Subject – Math (Standard Level)
Topic - Statistics and Probability
Year - Nov 2011 – Nov 2019
Paper -2

Question 1

(a) median = 174 (cm) *A1* *N1*
[1 mark]

(b) attempt to find number shorter than 161 *(M1)*
e.g. line on graph, 12 boys

$p = \frac{12}{200}$ $(= 0.06)$ *A1* *N2*
[2 marks]

(c) **METHOD 1**

18 % have a height less than h *(A1)*

$0.18 \times 200 = 36$ (36 may be seen as a line on the graph) *(A1)*

$h = 166$ (cm) *A1* *N2*
[3 marks]

METHOD 2

$0.82 \times 200 = 164$ (164 may be seen as a line on the graph) *(A1)*

$200 - 164 = 36$ *(A1)*

$h = 166$ (cm) *A1* *N2*
[3 marks]

Total [6 marks]

Question 2

Note: There may be slight differences in answers, depending on whether candidates use tables or GDCs, or their 3 sf answers in subsequent parts. Do not penalise answers that are consistent with **their** working and check carefully for ***FT***.

(a) evidence of recognizing binomial (seen anywhere in the question) ***(M1)***
e.g. ${}_nC_r\, p^r\, q^{n-r}$, B(n, p), ${}^{10}C_1\,(0.012)^1\,(0.988)^9$

$p = 0.108$ ***A1*** ***N2***

[2 marks]

(b) valid approach ***(M1)***
e.g. P$(X \leq 1)$, $0.88627\ldots + 0.10764\ldots$

$p = 0.994$ ***A1*** ***N2***

[2 marks]

(c) (i)

A1A1 ***N2***

Note: Award ***A1*** for vertical line to right of mean,
A1 for shading to left of **their** vertical line.

(ii) valid approach ***(M1)***
e.g. P$(X < 22.63)$

working to find standardized value ***(A1)***
e.g. $\dfrac{22.63 - 22}{0.3}$, 2.1

$p = 0.982$ ***A1*** ***N3***

[5 marks]

Question 3

(a) evidence of recognizing binomial distribution **(M1)**

e.g. $X \sim \text{B}(10, 0.57)$, $p = 0.57$, $q = 0.43$

EITHER

$\text{P}(X \leq 3) = 2.16 \times 10^{-4} + 0.00286 + 0.01709 + 0.06041 \quad (= 0.08057\ldots)$ **(A1)**

evidence of using complement **(M1)**

e.g. 1 – any probability, $\text{P}(X \geq 4) = 1 - \text{P}(X \leq 3)$

0.919423...

$\text{P}(X \geq 4) = 0.919$ **A1** **N3**

OR

summing the probabilities from $X = 4$ to $X = 10$ **(M1)**

correct expression or values **(A1)**

e.g. $\sum_{r=4}^{10} \binom{10}{r} (0.57)^r (0.43)^{10-r}$, $0.14013 + 0.2229 + \ldots + 0.02731 + 0.00362$

0.919424

$\text{P}(X \geq 4) = 0.919$ **A1** **N3**

[4 marks]

(b) evidence of valid approach **(M1)**

e.g. three tails in nine tosses, $\binom{9}{3} (0.57)^3 (0.43)^6$

correct calculation

e.g. $\binom{9}{3} (0.57)^3 (0.43)^6 \times 0.57$, 0.09834×0.57 **(A1)**

0.05605178…

$\text{P}(4^{\text{th}} \text{ tail on } 10^{\text{th}} \text{ toss}) = 0.0561$ **A1** **N2**

[3 marks]

Total [7 marks]

Question 4

(a) (i) $p = 17,\ q = 11$ **A1A1** **N2**

(ii) $75 \le T < 85$ **A1** **N1**

[3 marks]

(b) evidence of valid approach **(M1)**
e.g. adding frequencies

$\frac{76}{93} = 0.8172043\ldots$

$\mathrm{P}(T < 95) = \frac{76}{93} = 0.817$ **A1** **N2**

[2 marks]

(c) (i) 10 **A1** **N1**

(ii) 50 **A1** **N1**

[2 marks]

(d) (i) evidence of approach using mid-interval values (may be seen in part (ii)) **(M1)**
79.1397849
$\bar{x} = 79.1$ **A2** **N3**

(ii) 16.4386061
$\sigma = 16.4$ **A1** **N1**

[4 marks]

(e) evidence of valid approach **(M1)**
e.g. standardizing, $z = 0.9648...$

0.8326812
$\mathrm{P}(T < 95) = 0.833$ **A1** **N2**

[2 marks]

Total [13 marks]

Question 5

(a) evidence of appropriate method ***(M1)***

e.g. $z = \frac{122.5 - 117}{5}$, sketch of normal curve showing mean and 122.5, 1.1

$P(Z < 1.1) = 0.8643$ ***(A1)***

0.135666

P (H>122.5) = 0.136 ***A1*** ***N3***

[3 marks]

(b) $z = 0.3853$ ***(A1)***

set up equation ***(M1)***

e.g. $\frac{X - 117}{5} = 0.3853$, sketch

$k = 118.926602$

$k = 199$ ***A1*** ***N3***

[3 marks]

Total [6 marks]

Question 6

(a) evidence of recognizing binomial (seen anywhere) ***(M1)***
e.g. $B(n, p)$, 0.95^{30}

finding $P(X = 0) = 0.21463876$ ***(A1)***

appropriate approach ***(M1)***
e.g. complement, summing probabilities

0.785361
probability is 0.785 ***A1*** ***N3***
[4 marks]

(b) identifying correct outcomes (seen anywhere) ***(A1)***
e.g. $P(X = 1) + P(X = 2)$, 1 or 2 defective, $0.3389\ldots + 0.2586\ldots$

recognizing conditional probability (seen anywhere) ***R1***
e.g. $P(A|B)$, $P(X \leq 2|X \geq 1)$, P(at most 2|at least 1)

appropriate approach involving conditional probability ***(M1)***
e.g. $\frac{P(X = 1) + P(X = 2)}{P(X \geq 1)}$, $\frac{0.3389\ldots + 0.2586\ldots}{0.785\ldots}$, $\frac{1 \text{ or } 2}{0.785}$

0.760847
probability is 0.761 ***A1*** ***N2***
[4 marks]

Total [8 marks]

Question 7

correct z-values ***(A1)(A1)***
$-1.750686\ldots$, $0.524400\ldots$

attempt to set up **their** equations, must involve z-values, not % ***(M1)***
e.g. one correct equation

two correct equations ***A1A1***
e.g. $\mu - 1.750686\sigma = 5$, $0.5244005 = \frac{25 - \mu}{\sigma}$

attempt to solve **their** equations ***(M1)***
e.g. substitution, matrices, one correct value

$\mu = 20.39006\ldots$, $\sigma = 8.790874\ldots$

$\mu = 20.4\ [20.3, 20.4]$, $\sigma = 8.79\ [8.79, 8.80]$ ***A1A1*** ***N4***
[8 marks]

Question 8

(a) valid approach *(M1)*
e.g. Venn diagram with intersection, union formula,
$\mathrm{P}(S \cap F) = 0.75 + 0.40 - 1$

15 (accept 15 %) *A1* *N2*
[2 marks]

(b) valid approach involving subtraction *(M1)*
e.g. Venn diagram, 75 – 15

60 (accept 60 %) *A1* *N2*
[2 marks]

(c) (i) valid approach *(M1)*
e.g. tree diagram, multiplying probabilities, $\mathrm{P}(S \mid G) \times \mathrm{P}(G)$

correct calculation *(A1)*
e.g. 0.52×0.85

$\mathrm{P}(G \cap S) = 0.442$ (exact) *A1* *N3*

(ii) valid reasoning, with words, symbols or numbers (seen anywhere) *R1*
e.g. $\mathrm{P}(G) \times \mathrm{P}(S) \neq \mathrm{P}(G \cap S)$, $\mathrm{P}(S \mid G) \neq \mathrm{P}(S)$, not equal,

one correct value *A1*
e.g. $\mathrm{P}(G) \times \mathrm{P}(S) = 0.39$, $P(S|G) = 0.85$, $0.39 \neq 0.442$

G and S are not independent *AG* *N0*
[5 marks]

(d) **METHOD 1**

48 % are boys (seen anywhere) *A1*
e.g. $\mathrm{P}(B) = 0.48$

appropriate approach *(M1)*
e.g. P(girl and Spanish) + P(boy and Spanish) = P(Spanish)

correct approach to find P(boy and Spanish) *(A1)*
e.g. $\mathrm{P}(B \cap S) = \mathrm{P}(S) - \mathrm{P}(G \cap S)$, $\mathrm{P}(B \cap S) = \mathrm{P}(S \mid B) \times \mathrm{P}(B)$, 0.308

correct substitution *(A1)*
e.g. $0.442 + 0.48x = 0.75$, $0.48x = 0.308$

correct manipulation *(A1)*
e.g. $\mathrm{P}(S \mid B) = \dfrac{0.308}{0.48}$

$\mathrm{P}(\text{Spanish} \mid \text{boy}) = 0.641666\ldots, 0.641\overline{6}$

$\mathrm{P}(\text{Spanish} \mid \text{boy}) = 0.642$ [0.641, 0.642] *A1* *N3*
[6 marks]

METHOD 2

48 % are boys (seen anywhere) ***A1***
e.g. 0.48 used in tree diagram

appropriate approach ***(M1)***
e.g. tree diagram

correctly labelled branches on tree diagram ***(A1)***
e.g. first branches are boy/girl, second branches are Spanish/not Spanish

correct substitution ***(A1)***
e.g. $0.442 + 0.48x = 0.75$

correct manipulation ***(A1)***
e.g. $0.48x = 0.308$, $\mathrm{P}(S \mid B) = \dfrac{0.308}{0.48}$

$\mathrm{P}(\text{Spanish} \mid \text{boy}) = 0.641666\ldots, 0.641\overline{6}$

$\mathrm{P}(\text{Spanish} \mid \text{boy}) = 0.642\ [0.641, 0.642]$ ***A1*** ***N3***

[6 marks]

Total [15 marks]

Question 9

(a) valid approach ***(M1)***
eg $35 - 26$, $26 + p = 35$

$p = 9$ ***A1*** ***N2***

[2 marks]

