

eg BE is perpendicular to ship's path, angle $B=90$

correct working for BE **(A1)**

eg $\sin 46^{\circ}=\frac{BE}{146.034}$, $BE=146.034\sin 46^{\circ}$, 105.048

valid approach for expressing time **(M1)**

eg $t=\frac{d}{s}$, $t=\frac{d}{r}$, $t=\frac{192.612}{50}$

correct working equating time **(A1)**

eg $\frac{146.034\sin 46^{\circ}}{r}=\frac{192.612}{50}$, $\frac{s}{105.048}=\frac{50}{192.612}$

27.2694

27.3 (km per hour) **A1** **N3**

[5 marks]

[Total 15 marks]

Question 35

(a) evidence of choosing sine rule **(M1)**

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$

correct substitution **(A1)**

eg $\frac{\text{BC}}{\sin 50} = \frac{5}{\sin 112}$

4.13102

$\text{BC} = 4.13 \text{ (cm)}$ **A1** **N2**

[3 marks]

(b) correct working **(A1)**

eg $\hat{B} = 180 - 50 - 112$, $18°$, $\text{AC} = 1.66642$

correct substitution into area formula **(A1)**

eg $\frac{1}{2} \times 5 \times 4.13 \times \sin 18$, $0.5(5)(1.66642)\sin 50$, $\frac{1}{2}(4.13)(1.66642)\sin 112$

3.19139

area $= 3.19 \text{ (cm}^2\text{)}$ **A1** **N2**

[3 marks]

Total [6 marks]

Question 36

(a) substituting $x = 2\pi$ — *M1*

eg $2\pi + a\sin\left(2\pi - \frac{\pi}{2}\right) + a$

$2\pi + a\sin\left(\frac{3\pi}{2}\right) + a$ — *(A1)*

$2\pi - a + a$ — *A1*

$f(2\pi) = 2\pi$ — *AG* *N0*

[3 marks]

(b) (i) substituting the value of k — *(M1)*

$P_0(0, 0)$, $P_1(2\pi, 2\pi)$ — *A1A1* *N3*

(ii) attempt to find the gradient — *(M1)*

eg $\frac{2\pi - 0}{2\pi - 0}$, $m = 1$

correct working — *(A1)*

eg $\frac{y - 2\pi}{x - 2\pi} = 1$, $b = 0$, $y - 0 = 1(x - 0)$

$y = x$ — *A1* *N3*

[6 marks]

(c) subtracting x-coordinates of P_{k+1} and P_k (in any order) — *(M1)*

eg $2(k+1)\pi - 2k\pi$, $2k\pi - 2k\pi - 2\pi$

correct working (must be in correct order) — *A1*

eg $2k\pi + 2\pi - 2k\pi$, $\left|2k\pi - 2(k+1)\pi\right|$

distance is 2π — *AG* *N0*

[2 marks]

(d) **METHOD 1**

recognizing the toothed-edge as the hypotenuse *(M1)*

eg $300^2 = x^2 + y^2$, sketch

correct working (using their equation of L) *(A1)*

eg $300^2 = x^2 + x^2$

$x = \frac{300}{\sqrt{2}}$ (exact), 212.132 *(A1)*

dividing their value of x by 2π $\left(\text{do not accept } \frac{300}{2\pi}\right)$ *(M1)*

eg $\frac{212.132}{2\pi}$

33.7618 *(A1)*

33 (teeth) ***A1*** ***N2***

METHOD 2

vertical distance of a tooth is 2π (may be seen anywhere) *(A1)*

attempt to find the hypotenuse for one tooth *(M1)*

eg $x^2 = (2\pi)^2 + (2\pi)^2$

$x = \sqrt{8\pi^2}$ (exact), 8.88576 *(A1)*

dividing 300 by their value of x *(M1)*

eg

33.7618 *(A1)*

33 (teeth) ***A1*** ***N2***

[6 marks]

Total [17 marks]

Question 37

(a) valid approach to find k ***(M1)***

eg 8 minutes is half a turn, $k+$ diameter, $k+111=117$

$k=6$ ***A1*** ***N2***

[2 marks]