(b) (i) mean $= 26.7$ ***A2*** ***N2***

(ii) recognizing that variance is $(\text{sd})^2$ ***(M1)***
eg $11.021\ldots^2$, $\sigma = \sqrt{\text{var}}$, $11.158\ldots^2$

$\sigma^2 = 121$ ***A1*** ***N2***

[4 marks]

Total [6 marks]

Question 10

recognizing one quartile probability (may be seen in a sketch) ***(M1)***

eg $P(X < Q_3) = 0.75, 0.25$

finding standardized value for either quartile ***(A1)***

eg $z = 0.67448\ldots, z = -0.67448\ldots$

attempt to set up equation (must be with z – values) ***(M1)***

eg $0.67 = \frac{Q_3 - 150}{10}, -0.67448 = \frac{x - 150}{10}$

one correct quartile

eg $Q_3 = 156.74\ldots, Q_1 = 143.25\ldots$ ***(A1)***

correct working ***(A1)***

eg other correct quartile, $Q_3 - \mu = 6.744\ldots$

valid approach for IQR (seen anywhere) ***(A1)***

eg $Q_3 - Q_1, 2(Q_3 - \mu)$

$\text{IQR} = 13.5$ ***A1*** ***N4***

[7 marks]

Question 11

(a) evidence of appropriate approach ***(M1)***

eg $z = \frac{22.9 - 20}{5}$

$z = 0.58$ ***(A1)***

$P(X \leq 22.9) = 0.719$ ***A1*** ***N3***

[3 marks]

(b) z-score for 0.55 is 0.12566… ***(A1)***

valid approach (must be with z – values) ***(M1)***

eg using inverse normal, $0.1257 = \frac{k - 20}{5}$

$k = 20.6$ ***A1*** ***N3***

[3 marks]

Total [6 marks]

Question 12

(a) **METHOD 1**

(i) appropriate approach *(M1)*

eg $\frac{6}{10}\times\frac{6}{10}$, $\frac{6}{10}\times\frac{5}{9}$, $\frac{6}{10}\times\frac{5}{10}$

$P(X=0)=\frac{9}{25}=0.36$ *A1* *N2*

(ii) multiplying one pair of gold and silver probabilities *(M1)*

eg $\frac{6}{10}\times\frac{4}{10}$, $\frac{6}{10}\times\frac{4}{9}$, 0.24

adding the product of both pairs of gold and silver probabilities *(M1)*

eg $\frac{6}{10}\times\frac{4}{10}\times 2$, $\frac{6}{10}\times\frac{4}{9}+\frac{4}{10}\times\frac{6}{9}$

$P(X=1)=\frac{12}{25}=0.48$ *A1* *N3*

(iii) $P(X=2)=0.16$ (seen anywhere) *(A1)*

correct substitution into formula for $E(X)$ *(A1)*

eg $0\times 0.36+1\times 0.48+2\times 0.16$, $0.48+0.32$

$E(X)=\frac{4}{5}=0.8$ *A1* *N3*

[8 marks]

METHOD 2

(i) evidence of recognizing binomial (may be seen in part (ii)) *(M1)*

eg $X\sim B(2,0.6)$, $\binom{2}{0}(0.4)^2(0.6)^0$

correct probability for use in binomial *(A1)*

eg $p=0.4$, $X\sim B(2,0.4)$, ${}^2C_0(0.4)^0(0.6)^2$

$P(X=0)=\frac{9}{25}=0.36$ *A1* *N3*

(ii) correct set up *(A1)*

eg ${}_2C_1(0.4)^1(0.6)^1$

$P(X=1)=\frac{12}{25}=0.48$ *A1* *N2*

(iii) attempt to substitute into np ***(M1)***

eg 2×0.6

correct substitution into np ***(A1)***

eg 2×0.4

$\mathrm{E}(X) = \frac{4}{5} = 0.8$ ***A1*** ***N3***

[8 marks]

Let Y be the number of gold balls drawn from the bag in parts (b), (c), and (d).

(b) evidence of recognizing binomial (seen anywhere) ***(M1)***

eg ${}_{14}C_5 (0.4)^5 (0.6)^9$, $\mathrm{B}(14, 0.4)$

$\mathrm{P}(Y = 5) = 0.207$ ***A1*** ***N2***

[2 marks]

(c) recognize need to find $\mathrm{P}(Y \leq 5)$ ***(M1)***

$\mathrm{P}(Y \leq 5) = 0.486$ ***A1*** ***N2***

[2 marks]

(d) recognizing conditional probability ***(M1)***

eg $\mathrm{P}(A \mid B)$, $\mathrm{P}(Y = 5 \mid Y \leq 5)$, $\frac{\mathrm{P}(Y = 5)}{\mathrm{P}(Y \leq 5)}$, $\frac{0.207}{0.486}$

$\mathrm{P}(Y = 5 \mid Y \leq 5) = 0.42522518$ ***(A1)***

$\mathrm{P}(Y = 5 \mid Y \leq 5) = 0.43$ (to 2 dp) ***A1*** ***N2***

[3 marks]

Total [15 marks]

Question 13

(a) correct approach *(A1)*

eg $0.5 = 0.2 + \mathrm{P}(B)$, $\mathrm{P}(A \cap B) = 0$

$\mathrm{P}(B) = 0.3$ *A1* *N2*

[2 marks]

(b) Correct expression for $\mathrm{P}(A \cap B)$ (seen anywhere) *A1*

eg $\mathrm{P}(A \cap B) = 0.2\mathrm{P}(B)$, $0.2x$

attempt to substitute into correct formula for $\mathrm{P}(A \cup B)$ *(M1)*

eg $\mathrm{P}(A \cup B) = 0.2 + \mathrm{P}(B) - \mathrm{P}(A \cap B)$, $\mathrm{P}(A \cup B) = 0.2 + x - 0.2x$

correct working *(A1)*

eg $0.5 = 0.2 + \mathrm{P}(B) - 0.2\mathrm{P}(B)$, $0.8x = 0.3$

$\mathrm{P}(B) = \frac{3}{8}$ $(= 0.375$, exact) *A1* *N3*

[4 marks]

Total [6 marks]

Question 14

(a) attempt to standardize *(M1)*

eg $z = \frac{21.8 - 20}{1.25}$, 1.44

$\mathrm{P}(T < 21.8) = 0.925$ *A1* *N2*

[2 marks]

(b) attempt to subtract probabilities *(M1)*

eg $\mathrm{P}(T < 21.8) - \mathrm{P}(T < k) = 0.3$, 0.925–0.3

$\mathrm{P}(T < k) = 0.625$ *A1*

EITHER

finding the z-value for 0.625 *(A1)*

eg $z = 0.3186$ (from tables), $z = 0.3188$

attempt to set up equation using **their** z-value *(M1)*

eg $0.3186 = \frac{k - 20}{1.25}$, $-0.524 \times 1.25 = k - 20$

$k = 20.4$ *A1* *N3*

OR

$k = 20.4$ *A3* *N3*

[5 marks]

Total [7 marks]

Question 15

(a) appropriate approach **(M1)**
eg $\mathrm{P}(R \cap B) + \mathrm{P}(R' \cap B)$, tree diagram,

one correct multiplication **(A1)**
eg 0.2×0.5, 0.24

correct working **(A1)**
eg $0.2 \times 0.5 + 0.8 \times 0.3$, $0.1 + 0.24$

$\mathrm{P}(\text{bus}) = 0.34$ (exact) **A1** **N3**

[4 marks]

(b) recognizing conditional probability **(R1)**
eg $\mathrm{P}(A|B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

correct working **A1**
eg $\dfrac{0.2 \times 0.5}{0.34}$

$\mathrm{P}(R|B) = \dfrac{5}{17}$, 0.294 **A1** **N2**

[3 marks]

(c) recognizing binomial probability **(R1)**
eg $X \sim \mathrm{B}(n, p)$, $\binom{5}{3}(0.34)^3$, $(0.34)^3(1-0.34)^2$

$\mathrm{P}(X = 3) = 0.171$ **A1** **N2**

[2 marks]

METHOD 1

evidence of using complement (seen anywhere) **(M1)**
eg $1 - \mathrm{P}(\text{none})$, $1 - 0.95$

valid approach **(M1)**
eg $1 - \mathrm{P}(\text{none}) > 0.95$, $\mathrm{P}(\text{none}) < 0.05$, $1 - \mathrm{P}(\text{none}) = 0.95$

correct inequality (accept equation) **A1**
eg $1 - (0.66)^n > 0.95$, $(0.66)^n = 0.05$

$n > 7.209$ (accept $n = 7.209$) **(A1)**

$n = 8$ **A1** **N3**

METHOD 2

valid approach using guess and check/trial and error **(M1)**
eg finding $\mathrm{P}(X \geq 1)$ for various values of n

seeing the "cross over" values for the probabilities **A1A1**
$n = 7, \mathrm{P}(X \geq 1) = 0.9454$, $n = 8, \mathrm{P}(X \geq 1) = 0.9639$

recognising $0.9639 > 0.95$ **(R1)**

$n = 8$ **A1** **N3**

[5 marks]

Total [14 marks]

Question 16

(a)	(i)	$a = 0.486$ (exact)	*A1*	*N1*
		$b = -12.41$ (exact), -12.4	*A1*	*N1*
	(ii)	correct substitution *eg* $0.486(172) - 12.41$ 71.182 71.2 (kg)	*(A1)* *A1*	 *N2*
				[4 marks]
(b)	(i)	$r = 0.997276$ $r = 0.997$	 *A1*	 *N1*
	(ii)	strong, positive (must have both correct)	*A2*	*N2*
				[3 marks]
				Total [7 marks]

Question 17

(a) (i) 50 (g) *A1* *N1*

(ii) 65 rats weigh less than 70 grams *(A1)*

attempt to find a percentage *(M1)*

eg $\frac{65}{80}$, $\frac{65}{80}\times 100$

81.25 (%) (exact), 81.3 *A1* *N3*

[4 marks]

(b) (i) $p = 10$ *A2* *N2*

(ii) subtracting to find q *(M1)*

eg $75-45-10$

$q = 20$ *A1* *N2*

[4 marks]

(c) evidence of mid-interval values *(M1)*

eg 15, 45, 75, 105

$\bar{x} = 52.5$ (exact), $\sigma = 22.5$ (exact) *A1A1* *N3*

[3 marks]

(d) 0.781650

78.2 (%) *A2* *N2*

[2 marks]

(e) recognize binomial probability *(M1)*

eg $X \sim \mathrm{B}(n, p)$, $\binom{5}{r}\times 0.782^{r}\times 0.218^{5-r}$

valid approach *(M1)*

eg $\mathrm{P}(X \leq 3)$

0.30067

0.301 *A1* *N2*

[3 marks]

Total [16 marks]

Question 18

(a) (i) $a = 0.0823604$, $b = 0.306186$

$a = 0.0824$, $b = 0.306$ **A1A1** **N2**

(ii) correct explanation with reference to number of litres required for 1 km **A1** **N1**

eg a represents the (average) amount of fuel (litres) required to drive 1 km, (average) litres per kilometre, (average) rate of change in fuel used for each km travelled

[3 marks]

(b) valid approach ***(M1)***

eg $y = 0.0824(110) + 0.306$, sketch

9.36583

9.37 (litres) **A1** **N2**

[2 marks]

Total [5 marks]

Question 19

(a) correct substitution ***(A1)***

eg 0.3×0.6

$P(A \cap B) = 0.18$ **A1** **N2**

[2 marks]

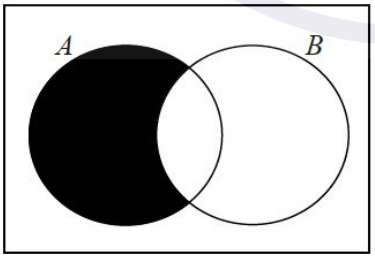
(b) correct substitution ***(A1)***

eg $P(A \cup B) = 0.3 + 0.6 - 0.18$

$P(A \cup B) = 0.72$ **A1** **N2**

[2 marks]

(c) (i)



A1 **N1**

(ii) appropriate approach ***(M1)***

eg $0.3 - 0.18$, $P(A) \times P(B')$

$P(A \cap B') = 0.12$ (may be seen in Venn diagram) **A1** **N2**

[3 marks]

Total [7 marks]

Question 20

(a) (i) valid approach **(M1)**

eg $\mathrm{P}(G) = \mathrm{P}(H > 60)$, $z = 0.875$, $\mathrm{P}(H > 60) = 1 - 0.809$, $\mathrm{N}(53, 8^2)$

0.190786

$\mathrm{P}(G) = 0.191$ **A1** **N2**

(ii) finding $\mathrm{P}(H > 70) = 0.01679$ (seen anywhere) **(A1)**

recognizing conditional probability **(R1)**

eg $\mathrm{P}(A \mid B)$, $\mathrm{P}(H > 70 \mid H > 60)$

correct working **(A1)**

eg $\dfrac{0.01679}{0.191}$

0.0880209

$\mathrm{P}(X > 70 \mid G) = 0.0880$ **A1** **N3**

[6 marks]

(b) attempt to square their $\mathrm{P}(G)$ **(M1)**

eg 0.191^2

0.0363996

$\mathrm{P}(\text{both } G) = 0.0364$ **A1** **N2**

[2 marks]

(c) (i) correct substitution into formula for $\mathrm{E}(X)$ **(A1)**

eg $100(0.191)$

$\mathrm{E}(G) = 19.1$ [19.0, 19.1] **A1** **N2**

(ii) recognizing binomial probability (may be seen in part (c)(i)) **(R1)**

eg $X \sim \mathrm{B}(n, p)$

valid approach (seen anywhere) **(M1)**

eg $\mathrm{P}(X \geq 25) = 1 - \mathrm{P}(X \leq 24)$, $1 - \mathrm{P}(X < a)$

correct working **(A1)**

eg $\mathrm{P}(X \leq 24) = 0.913\ldots$, $1 - 0.913\ldots$

0.0869002

$P(X \geq 25) = 0.0869$ **A1** **N2**

[6 marks]

Total [14 marks]

Question 21

(a) evidence of set up **(M1)**
eg correct value for r (or for a or b, seen in (b))

0.996010
$r = 0.996$ [0.996, 0.997] **A1** **N2**
[2 marks]

(b) $a = 3.15037$, $b = -15.4393$
$a = 3.15$ [3.15, 3.16], $b = -15.4$ [−15.5, −15.4] **A1A1** **N2**
[2 marks]

(c) substituting 26 into **their** equation **(M1)**
eg $y = 3.15(26) - 15.4$

66.4704
66.5 [66.4, 66.5] **A1** **N2**
[2 marks]

Total [6 marks]

Question 22

(a) recognizing that the median is at half the total frequency *(M1)*

eg $\frac{2000}{2}$

$m = 2500$ (dollars) *A1* *N2*

[2 marks]

(b) (i) 500 families have a monthly income less than 2000 *A1* *N1*

(ii) correct cumulative frequency, 1850 *(A1)*

subtracting **their** cumulative frequency from 2000 *(M1)*

eg $2000 - 1850$

150 families have a monthly income of more than 4000 dollars *A1* *N2*

Note: If working shown, award ***M1A1A1*** for $128 + 22 = 150$, using the table.

[4 marks]

(c) correct calculation *(A1)*

eg $2000 - (436 + 64 + 765 + 28 + 122)$, $1850 - 500 - 765$

$p = 585$ *A1* *N2*

[2 marks]

(d) (i) correct working *(A1)*

eg $436 + 765 + 28$

0.6145 (exact)

$\frac{1229}{2000}$, 0.615 [0.614, 0.615] *A1* *N2*

(ii) correct working/probability for number of families *(A1)*

eg $122 + 28$, $\frac{150}{2000}$, 0.075

0.186666

$\frac{28}{150}\left(=\frac{14}{75}\right)$, 0.187 [0.186, 0.187] *A1* *N2*

[4 marks]

(e) evidence of using correct mid-interval values (1500, 3000, 4500) *(A1)*

attempt to substitute into $\frac{\sum fx}{\sum f}$ *(M1)*

eg $\frac{1500 \times 64 + 3000 \times p + 4500 \times 122}{64 + 585 + 122}$

3112.84

3110 [3110, 3120] (dollars) *A1* *N2*

[3 marks]

Total [15 marks]

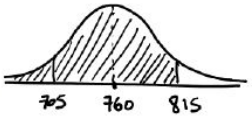
Question 23

(a) (i) $P(X > 760) = 0.5$ (exact), [0.499, 0.500] ***A1*** ***N1***

(ii) evidence of valid approach ***(M1)***

recognising symmetry, $\frac{0.7887}{2}$, $1 - P(W < 815)$, $\frac{21.13}{2} + 78.87\%$

correct working ***(A1)***

eg $0.5 + 0.39435$, $1 - 0.10565$, 

0.89435 (exact), 0.894 [0.894, 0.895] ***A1*** ***N2***

[4 marks]

(b) (i) 1.24999

$z = 1.25$ [1.24, 1.25] ***A1*** ***N1***

(ii) evidence of appropriate approach ***(M1)***

eg $\sigma = \frac{x - \mu}{1.25}$, $\frac{815 - 760}{\sigma}$

correct substitution ***(A1)***

eg $1.25 = \frac{815 - 760}{\sigma}$, $\frac{815 - 760}{1.24999}$

44.0003

$\sigma = 44.0$ [44.0, 44.1] (g) ***A1*** ***N2***

[4 marks]

(c) correct working ***(A1)***

eg $760 - 1.5 \times 44$

693.999

694 [693, 694] (g) ***A1*** ***N2***

[2 marks]

(d) 0.0668056

$P(X < 694) = 0.0668$ [0.0668, 0.0669] ***A2*** ***N2***

[2 marks]

(e) recognizing conditional probability (seen anywhere) ***(M1)***

eg $\mathrm{P}(\mathrm{A}|\mathrm{B})$, $\frac{0.025}{0.0668}$

appropriate approach involving conditional probability ***(M1)***

eg $\mathrm{P}(S \mid T) = \frac{\mathrm{P}(S \text{ and } T)}{\mathrm{P}(T)}$,

correct working

eg $\mathrm{P}(\text{salmon and tiddler}) = 0.25 \times 0.1$, $\frac{0.25 \times 0.1}{0.0668}$ ***(A1)***

0.374220
0.374 [0.374, 0.375] ***A1*** ***N2***

[4 marks]

Total [16 marks]

Question 24

(a) (i) evidence of valid approach ***(M1)***

eg 1 correct value for r, (or for a or b, seen in (ii))

0.946591
$r = 0.947$ ***A1*** ***N2***

(ii) $a = 0.500957$, $b = 0.803544$

$a = 0.501$, $b = 0.804$ ***A1A1*** ***N2***

[4 marks]

(b) substituting $x = 3.7$ into **their** equation ***(M1)***

eg $0.501(3.7) + 0.804$

2.65708 (2 hours 39.4252 minutes) ***(A1)***

$y = 2.7$ (hours)(**must** be correct 1 dp, accept 2 hours 39.4 minutes) ***A1*** ***N3***

[3 marks]

Total [7marks]

Question 25

(a) 0.0477903

probability $= \mathbf{0.0478}$ **A2** **N2**

[2 marks]

(b) $\mathrm{P}(\text{volume} < 250) = 0.02$ **(M1)**

$z = -2.05374$ (may be seen in equation) **A1**

attempt to set up equation with z **(M1)**

eg $\frac{\mu - 260}{\sigma} = z$, $260 - 2.05(\sigma) = 250$

4.86914

$\sigma = 4.87$ (ml) **A1** **N3**

[4 marks]

(c) (i) 0.968062

$\mathrm{P}(250 < \text{Vol} < 271) = 0.968$ **A2** **N2**

(ii) recognizing conditional probability (seen anywhere, including in correct working) **R1**

eg $\mathrm{P}(A|B)$, $\frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$, $\mathrm{P}(A \cap B) = \mathrm{P}(A|B)\mathrm{P}(B)$

correct value or expression for P(not underfilled) **(A1)**

eg $0.98, 1 - 0.02, 1 - P(X < 250)$

probability $= \frac{0.968}{0.98}$ **A1**

0.987818

probability $= \mathbf{0.988}$ **A1** **N2**

[6 marks]

(d) **METHOD 1**

evidence of recognizing binomial distribution (seen anywhere) ***(M1)***
eg $X \sim \mathrm{B}(50, 0.968)$, binomial cdf, $p = 0.968$, $r = 47$