(b) **METHOD 1**

valid approach ***(M1)***

eg $\frac{\max - \min}{2}$, $a=$ radius

$|a|=\frac{117-6}{2}, 55.5$ ***(A1)***

$a=-55.5$ ***A1*** ***N2***

METHOD 2

attempt to substitute valid point into equation for f ***(M1)***

eg $h(0)=6,\ h(8)=117$

correct equation ***(A1)***

eg $6=61.5+a\cos\left(\frac{\pi}{8}\times 0\right),\ 117=61.5+a\cos\left(\frac{\pi}{8}\times 8\right),\ 6=61.5+a$

$a=-55.5$ ***A1*** ***N2***

[3 marks]

(c) valid approach ***(M1)***

eg sketch of h and $y=30$, $h=30$, $61.5-55.5\cos\left(\frac{\pi}{8}t\right)=30$, $t=2.46307$, $t=13.5369$

18.4630

$t=18.5$ (minutes) ***A2*** ***N3***

[3 marks]

[Total: 8 marks]

Question 38

(a) evidence of choosing sine rule **(M1)**

eg $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

correct substitution **(A1)**

eg $\frac{\text{DB}}{\sin 59^\circ} = \frac{11}{\sin 100^\circ}$

9.57429

$\text{DB} = 9.57$ (cm) **A1** **N2**

[3 marks]

(b) evidence of choosing cosine rule **(M1)**

eg $a^2 = b^2 + c^2 - 2bc\cos(A)$, $\text{DC}^2 = \text{DB}^2 + \text{BC}^2 - 2\text{DB} \times \text{BC} \times \cos(\text{D}\hat{\text{B}}\text{C})$

correct substitution into RHS **(A1)**

eg $9.57^2 + 6^2 - 2 \times 9.57 \times 6 \times \cos 82^\circ$, 111.677

10.5677

$\text{DC} = 10.6$ (cm) **A1** **N2**

[3 marks]

[Total: 6 marks]

Question 39

(a) −0.394791, 13

A(−0.395, 13) **A1A1** **N2**

[2 marks]

(b) (i) 13 **A1** **N1**

(ii) 2π, 6.28 **A1** **N1**

[2 marks]

(c) valid approach **(M1)**

eg recognizing that amplitude is p or shift is r

$f(x) = 13\cos(x + 0.395)$ (accept $p = 13$, $r = 0.395$) **A1A1** **N3**

Note: Accept any value of r of the form $0.395 + 2\pi k$, $k \in \mathbb{Z}$

[3 marks]

Question 40

correct substitution into the formula for area of a triangle (A1)

eg $15 = \frac{1}{2} \times 8.1 \times 12.3 \times \sin C$

correct working for angle C (A1)

eg $\sin C = 0.301114$, $17.5245\ldots$, 0.305860

recognizing that obtuse angle needed (M1)

eg 162.475, 2.83573, $\cos C < 0$

evidence of choosing the cosine rule (M1)

eg $a^2 = b^2 + c^2 - 2bc\cos(A)$

correct substitution into cosine rule to find c (A1)

eg $c^2 = (8.1)^2 + (12.3)^2 - 2(8.1)(12.3)\cos C$

$c = 20.1720$ (A1)

$8.1 + 12.3 + 20.1720 = 40.5720$

perimeter $= 40.6$ A1 N4

Total [7 marks]

Question 41

(a) valid approach to find area of segment (M1)

eg area of sector – area of triangle, $\frac{1}{2}r^2(\theta - \sin\theta)$

correct substitution (A1)

eg $\frac{1}{2}(4)^2\theta - \frac{1}{2}(4)^2\sin\theta$, $\frac{1}{2}\times 16[\theta - \sin\theta]$

area $= 8\theta - 8\sin\theta$, $8(\theta - \sin\theta)$ A1 N2

[3 marks]

(b) setting **their** area expression equal to 12 (M1)

eg $12 = 8(\theta - \sin\theta)$

2.26717

$\theta = 2.27$ (do not accept an answer in degrees) A2 N3

[3 marks]

Total [6 marks]

Question 42

(a) recognizing $\text{TR} = 32$ (seen anywhere, including diagram) ***A1***

correct working ***A1***

eg $32^2 = x^2 + 38^2 - 2(x)(38)\cos 43^\circ$, $1024 = 1444 + x^2 - 76(x)\cos 43^\circ$

$x^2 - (76\cos 43^\circ)x + 420 = 0$ ***AG*** ***N0***

[2 marks]

(b)

Note: There are many approaches to this question, depending on which triangle the candidate has used, and whether they used the cosine rule and/or the sine rule. Please check working carefully and award marks in line with the markscheme.