$\mathrm{P}(X \leq 47) = 0.214106$ ***(A1)***

evidence of using complement ***(M1)***
eg $1 - \mathrm{P}(X \leq 47)$

0.785894
probability $= \mathbf{0.786}$ ***A1*** ***N3***

METHOD 2

evidence of recognizing binomial distribution (seen anywhere) ***(M1)***
eg $X \sim \mathrm{B}(50, 0.968)$, binomial cdf, $p = 0.968$, $r = 47$

$\mathrm{P}(\text{not pass}) = 1 - \mathrm{P}(\text{pass}) = 0.0319378$ ***(A1)***
evidence of attempt to find P (2 or fewer fail) ***(M1)***
eg 0, 1, or 2 not pass, $\mathrm{B}(50, 2)$

0.785894
probability $= \mathbf{0.786}$ ***A1*** ***N3***

METHOD 3

evidence of recognizing binomial distribution (seen anywhere) ***(M1)***
eg $X \sim \mathrm{B}(50, 0.968)$, binomial cdf, $p = 0.968$, $r = 47$

evidence of summing probabilities ***(M1)***
eg $\mathrm{P}(X = 48) + \mathrm{P}(X = 49) + \mathrm{P}(X = 50)$

correct working
eg $0.263088 + 0.325488 + 0.197317$ ***(A1)***

0.785894
probability $= \mathbf{0.786}$ ***A1*** ***N3***

[4 marks]

Total [16 marks]

Question 26

(a) (i) evidence of set up **(M1)**
eg correct value for a, b or r

$a = 4.8$, $b = 1.2$ **A1A1** **N3**

(ii) $r = 0.988064$
$r = 0.988$ **A1** **N1**

[4 marks]

(b) correct substitution into **their** regression equation **(A1)**
eg $4.8 \times 7 + 1.2$

34.8 (millions of dollars) (accept 35 and 34 800 000) **A1** **N2**

[2 marks]

Total [6 marks]

Question 27

(a) valid approach **(M1)**
eg $\frac{L-\mu}{\sigma}$, using a value for σ, using 68% and 95%

correct working
$P(-1 < Z < 2)$, correct probabilities $(0.6826\ldots + 0.1359\ldots)$ **(A1)**

$P(50 - \sigma < L < 50 + 2\sigma) = 0.818594$
$P(50 - \sigma < L < 50 + 2\sigma) = 0.819$ **A1** **N2**

[3 marks]

(b) $z = 1.95996$ **(A1)**
correct equation **A1**
eg $\frac{53.92 - 50}{\sigma} = 1.95996$, $\sigma = 2.00004$

$\sigma = 2.00$ **AG** **N0**

[2 marks]

(c) valid set up **M1**

eg $P(L > t) = 0.75$, right tail, 

, 0.25

$t = 48.6510$
$t = 48.7$ (do not accept 48.5 from using $z = -0.75$) **A2** **N2**

[3 marks]

(d) (i) correct approach **(A1)**
eg from t to 50.1, $\mathrm{P}(48.7 < X < 50.1)$, 0.269942

recognize conditional probability (seen anywhere, including in correct working) **R1**
eg $\mathrm{P}(\mathrm{A}|\mathrm{B})$

correct substitution **(A1)**
eg $\frac{\mathrm{P}(48.7 < X < 50.1)}{\mathrm{P}(X > 48.7)}, \frac{0.269942}{0.75}$

0.359923
0.360 **A1** **N3**

(ii) $\mathrm{P}(X \geq 2)$ **(A1)**

attempt to find $\mathrm{P}(X \geq 2)$ **(M1)**
eg $1 - \mathrm{P}(X = 0) - \mathrm{P}(X = 1), \mathrm{P}(X = 2) + \mathrm{P}(X = 3) + \ldots$

recognize binomial distribution **(M1)**
eg $X \sim \mathrm{B}(n, p)$

0.923741
0.924 **A1** **N2**

[8 marks]

Total [16 marks]

Question 28

(a) evidence of using $\sum p_i = 1$ **(M1)**

correct substitution **A1**
eg $0.15 + k + 0.1 + 2k = 1$, $3k + 0.25 = 1$

$k = 0.25$ **A1** **N2**

[3 marks]

(b) correct substitution **(A1)**
eg $0 \times 0.15 + 1 \times 0.25 + 2 \times 0.1 + 3 \times 0.5$

$\mathrm{E}(X) = 1.95$ **A1** **N2**

[2 marks]

Total [5 marks]

Question 29

(a) $\mathrm{P}(C \cap D) = 2k \times 3k^2$ **(A1)**

$\mathrm{P}(C \cap D) = 6k^3$ **A1** **N2**

[2 marks]

(b) **their** correct equation **(A1)**

eg $2k \times 3k^2 = 0.162$, $6k^3 = 0.162$

$k = 0.3$ **A1** **N2**

[2 marks]

(c) **METHOD 1**

finding **their** $\mathrm{P}(C' \cap D)$ (seen anywhere) **(A1)**

eg 0.4×0.27, $0.27 - 0.162$, 0.108

correct substitution into conditional probability formula **(A1)**

eg $\mathrm{P}(C' \mid D) = \frac{\mathrm{P}(C' \cap D)}{0.27}$, $\frac{(1-2k)(3k^2)}{3k^2}$

$\mathrm{P}(C' \mid D) = 0.4$ **A1** **N2**

METHOD 2

recognizing $\mathrm{P}(C' \mid D) = \mathrm{P}(C')$ **A1**

finding **their** $\mathrm{P}(C') = 1 - \mathrm{P}(C)$ (only if first line seen) **(A1)**

eg $1 - 2k$, $1 - 0.6$

$\mathrm{P}(C' \mid D) = 0.4$ **A1** **N2**

[3 marks]

Total [7 marks]

Question 30

(a) evidence of setup **(M1)**
eg correct value for a or b

$13.3823,\ 137.482$
$a = 13.4,\ b = 137$ **A1A1** **N3**

[3 marks]

(b) correct substitution into **their** regression equation
eg $13.3823 \times 7 + 137.482$ **(A1)**

correct calculation
231.158 **(A1)**

231 (coyotes) (must be an integer) **A1** **N2**

[3 marks]

(c) recognizing $t = 0$ **(M1)**
eg $f(0)$

correct substitution into the model
eg $\frac{2000}{1+99e^{-k(0)}},\ \frac{2000}{100}$ **(A1)**

20 (foxes) **A1** **N2**

[3 marks]

(d) recognizing $(5,\ 64)$ satisfies the equation **(M1)**
eg $f(5) = 64$

correct substitution into the model
eg $64 = \frac{2000}{1+99e^{-k(5)}},\ 64\left(1+99e^{-5k}\right) = 2000$ **(A1)**

0.237124
$k = -\frac{1}{5}\ln\left(\frac{11}{36}\right)$ (exact), 0.237 **A1** **N2**

[3 marks]

(e) valid approach **(M1)**
eg $c = f$, sketch of graphs

correct working **(A1)**
eg $\frac{2000}{1+99e^{-0.237124t}} = 13.382t + 137.482$, sketch of graphs, table of values

$t = 12.0403$ **(A1)**
2007 **A1** **N2**

Note: Exception to the ***FT*** rule. Award ***A1FT*** on their value of t.

[4 marks]

Total [16 marks]

Question 31

(a) finding standardized value for $4\,\text{kg}$ (seen anywhere) **(A1)**

eg $z = -1.64485$

attempt to standardize **(M1)**

eg $\sigma = \frac{x-\mu}{z}, \frac{4-10}{\sigma}$

correct substitution **(A1)**

eg $-1.64 = \frac{4-10}{\sigma}, \frac{4-10}{-1.64}$

$\sigma = 3.64774$

$\sigma = 3.65$ **A1** **N2**

[4 marks]

(b) valid approach **(M1)**

eg $1-p$, 0.62, $\frac{w-10}{3.65} = 0.305$

$w = 11.1143$

$w = 11.1$ **A1** **N2**

[2 marks]

(c) attempt to restrict melon population **(M1)**

eg 95% are delivered, $\text{P}(\text{medium} \mid \text{delivered})$, $57 + 38$

correct probability for medium watermelons **(A1)**

eg $\frac{0.57}{0.95}$

$\frac{57}{95}$, 0.6, 60% **A1** **N3**

[3 marks]

(d) proportion of large watermelons (seen anywhere) **(A1)**

eg $\text{P}(\text{large}) = 0.4$, 40%

correct approach to find total sales (seen anywhere) **(A1)**

eg $150 = \text{sales} - 300$, total sales $= \$450$

correct expression **(A1)**

eg $1.75(0.6x) + 3(0.4x)$, $1.75(0.6) + 3(0.4)$

evidence of correct working **(A1)**

eg $1.75(0.6x) + 3(0.4x) = 450$, $2.25x = 450$

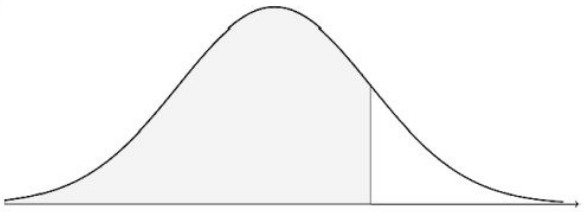
200 watermelons in the delivery **A1** **N2**

[5 marks]

Total [14 marks]

Question 32

(a)



A1A1 ***N2***

Note: Award ***A1*** for vertical line clearly to right of mean, ***A1*** for shading to left of their vertical line.

[2 marks]

(b) $P(X \leq 25) = 0.894350$ ***(A1)***

$P(X \leq 25) = 0.89$ (must be 2 d.p.) ***A1*** ***N2***

[2 marks]

(c) $c = 22.0976$

$c = 22.1$ ***A2*** ***N2***

[2 marks]

Total [6 marks]

Question 33

(a) strong, negative (both required) **A2** **N2**

[2 marks]

(b) **METHOD 1**

valid approach **(M1)**

eg $e^{\ln M} = e^{-0.12t+4.67}$

correct use of exponent laws for $e^{-0.12t+4.67}$ **(A1)**

eg $e^{-0.12t} \times e^{4.67}$

comparing coefficients/terms **(A1)**

eg $b^t = e^{-0.12t}$

$b = e^{-0.12}$ (exact), 0.887 **A1** **N3**

METHOD 2

valid approach **(M1)**

eg $\ln\left(a \times b^t\right) = -0.12t + 4.67$

correct use of log laws for $\ln\left(ab^t\right)$ **(A1)**

eg $\ln a + t \ln b$

comparing coefficients **(A1)**

eg $-0.12 = \ln b$

$b = e^{-0.12}$ (exact), 0.887 **A1** **N3**

[4 marks]

Total [6 marks]

Question 34

(a) evidence of summing to 1 **(M1)**
eg $0.55+0.3+0.1+k=1$

$k=0.05$ (exact) **A1** **N2**
[2 marks]

(b) (i) 0.55 **A1** **N1**

(ii) recognizing binomial probability **(M1)**
eg $X:\ B(n,\ p),\ \binom{5}{4},\ (0.55)^4(1-0.55),\ \binom{n}{r}p^r q^{n-r}$

$\mathrm{P}(X=4)=0.205889$
$\mathrm{P}(X=4)=0.206$ **A1** **N2**
[3 marks]

(c) correct substitution into formula for $\mathrm{E}(X)$ **(A1)**
eg $0.2+(2\times0.08)+(3\times0.02)$

$\mathrm{E}(B)=0.42$ (exact) **A1** **N2**
[2 marks]

(d) (i) valid attempt to find one possible way of having 2 breakdowns **(M1)**
eg $2A,\ 2B,\ 1A$ and $1B$, tree diagram

one correct calculation for 1 way (seen anywhere) **(A1)**
eg $0.1\times0.7,\ 0.55\times0.08,\ 0.3\times0.2$

recognizing there are 3 ways of having 2 breakdowns **(M1)**
eg A twice or B twice or one breakdown each

correct working **(A1)**
eg $(0.1\times0.7)+(0.55\times0.08)+(0.3\times0.2)$

$\mathrm{P}(2\text{ breakdowns})=0.174$ (exact) **A1** **N3**

(ii) recognizing conditional probability **(M1)**
eg $\mathrm{P}(A|B),\ \mathrm{P}(2A|2\text{ breakdowns})$

correct working **(A1)**
eg $\dfrac{0.1\times0.7}{0.174}$

$\mathrm{P}(A=2|\text{two breakdowns})=0.402298$
$\mathrm{P}(A=2|\text{two breakdowns})=0.402$ **A1** **N2**
[8 marks]

Total [15 marks]

Question 35

(a) 0.0668072

$\mathrm{P}(S < 50) = 0.0668$ (accept $\mathrm{P}(S \leq 49) = 0.0548$) ***A2*** ***N2***

[2 marks]

(b) valid approach ***(M1)***

Eg $\mathrm{P}(S < 50) \times \mathrm{P}(R < x)$

correct equation (accept any variable) ***A1***

eg $\mathrm{P}(S < 50) \times \mathrm{P}(R < x) = 1\%,\ 0.0668072 \times p = 0.01,\ \mathrm{P}(R < x) = \dfrac{0.01}{0.0668}$

finding the value of $\mathrm{P}(R < x)$ ***(A1)***

eg $\dfrac{0.01}{0.0668},\ 0.149684$

9.40553

$x = 9.41$ (accept $x = 9.74$ from 0.0548) ***A1*** ***N3***

Total [6 marks]

Question 36

(a) (i) valid approach **(M1)**
eg correct value for r (or for a or b seen in (ii))

-0.994347
$r = -0.994$ **A1** **N2**

(ii) $-1.58095,\ 33480.3$
$a = -1.58,\quad b = 33500$ **A1A1** **N2**
[4 marks]

(b) correct substitution into **their** regression equation
eg $-1.58095(11000) + 33480.3$ **(A1)**

16089.85 (16120 from 3sf) **(A1)**

price $= 16100$ (dollars) (must be rounded to the nearest 100 dollars) **A1** **N3**
[3 marks]

(c) **METHOD 1**

valid approach **(M1)**
eg $P \times (\text{rate})^t$

rate $= 0.95$ (may be seen in their expression) **(A1)**

correct expression **(A1)**
eg 16100×0.95^6

11834.97
11 800 (dollars) **A1** **N2**

METHOD 2

attempt to find all six terms **(M1)**
eg $\big(\big((16100 \times 0.95) \times 0.95\big)\ldots\big) \times 0.95$, table of values

5 correct values (accept values that round correctly to the nearest dollar)
15 295, 14 530, 13 804, 13 114, 12 458 **A2**

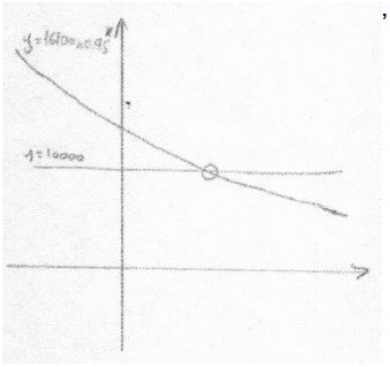
11 835
11 800 (dollars) **A1** **N2**
[4 marks]

(d) **METHOD 1**

correct equation ***(A1)***

eg $16100 \times 0.95^x = 10000$

valid attempt to solve ***(M1)***

eg  , using logs

9.28453 ***(A1)***

year 2019 ***A1*** ***N2***

METHOD 2

valid approach using table of values ***(M1)***

both crossover values (accept values that round correctly to the nearest dollar) ***A2***

eg $P = 10147$ (1 Jan 2019), $P = 9639.7$ (1 Jan 2020)

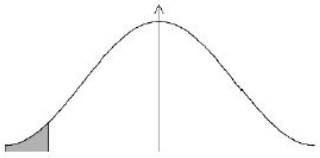
year 2019 ***A1*** ***N2***

[4 marks]

Total [15 marks]

Question 37

(a) valid approach **(M1)**



eg $z = -1.61643$,

2.48863

$w = 2.49$ (kg) **A2 N3**

[3 marks]

(b) correct value or expression (seen anywhere)

eg $0.053 - \mathrm{P}(X \leq 2.15)$, 0.039465 **(A1)**

evidence of conditional probability **(M1)**

eg $\dfrac{\mathrm{P}(2.15 \leq X \leq w)}{\mathrm{P}(X \leq w)}$, $\dfrac{0.039465}{0.053}$

0.744631

0.745 **A1 N2**

[3 marks]

[Total 6 marks]

Question 38

(a) $\mathrm{P}(\text{red}) = \dfrac{5}{15+m}$ **A1 N1**

[1 mark]

(b) recognizing binomial distribution **(M1)**

eg $X \sim \mathrm{B}(n, p)$

correct value for the complement of **their** p (seen anywhere) **A1**

eg $1 - \dfrac{5}{15+m}, \dfrac{10+m}{15+m}$

correct substitution into $\mathrm{Var}(X) = np(1-p)$ **(A1)**

eg $4\left(\dfrac{5}{15+m}\right)\left(\dfrac{10+m}{15+m}\right), \dfrac{20(10+m)}{(15+m)^2} < 0.6$

$m > 12.2075$ **(A1)**

$m = 13$ **A1 N3**

[5 marks]

[Total 6 marks]

Question 39

(a) attempt to substitute into formula for mean **(M1)**

eg $\frac{\sum x}{10}, \frac{252}{n}, \frac{252}{10}$

mean $= 25.2$ (hours) **A1** **N2**

[2 marks]

(b) (i) mean $= 30.2$ (hours) **A1** **N1**

(ii) $\sigma = 5$ (hours) **A1** **N1**

[2 marks]

(c) (i) valid approach **(M1)**

eg 95%, 5% of 27

correct working **(A1)**

eg 0.95×27, $27 - (5\% \text{ of } 27)$

median $= 25.65$ (exact), 25.7 (hours) **A1** **N2**

(ii) **METHOD 1**

variance = (standard deviation)^2 (seen anywhere) **(A1)**

valid attempt to find new standard deviation **(M1)**

eg $\sigma_{new} = 0.95 \times 5$, 4.75

variance $= 22.5625$ (exact), 22.6 **A1** **N2**

METHOD 2

variance = (standard deviation)^2 (seen anywhere) **(A1)**

valid attempt to find new variance **(M1)**

eg 0.95^2, $0.9025 \times \sigma^2$

new variance $= 22.5625$ (exact), 22.6 **A1** **N2**

[6 marks]

(d) (i) both correct frequencies **(A1)**

eg 80, 150

subtracting **their** frequencies in either order **(M1)**

eg $150 - 80$, $80 - 150$

70 (students) **A1** **N2**

(ii) evidence of a valid approach **(M1)**

eg 10% of 200, 90%

correct working **(A1)**

eg 0.90×200, $200 - 20$, 180 students

$k = 35$ **A1** **N3**

[6 marks]

[Total 16 marks]

Question 40

(a) (i) mode = 10 **A1** **N1**

(ii) valid approach **(M1)**
eg $x_{\max} - x_{\min}$, interval 2 to 11

range = 9 **A1** **N2**
[3 marks]

(b) (i) 7.14666
mean = 7.15 **A2** **N2**

(ii) recognizing that variance is $(\text{sd})^2$ **(M1)**
eg $\text{var} = \sigma^2$, 2.90605^2, 2.92562^2

$\sigma^2 = 8.44515$
$\sigma^2 = 8.45$ **A1** **N2**
[4 marks]

Total [7 marks]

Question 41

(a) evidence of binomial distribution (may be seen in part (b)) **(M1)**
eg np, 150×0.08

$k = 12$ **A1** **N2**
[2 marks]

(b) (i) $\text{P}(X = 12) = \binom{150}{12}(0.08)^{12}(0.92)^{138}$ **(A1)**

0.119231
probability = 0.119 **A1** **N2**

(ii) recognition that $X \leq 11$ **(M1)**

0.456800
$\text{P}(X < 12) = 0.457$ **A1** **N2**
[4 marks]

Total [6 marks]

Question 42

(a) valid approach **(M1)**
eg $\mathrm{P}(X < \mu) = 0.5,\ 0.5 - 0.3$

$\mathrm{P}(X < 9) = 0.2$ (exact) **A1** **N2**
[2 marks]

(b) $z = -0.841621$ (may be seen in equation) **(A1)**

valid attempt to set up an equation with **their** z **(M1)**
eg $-0.842 = \frac{\mu - X}{\sigma}, -0.842 = \frac{X - \mu}{\sigma},\ z = \frac{9 - \mu}{2.1}$