METHOD 1

correct values for x (seen anywhere) ***A1A1***

$x = 9.02007$, 46.5628

recognizing the need to find difference in values of x ***(M1)***

eg $46.5 - 9.02$, $x_1 - x_2$

37.5427

37.5 (km) ***A1*** ***N2***

METHOD 2

correct use of sine rule in ΔSRT

eg $\frac{\sin \text{S}\hat{\text{R}}\text{T}}{38} = \frac{\sin 43^\circ}{32}$, $\text{S}\hat{\text{R}}\text{T} = 54.0835^\circ$ ***(A1)***

recognizing isosceles triangle (seen anywhere) ***(M1)***

eg $\hat{T} = 180^\circ - 2 \cdot 54.0835^\circ$, two sides of 32

correct working to find distance ***A1***

eg $\sqrt{32^2 + 32^2 - 2 \cdot 32 \cdot 32\cos(180^\circ - 2 \cdot 54.0835^\circ)}$,

$\frac{\sin 71.8329^\circ}{d} = \frac{\sin 54.0835^\circ}{32}$, $32^2 = 32^2 + x^2 - 2 \cdot 32x\cos(0.944)$

37.5427

37.5 (km) ***A1*** ***N2***

[4 marks]

Total [6 marks]

Question 43

(a) $\cos\theta = \frac{\text{OC}}{r}$ **A1**

$\text{OC} = r\cos\theta$ **AG** **N0**

[1 mark]

(b) valid approach **(M1)**

eg $\frac{1}{2}\text{OC}\times\text{OB}\sin\theta$, $\text{BC} = r\sin\theta$, $\frac{1}{2}r\cos\theta\times\text{BC}$, $\frac{1}{2}r\sin\theta\times\text{OC}$

area $= \frac{1}{2}r^2\sin\theta\cos\theta$ $\left(= \frac{1}{4}r^2\sin(2\theta)\right)$ (must be in terms of r and θ) **A1** **N2**

[2 marks]

(c) valid attempt to express the relationship between the areas (seen anywhere) **(M1)**

eg $\text{OCB} = \frac{3}{5}\text{OBA}$, $\frac{1}{2}r^2\sin\theta\cos\theta = \frac{3}{5}\times\frac{1}{2}r^2\theta$, $\frac{1}{4}r^2\sin 2\theta = \frac{3}{10}r^2\theta$

correct equation in terms of θ only **A1**

eg $\sin\theta\cos\theta = \frac{3}{5}\theta$, $\frac{1}{4}\sin 2\theta = \frac{3}{10}\theta$

valid attempt to solve **their** equation **(M1)**

eg sketch, -0.830017 , 0

0.830017

$\theta = 0.830$ **A1** **N2**

Note: Do not award final ***A1*** if additional answers given.

[4 marks]

Total [7 marks]

Question 44

(a) choosing cosine rule **(M1)**

eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution into RHS **(A1)**

eg $4.83^2 + 3.80^2 - 2\times 4.83\times 3.80\times\cos 78.2$, 30.2622,
$4.83^2 + 3.80^2 - 2(4.83)(3.80)\cos 1.36$

5.50111

5.50 (cm) **A1** **N2**

[3 marks]

(b) correct substitution for area of triangle ABD **(A1)**

eg $\frac{1}{2}\times 6.73\times 5.50111\sin\theta$

correct equation **A1**

eg $\frac{1}{2}\times 6.73\times 5.50111\sin\theta = 18.5$, $\sin\theta = 0.999393$

$88.0023, 91.9976, 1.53593, 1.60566$

$\theta = 88.0$ (degrees) or 1.54 (radians)