10.7674

$\mu = 10.8$ **A1** **N3**
[3 marks]

(c) $\mathrm{P}(X > 9) = 0.8$ (seen anywhere) **(A1)**

valid approach **(M1)**
eg $\mathrm{P}(A) \times \mathrm{P}(B)$

correct equation **(A1)**
eg $0.8 \times \mathrm{P}(Y > 9) = 0.4$

$\mathrm{P}(Y > 9) = 0.5$ **A1**

$\lambda = 9$ **A1** **N3**
[5 marks]

(d) finding $\mathrm{P}(9 < Y < 13) = 0.373450$ (seen anywhere) **(A2)**

recognizing conditional probability **(M1)**
eg $\mathrm{P}(A \mid B),\ \mathrm{P}(Y < 13 \mid Y > 9)$

correct working **(A1)**
eg $\frac{0.373}{0.5}$

0.746901

0.747 **A1** **N3**
[5 marks]

Total [15 marks]

Question 43

(a) (i) evidence of set up *(M1)*
eg correct value for a or b

$0.667315, 22.2117$
$a = 0.667,\ b = 22.2$ *A1A1* *N3*

(ii) 0.922958
$r = 0.923$ *A1* *N1*
[4 marks]

(b) valid approach *(M1)*
eg $0.667(15) + 22.2,\ N(15)$

32.2214 *(A1)*
32 (visitors) (must be an integer) *A1* *N2*
[3 marks]

[Total 7 marks]

Question 44

(a) (i) correct substitution into $\mathrm{E}(X)$ formula **(A1)**

eg $0(p)+1(0.5)+2(0.3)+3(q)=1.2$

$q=\frac{1}{30}$, 0.0333 **A1** **N2**

(ii) evidence of summing probabilities to 1 **(M1)**

eg $p+0.5+0.3+q=1$

$p=\frac{1}{6}$, 0.167 **A1** **N2**

[4 marks]

(b) (i) $\mathrm{P}(3 \text{ blue})=\frac{1}{30}$, 0.0333 **A1** **N1**

(ii) valid reasoning **R1**

eg P (3 white) = P(0 blue)

$\mathrm{P}(3 \text{ white})=\frac{1}{6}$ **AG** **N0**

(iii) valid method **(M1)**

eg $\mathrm{P}(3 \text{ white})=\frac{w}{10}\times\frac{w-1}{9}\times\frac{w-2}{8}$, $\frac{{}_{w}C_{3}}{{}_{10}C_{3}}$

correct equation **A1**

eg $\frac{w}{10}\times\frac{w-1}{9}\times\frac{w-2}{8}=\frac{1}{6}$, $\frac{{}_{w}C_{3}}{{}_{10}C_{3}}=0.167$

$w=6$ **A1** **N2**

[5 marks]

(c) valid approach **(M1)**

eg $\mathrm{B}(n,\ p)$, $\binom{n}{r}p^{r}q^{n-r}$, $(0.167)^{2}(0.833)^{7}$, $\binom{9}{2}$

0.279081

0.279 **A1** **N2**

[2 marks]

(d) recognizing one prize in first seven attempts **(M1)**

eg $\binom{7}{1}, \left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^6$

correct working **(A1)**

eg $\binom{7}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^6, 0.390714$

correct approach **(A1)**

eg $\binom{7}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^6 \times \frac{1}{6}$

0.065119

0.0651 **A1** **N2**

[4 marks]

[Total 15 marks]

Question 45

(a) valid approach **(M1)**

eg total probability $= 1$

correct equation **(A1)**

eg $0.475 + 2k^2 + \frac{k}{10} + 6k^2 = 1,\ 8k^2 + 0.1k - 0.525 = 0$

$k = 0.25$ **A2** **N3**

[4 marks]

(b) $\mathrm{P}(X = 2) = 0.025$ **A1** **N1**

[1 mark]

(c) valid approach for finding $\mathrm{P}(X > 0)$ **(M1)**

eg $1 - 0.475,\ 2(0.25^2) + 0.025 + 6(0.25^2),\ 1 - \mathrm{P}(X = 0),\ 2k^2 + \frac{k}{10} + 6k^2$

correct substitution into formula for conditional probability **(A1)**

eg $\frac{0.025}{1 - 0.475}, \frac{0.025}{0.525}$

0.0476190

$\mathrm{P}(X = 2 \mid X > 0) = \frac{1}{21}$ (exact), 0.0476 **A1** **N2**

[3 marks]

Total [8 marks]

Question 46

finding the z-value for 0.17 **(A1)**

eg $z = -0.95416$

setting up equation to find σ, **(M1)**

eg $z = \frac{168-180}{\sigma}$, $-0.954 = \frac{-12}{\sigma}$

$\sigma = 12.5765$ **(A1)**

EITHER (Properties of the Normal curve)

correct value (seen anywhere) **(A1)**

eg $\mathrm{P}(X < 192) = 0.83$, $\mathrm{P}(X > 192) = 0.17$

correct working **(A1)**

eg $\mathrm{P}(X < 192 - h) = 0.83 - 0.8, \mathrm{P}(X < 192 - h) = 1 - 0.8 - 0.17$,
$\mathrm{P}(X > 192 - h) = 0.8 + 0.17$

correct equation in h

eg $\frac{(192-h)-180}{12.576} = -1.88079$, $192 - h = 156.346$ **(A1)**

35.6536

$h = 35.7$ **A1** **N3**

OR (Trial and error using different values of h)

two correct probabilities whose 2 sf will round up **and** down, respectively, to 0.8 **A2**

eg $\mathrm{P}(192 - 35.6 < X < 192) = 0.799706$, $\mathrm{P}(157 < X < 192) = 0.796284$,
$\mathrm{P}(192 - 36 < X < 192) = 0.801824$

$h = 35.7$ **A2**

[7 marks]

Question 47

(a) evidence of setup **(M1)**
eg correct value for a or b

$a = 6.96103,\ b = -454.805$
$a = 6.96,\ b = -455$ (accept $6.96x - 455$) **A1A1** **N3**
[3 marks]

(b) substituting $N = 270$ into **their** equation **(M1)**
eg $6.96(270) - 455$

1424.67
$P = 1420$ (g) **A1** **N2**
[2 marks]

(c) 40 (hives) **A1** **N1**
[1 mark]

(d) (i) valid approach **(M1)**
eg $128 + 40$

168 hives have a production less than k **(A1)**

$k = 1640$ **A1** **N3**

(ii) valid approach **(M1)**
eg $200 - 168$

32 (hives) **A1** **N2**
[5 marks]

(e) recognize binomial distribution (seen anywhere) **(M1)**
eg $X \sim \mathrm{B}(n,\ p),\ \binom{n}{r} p^r (1-p)^{n-r}$

correct values **(A1)**
eg $n = 40$ (check **FT**) and $p = 0.75$ and $r = 30,\ \binom{40}{30} 0.75^{30} (1 - 0.75)^{10}$

0.144364
0.144 **A1** **N2**
[3 marks]

Total [14 marks]

Question 48

(a) (i) evidence of using $\sum p_i = 1$ **(M1)**

eg $k + 0.98 + 0.01 = 1$

$k = 0.01$ **A1** **N2**

(ii) recognizing that 93 and 119 are symmetrical about μ **(M1)**

eg μ is midpoint of 93 and 119

correct working to find μ **A1**

$\frac{119 + 93}{2}$

$\mu = 106$ **AG** **N0**

[4 marks]

(b) finding standardized value for 93 or 119 **(A1)**

eg $z = -2.32634,\ z = 2.32634$

correct substitution using **their** z value **(A1)**

eg $\frac{93 - 106}{\sigma} = -2.32634,\ \frac{119 - 106}{2.32634} = \sigma$

$\sigma = 5.58815$ **(A1)**

0.024508

$\mathrm{P}(X < 95) = 0.0245$ **A2** **N3**

[5 marks]

(c) evidence of recognizing binomial **(M1)**

eg ${}_nC_a \times p^a \times q^{n-a},\ n = 10$ and $p = 0.0245, \mathrm{B}(n, p)$

valid approach **(M1)**

eg $\mathrm{P}(X \leq 1),\ \mathrm{P}(X = 0) + \mathrm{P}(X = 1)$

0.976285

0.976 **A1** **N2**

[3 marks]

(d) (i) recognizing **new** binomial probability *(M1)*
eg $\mathrm{B}(50,\ 0.976)$

correct substitution *(A1)*
eg $\mathrm{E}(X) = 50(0.976285)$

48.81425
48.8 *A1* *N2*

(ii) valid approach *(M1)*
eg $\mathrm{P}(\mathrm{X} \geq 48),\ 1 - \mathrm{P}(\mathrm{X} \leq 47)$

0.884688
0.885 *A1* *N2*

[5 marks]

[Total: 17 marks]

Question 49

(a) (i) valid approach *(M1)*
eg correct value for a or b (or for r seen in (ii))

$a = 1.91966\ \ b = 7.97717$
$a = 1.92,\ b = 7.98$ *A1A1* *N3*

(ii) 0.984674
$r = 0.985$ *A1* *N1*
[4 marks]

(b) correct substitution into their equation *(A1)*
eg $1.92 \times 1.95 + 7.98$

11.7205
11.7 (kg) *A1* *N2*
[2 marks]

[Total: 6 marks]

Question 50

(a) correct approach indicating subtraction *(A1)*
eg $0.79 - 0.095$, appropriate shading in diagram

$\mathrm{P}(289 < w < 310) = 0.695$ (exact), $69.5\,\%$ *A1* *N2*
[2 marks]

(b) **METHOD 1**

(i) valid approach ***(M1)***

eg $1-p$, 21

-0.806421

$z=-0.806$ ***A1*** ***N2***

(ii) attempt to standardize ***(M1)***

eg $\sigma=\frac{289-297}{z}$, $\frac{289-297}{\sigma}$

correct substitution with their z (do not accept a probability) ***A1***

eg $-0.806=\frac{289-297}{\sigma}$, $\frac{289-297}{-0.806}$

9.92037

$\sigma=9.92$ ***A1*** ***N2***

METHOD 2

(i) & (ii)

correct expression for z (seen anywhere) ***(A1)***

eg $\frac{289-\mu}{\sigma}$

valid approach ***(M1)***

eg $1-p$, 21

-0.806421

$z=-0.806$ (seen anywhere) ***A1*** ***N2***

valid attempt to set up an equation with **their** z
(do not accept a probability) ***(M1)***

eg $-0.806=\frac{289-297}{\sigma}$, $\frac{289-297}{-0.806}$

9.92037

$\sigma=9.92$ ***A1*** ***N2***

[5 marks]

continued…

(c) valid approach **(M1)**

eg $\mathrm{P}(W < w) = 0.35, -0.385320$ (accept 0.385320), diagram showing values in a standard normal distribution

correct score at the 35th percentile **(A1)**

eg 293.177

294 (g) **A1** **N2**

Note: If working shown, award ***(M1)(A1)A0*** for 293.
If no working shown, award ***N1*** for 293.177, ***N1*** for 293.

Exception to the ***FT*** rule: If the score is incorrect, and working shown, award ***A1FT*** for correctly finding their minimum weight (by rounding up)

[3 marks]

(d) evidence of recognizing binomial (seen anywhere) **(M1)**

eg $X \sim \mathrm{B}(36, p)$, ${}_n\mathrm{C}_a \times p^a \times q^{n-a}$

correct probability (seen anywhere) **(A1)**

eg 0.65

EITHER

finding $\mathrm{P}(X \leq 18)$ from GDC **(A1)**

eg 0.045720

evidence of using complement **(M1)**

eg $1 - \mathrm{P}(X \leq 18)$

0.954279

$\mathrm{P}(X > 18) = 0.954$ **A1** **N2**

OR

recognizing $\mathrm{P}(X > 18) = \mathrm{P}(X \geq 19)$ **(M1)**

summing terms from 19 to 36 **(A1)**

eg $\mathrm{P}(X = 19) + \mathrm{P}(X = 20) + \ldots + \mathrm{P}(X = 36)$

0.954279

$\mathrm{P}(X > 18) = 0.954$ **A1** **N2**

[5 marks]

(e) correct calculation **(A1)**

0.954^2, $\binom{2}{2} 0.954^2 (1 - 0.954)^0$

0.910650

0.911 **A1** **N2**

[2 marks]

Total [17 marks]

Question 51

(a) $-0.394791, 13$

$A(-0.395, 13)$ **A1A1** **N2**

[2 marks]

(b) (i) 13 **A1** **N1**

(ii) 2π, 6.28 **A1** **N1**

[2 marks]

(c) valid approach **(M1)**

eg recognizing that amplitude is p or shift is r

$f(x) = 13\cos(x + 0.395)$ (accept $p = 13$, $r = 0.395$) **A1A1** **N3**

Note: Accept any value of r of the form $0.395 + 2\pi k$, $k \in \mathbb{Z}$

[3 marks]

(d) recognizing need for $d'(t)$ **(M1)**

eg $-12\sin(t) - 5\cos(t)$

correct approach (accept any variable for t) **(A1)**

eg $-13\sin(t + 0.395)$, sketch of d', $(1.18, -13)$, $t = 4.32$

maximum speed $= 13\ (\text{cm s}^{-1})$ **A1** **N2**

[3 marks]

(e) recognizing that acceleration is needed **(M1)**

eg $a(t)$, $d''(t)$

correct equation (accept any variable for t) **(A1)**

eg $a(t) = -2$, $\left|\frac{\mathrm{d}}{\mathrm{d}t}(d'(t))\right| = 2$, $-12\cos(t) + 5\sin(t) = -2$

valid attempt to solve **their** equation **(M1)**

eg sketch, 1.33

1.02154

1.02 **A2** **N3**

[5 marks]

Total [15 marks]

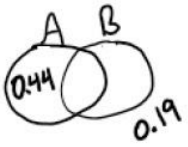
Question 52

(a) valid approach
eg Venn diagram, $\mathrm{P}(A)-\mathrm{P}(A\cap B)$, $0.62-0.18$ **(M1)**

$\mathrm{P}(A\cap B')=0.44$ **A1** **N2**

[2 marks]

(b) valid approach to find either $\mathrm{P}(B')$ or $\mathrm{P}(B)$ **(M1)**

eg 

(seen anywhere), $1-\mathrm{P}(A\cap B')-\mathrm{P}\left((A\cup B)'\right)$

correct calculation for $\mathrm{P}(B')$ or $\mathrm{P}(B)$ **(A1)**
eg $0.44+0.19$, $0.81-0.62+0.18$

correct substitution into $\dfrac{\mathrm{P}(A\cap B')}{\mathrm{P}(B')}$ **(A1)**

eg $\dfrac{0.44}{0.19+0.44}$, $\dfrac{0.44}{1-0.37}$

0.698412

$\mathrm{P}(A\,|\,B')=\dfrac{44}{63}$ (exact), 0.698 **A1** **N3**

[4 marks]

Total [6 marks]

Question 53

(a) evidence of summing to 1 **(M1)**
eg $0.28+k+0.15+0.3=1$, $0.73+k=1$

$k=0.27$ **A1** **N2**
[2 marks]

(b) correct substitution into formula for $\mathrm{E}(X)$ **(A1)**
eg $1\times0.28+2\times k+3\times0.15+4\times0.3$

$\mathrm{E}(X)=2.47$ (exact) **A1** **N2**
[2 marks]

(c) valid approach **(M1)**
eg np, 80×0.15

12 **A1** **N2**
[2 marks]

Total [6 marks]

Question 54

(a) 0.010724
0.0107 **A2** **N2**
[2 marks]

(b) correct z-value **(A1)**
0.263714…

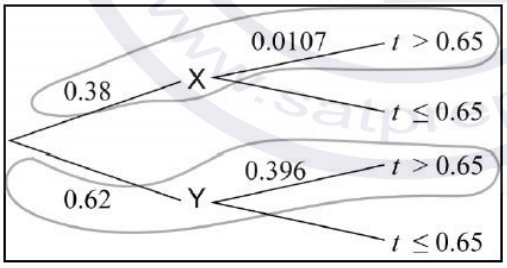
evidence of appropriate approach **(M1)**
eg $\frac{0.65-0.592}{\sigma}$, $0.264=\frac{x-\mu}{\sigma}$

correct substitution **(A1)**
eg $0.263714=\frac{0.65-0.592}{\sigma}$, $\sigma=\frac{0.65-0.592}{0.264}$

0.219934
$\sigma=0.220$ **A1** **N3**
[4 marks]

(c) correct work for $\mathrm{P}(\text{group X and } t>0.65)$ or $\mathrm{P}(\text{group Y and } t>0.65)$ (may be seen anywhere) **(A1)**
eg $\mathrm{P}(\text{group X})\times\mathrm{P}(t>0.65\,|\,\mathrm{X})$, $\mathrm{P}(X\cap t>0.65)=0.0107\times0.38\,(=0.004075)$, $\mathrm{P}(Y\cap t>0.65)=0.396\times0.62$

recognizing conditional probability (seen anywhere) **(M1)**
eg $\mathrm{P}(\mathrm{X}\,|\,t>0.65)$, $\mathrm{P}(A\,|\,B)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$

valid approach to find $\mathrm{P}(t>0.65)$ **(M1)**
eg 

, $\mathrm{P}(\mathrm{X}\text{ and } t>0.65)+\mathrm{P}(\mathrm{Y}\text{ and } t>0.65)$

correct work for $\mathrm{P}(t>0.65)$ **(A1)**
eg $0.0107\times0.38+0.396\times0.62$, 0.249595

correct substitution into conditional probability formula **A1**
eg $\frac{0.0107\times0.38}{0.0107\times0.38+0.396\times0.62}$, $\frac{0.004075}{0.249595}$

0.016327
$\mathrm{P}(\mathrm{X}\,|\,t>0.65)=0.0163270$ **A1** **N3**
[6 marks]

(d) recognizing binomial probability **(M1)**

eg $X \sim \mathrm{B}(n,\ p)$, $\binom{n}{r}p^r q^{n-r}$, $(0.016327)^2(0.983672)^8$, $\binom{10}{2}$

valid approach **(M1)**

eg $\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X \leq 1)$, $1 - \mathrm{P}(X < a)$, summing terms from 2 to 10

(accept binomcdf(10, 0.0163, 2, 10))

0.010994

$\mathrm{P}(X \geq 2) = 0.0110$ **A1** **N2**

[3 marks]

Total [15 marks]

Question 55

(a) (i) evidence of set up **(M1)**

eg correct value for a or b or r (seen in (ii)) or $r^2 (= 0.973)$

9.91044, –31.3194

$a = 9.91$, $b = -31.3$, $y = 9.91x - 31.3$ **A1A1** **N3**

(ii) 0.986417

$r = 0.986$ **A1** **N1**

[4 marks]

(b) substituting $x = 21.5$ into **their** equation **(M1)**

eg $9.91(21.5) - 31.3$

181.755

182 (cm) **A1** **N2**

[2 marks]

Total [6 marks]

Question 56

(a) valid approach **(M1)**

$(A \cap M') + (A \cap M)$, $\frac{17}{35}$, $11 + 6$

number of students taking art class $= 17$ **A1** **N2**

[2 marks]

(b) (i) valid approach **(M1)**

$13 + 5$, $35 - 17$, 18, $1 - \mathrm{P}(A)$

0.514285

$\mathrm{P}(A') = \frac{18}{35}$ (exact), 0.514 **A1** **N2**

(ii) valid approach **(M1)**

$11 + 13$, $35 - 6 - 5$, 24

0.685714

$\mathrm{P}(A \text{ or } M \text{ but not both}) = \frac{24}{35}$ (exact), 0.686 **A1** **N2**

[4 marks]

Total [6 marks]

Question 57

(a) **METHOD 1**

multiplication of $\mathrm{P}(A)$ and $\mathrm{P}(D)$ **(A1)**

eg 0.70×0.85, 0.595

correct reasoning for their probabilities **R1**

eg $0.595 \neq 0.65$, $0.70 \times 0.85 \neq \mathrm{P}(A \cap D)$

A and D are not independent **AG** **N0**

METHOD 2

calculation of $\mathrm{P}(D \mid A)$ **(A1)**

eg $\frac{13}{14}$, 0.928

correct reasoning for their probabilities **R1**

eg $0.928 \neq 0.85$, $\frac{0.65}{0.7} \neq \mathrm{P}(D)$

A and D are not independent **AG** **N0**

[2 marks]