$\theta = 92.0$ (degrees) or 1.61 (radians)

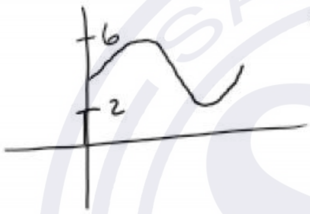
A1A1 **N2**

[4 marks]

Total [7 marks]

Question 45

(a) valid attempt to find range **(M1)**

eg 

, max $= 6$ min $= 2$,

$2\sin\left(3\times\frac{\pi}{6}\right)+4$ and $2\sin\left(3\times\frac{\pi}{2}\right)+4$, $2(1)+4$ and $2(-1)+4$,

$k = 2,\ m = 6$ **A1A1** **N3**

[3 marks]

(b) $10 \le y \le 30$ **A2** **N2**

[2 marks]

(c) (i) evidence of substitution (may be seen in part (b)) **(M1)**

eg $5\left(2\sin(3(2x))+4\right)$, $3(2x)$

$b = 6,\ c = 20$ (accept $10\sin(6x)+20$) **A1A1** **N3**

Note: If no working shown, award ***N2*** for one correct value.

(ii) correct working **(A1)**

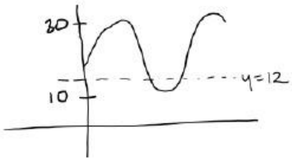
eg $\frac{2\pi}{b}$

1.04719

$\frac{2\pi}{6}\ \left(=\frac{\pi}{3}\right)$, 1.05 **A1** **N2**

[5 marks]

(d) valid approach *(M1)*

eg 

, $\sin^{-1}\left(-\frac{8}{10}\right)$, $6x = -0.927$, -0.154549, $x = 0.678147$

Note: Award ***M1*** for any correct value for x or $6x$ which lies outside the domain of f.

3.81974, 4.03424

$x = 3.82$, $x = 4.03$ (do not accept answers in degrees) ***A1A1*** ***N3***

Total [13 marks]

Question 46

evidence of choosing cosine rule *(M1)*

eg $a^2 = b^2 + c^2 - 2bc\cos A$

correct substitution to find AB *(A1)*

eg $28.4^2 = x^2 + (x+2)^2 - 2x(x+2)\cos(0.667)$

$x = 42.2822$ ***A2***

appropriate approach to find AD *(M1)*

eg $\text{AD} = x\cos(0.611)$, $\cos(0.611) = \frac{\text{AD}}{42.2822}$

34.6322

$\text{AD} = 34.6$ ***A1*** ***N3***

Total [6 marks]

Question 47

(a) correct working *(A1)*

eg $\sin\alpha = \frac{8}{10}$, $\cos\theta = \frac{6}{10}$, $\cos B\hat{A}C = \frac{6^2 + 10^2 - 8^2}{2 \times 6 \times 10}$

0.927295

$B\hat{A}C = 0.927 \quad (= 53.1^\circ)$ *(A1)* **N2**

[2 marks]

(b)

Note: There may be slight differences in the final answer, depending on the approach the candidate uses in part (b). Accept a final answer that is consistent with their working.

correct area of sector ABF (seen anywhere) *(A1)*

eg $\frac{1}{2} \times 6^2 \times 0.927$, $\frac{53.1301^\circ}{360^\circ} \times \pi \times 6^2$, 16.6913

correct expression (or value) for either [AD] or [BD] (seen anywhere) *(A1)*

eg $AD = 6\cos(B\hat{A}C)$ (=3.6)

$BD = 6\sin(53.1^\circ)$ (=4.8)

correct area of triangle ABD (seen anywhere) *(A1)*

eg $\frac{1}{2} \times 6\cos B\hat{A}D \times 6\sin B\hat{A}D$, $9\sin(2B\hat{A}C)$, 8.64 (exact)

appropriate approach (seen anywhere) *(M1)*

eg $A_{\text{triangle ABD}} - A_{\text{sector}}$, their sector - their triangle ABD

8.05131

area of shaded region = 8.05 (cm^2) **A1** **N2**

[5 marks]

Total [7 marks]