(b) (i) correct working **(A1)**

eg $\mathrm{P}(A)-\mathrm{P}(A\cap D)$, $0.7-0.65$, correct shading and/or value on Venn diagram

$\mathrm{P}(A\cap D')=0.05$ **A1** **N2**

(ii) recognizing conditional probability (seen anywhere) **(M1)**

eg $\frac{\mathrm{P}(D'\cap A)}{\mathrm{P}(A)}$, $\mathrm{P}(A\,|\,B)$

correct working **(A1)**

eg $\frac{0.05}{0.7}$

0.071428

$\mathrm{P}(D'\,|\,A)=\frac{1}{14}$, 0.0714 **A1** **N2**

[5 marks]

(c) finding standardized value for 28 hours (seen anywhere) **(A1)**

eg $z=1.28155$

correct working to find σ **(A1)**

eg $1.28155=\frac{28-25}{\sigma}$, $\frac{28-25}{1.28155}$

2.34091

$\sigma=2.34$ **A1** **N2**

[3 marks]

(d) $\mathrm{P}(X>30)=0.0163429$ **(A1)**

valid approach (seen anywhere) **(M1)**

eg $\left[\mathrm{P}(X>30)\right]^2$, $(0.01634)^2$, B(2, 0.0163429), 2.67E-4, 2.66E-4

0.0267090

0.0267% **A2** **N3**

[4 marks]

Total [14 marks]

Question 58

(a) (i) valid approach **(M1)**

eg correct value for a or b (or for correct r or $r^2 = 0.955631$ seen in (ii))

$0.141120,\ 11.1424$

$a = 0.141,\ b = 11.1$ **A1A1** **N3**

(ii) 0.977563

$r = 0.978$ **A1** **N1**

[4 marks]

(b) correct substitution into **their** regression equation **(A1)**

eg $0.141(95) + 11.1$

24.5488

24.5 **A1** **N2**

[2 marks]

Total [6 marks]

Question 59

(a) (i) valid approach to find $\mathrm{P}(\text{one red})$ **(M1)**

eg ${}_nC_a \times p^a \times q^{n-a}$, $\mathrm{B}(n,\ p)$, $3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2$, $\binom{3}{1}$

listing all possible cases for exactly one red (may be indicated on tree diagram)

$\mathrm{P}(1 \text{ red}) = 0.444\left(=\frac{4}{9}\right)$ $[0.444,\ 0.445]$ **A1** **N2**

(ii) valid approach **(M1)**

eg $\mathrm{P}(X = 2) + \mathrm{P}(X = 3)$, $1 - \mathrm{P}(X \le 1)$, $\text{binomcdf}\left(3, \frac{1}{3}, 2, 3\right)$

correct working **(A1)**

eg $\frac{2}{9} + \frac{1}{27}$, $0.222 + 0.037$, $1 - \left(\frac{2}{3}\right)^3 - \frac{4}{9}$

0.259259

$\mathrm{P}(\text{at least two red}) = 0.259\left(=\frac{7}{27}\right)$ **A1** **N3**

[5 marks]

(b) recognition that winning $\$10$ means rolling exactly one green **(M1)**

recognition that winning $\$10$ also means rolling at most 1 red **(M1)**
eg "cannot have 2 or more reds"

correct approach **A1**
eg $\mathrm{P}(1\mathrm{G} \cap 0\mathrm{R}) + \mathrm{P}(1\mathrm{G} \cap 1\mathrm{R})$, $\mathrm{P}(1\mathrm{G}) - \mathrm{P}(1\mathrm{G} \cap 2\mathrm{R})$,
"one green and two yellows or one of each colour"

Note: Because this is a "show that" question, do not award this ***A1*** for purely numerical expressions.

one correct probability for their approach **(A1)**
eg $3\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^2$, $\frac{6}{27}$, $3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2$, $\frac{1}{9}$, $\frac{2}{9}$

correct working leading to $\frac{1}{3}$ **A1**
eg $\frac{3}{27}+\frac{6}{27}$, $\frac{12}{27}-\frac{3}{27}$, $\frac{1}{9}+\frac{2}{9}$

probability $=\frac{1}{3}$ **AG** **N0**

[5 marks]

(c) (i) $x=\frac{7}{27}$, 0.259 (check ***FT*** from (a)(ii)) **A1** **N1**

(ii) evidence of summing probabilities to 1 **(M1)**
eg $\Sigma = 1$, $x+y+\frac{1}{3}+\frac{2}{9}+\frac{1}{27}=1$, $1-\frac{7}{27}-\frac{9}{27}-\frac{6}{27}-\frac{1}{27}$

0.148147 (0.148407 if working with **their** x value to 3 sf)
$y=\frac{4}{27}$ (exact), 0.148 **A1** **N2**

[3 marks]

(d) correct substitution into the formula for expected value **(A1)**
eg $-w\cdot\frac{7}{27}+10\cdot\frac{9}{27}+20\cdot\frac{6}{27}+30\cdot\frac{1}{27}$

correct critical value (accept inequality) **A1**
eg $w=34.2857\left(=\frac{240}{7}\right)$, $w>34.2857$

$\$40$ **A1** **N2**
[3 marks]

Total [16 marks]

Question 60

(a) valid approach **(M1)**

eg correct value for a or b (ignore incorrect labels)

$a = 6.92986,\ b = 8.80769$

$a = 6.93,\ b = 8.81$ (accept $y = 6.93x + 8.81$) **A1A1** **N3**

[3 marks]

(b) valid approach **(M1)**

eg $750 = x + y$, edge + interior = 750

correct working **(A1)**

eg $750 - x = 6.9298x + 8.807$, 93.4684

93 (pieces) (accept 94) **A1** **N3**

[3 marks]

Total [6 marks]

Question 61

(a) evidence of finding $\frac{\sum x}{n}$ **(M1)**

eg $\frac{0.3 + 0.4 + 3 + \ldots + 10}{10}, \frac{48.2}{10}$

$\overline{x} = 4.82$ (exact) **A1** **N2**

[2 marks]

(b) $p = 4.25$ (exact) **A1** **N1**

[1 mark]

(c) valid approach **(M1)**

eg $Q_3 - Q_1$ $3 - 8$, 3 to 8

$\text{IQR} = 5$ **A1** **N2**

[2 marks]

Total [5 marks]

Question 62

(a) valid approach **(M1)**
eg $\mathrm{P}(X < 275)$, $1 - 0.158655$

0.841344
0.841 **A1** **N2**
[2 marks]

(b) valid approach **(M1)**
eg $\mathrm{P}(X < 275) - \mathrm{P}(X < m) = 0.830$

correct working **(A1)**
eg $\mathrm{P}(X < m) = 0.0113447$

225.820
226 (minutes) **A1** **N3**
[3 marks]

(c) (i) evidence of recognizing binomial distribution (seen anywhere) **(M1)**
eg ${}_nC_a \times p^a \times q^{n-a}$, $\mathrm{B}(n, p)$

evidence of summing probabilities from 7 to 12 **(M1)**
eg $\mathrm{P}(X = 7) + \mathrm{P}(X = 8) + \ldots + \mathrm{P}(X = 12)$, $1 - \mathrm{P}(X \leq 6)$

0.991248
0.991 **A1** **N2**

(ii) finding $\mathrm{P}(X=10)$ (seen anywhere) ***A1***

eg $\binom{12}{10}\times 0.83^{10}\times 0.17^{2}\ \ (=0.295952)$

recognizing conditional probability ***(M1)***

eg $\mathrm{P}(A\,|\,B),\ \mathrm{P}(X=10\,|\,X\geq 7),\ \dfrac{\mathrm{P}(X=10\cap X\geq 7)}{\mathrm{P}(X\geq 7)}$

correct working ***(A1)***

eg $\dfrac{0.295952}{0.991248}$

0.298565
0.299 ***A1*** ***N1***

Note: **Exception to the *FT* rule:** if the candidate uses an incorrect value for the probability that a flight is on time in (i) and working shown, award full ***FT*** in (ii) as appropriate.

[7 marks]

(d) correct equation ***(A1)***

eg $\binom{20}{19}p^{19}(1-p)+p^{20}=0.788$

valid attempt to solve ***(M1)***
eg graph

0.956961
0.957 ***A1*** ***N1***

[3 marks]

Total [15 marks]

Question 63

(a) correct approach **A1**
eg $0.2+0.5+b+a=1$, $0.7+a+b=1$

$b=0.3-a$ **AG** **N0**
[1 mark]

(b) correct substitution into $\mathrm{E}(X)$ **(A1)**
eg $0.2+4\times0.5+a\times b+(a+b-0.5)\times a$, $0.2+2+a\times b-0.2a$

valid attempt to express $\mathrm{E}(X)$ in one variable **M1**
eg $0.2+4\times0.5+a\times(0.3-a)+(-0.2)\times a$, $2.2+0.1a-a^2$,
$0.2+4\times0.5+(0.3-b)\times b+(-0.2)\times(0.3-b)$, $2.14+0.5b-b^2$

correct value of greatest $\mathrm{E}(X)$ **(A1)**
2.2025 (exact)

valid attempt to find least value **(M1)**
eg graph with minimum indicated, $\mathrm{E}(0)$ **and** $\mathrm{E}(0.3)$,
$(0,\ 2.2)$ **and** $(0.3,\ 2.14)$ if $\mathrm{E}(X)$ in terms of a
$(0,\ 2.14)$ **and** $(0.3,\ 2.2)$ if $\mathrm{E}(X)$ in terms of b

correct value of least $\mathrm{E}(X)$ **(A1)**
eg 2.14 (exact)

difference = 0.0625 (exact) **A1** **N2**

[6 marks]

Total [7 marks]

Question 64

(a) evidence of set up **(M1)**
eg correct value for a or b (accept $r=0.966856$)

4.30161, 163.330
$a=4.30$, $b=163$ (accept $y=4.30x+163$) **A1A1** **N3**
[3 marks]

(b) valid approach **(M1)**
eg $4.30(154)+163$

825.778 (825.2 from 3 sf values) **(A1)**

number of messages = 826 (must be an integer) **A1** **N3**
[3 marks]

Total [6 marks]