Subject – Math (Standard Level)
Topic - Statistics and Probability
Year - Nov 2011 – Nov 2019
Paper -1

Question 1

Note: In this question, method marks may be awarded for selecting without replacement, as noted in the examples.

(a) $P(R)=\frac{6}{8}\left(=\frac{3}{4}\right)$ ***A1*** ***N1***

[1 mark]

(b) attempt to find $P(\text{Red})\times P(\text{Red})$ ***(M1)***

e.g. $P(R)\times P(R), \frac{3}{4}\times\frac{3}{4}, \frac{6}{8}\times\frac{5}{7}$

$P(2R)=\frac{36}{64}\left(=\frac{9}{16}\right)$ ***A1*** ***N2***

[2 marks]

(c) **METHOD 1**

attempt to find $P(\text{Red})\times P(\text{Blue})$ ***(M1)***

e.g. $P(R)\times P(B), \frac{6}{8}\times\frac{2}{8}, \frac{6}{8}\times\frac{2}{7}$

recognizing two ways to get one red, one blue ***(M1)***

e.g. $P(RB)+P(BR), 2\left(\frac{12}{64}\right), \frac{6}{8}\times\frac{2}{7}+\frac{2}{8}\times\frac{6}{7}$

$P(1R,1B)=\frac{24}{64}\left(=\frac{3}{8}\right)$ ***A1*** ***N2***

[3 marks]

METHOD 2

recognizing that $P(1R,1B)$ is $1-P(2B)-P(2R)$ ***(M1)***

attempt to find $P(2R)$ and $P(2B)$ ***(M1)***

e.g. $P(2R)=\frac{3}{4}\times\frac{3}{4}, \frac{6}{8}\times\frac{5}{7}$; $P(2B)=\frac{1}{4}\times\frac{1}{4}, \frac{2}{8}\times\frac{1}{7}$

$P(1R,1B)=\frac{24}{64}\left(=\frac{3}{8}\right)$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

Question 2

correct substitution into $\mathrm{E}(X)=\sum px$ (seen anywhere) **A1**

e.g. $1s+2\times0.3+3q=1.7$, $s+3q=1.1$

recognizing $\sum p=1$ (seen anywhere) **(M1)**

correct substitution into $\sum p=1$ **A1**

e.g. $s+0.3+q=1$

attempt to solve simultaneous equations **(M1)**

correct working **(A1)**

e.g. $0.3+2q=0.7, 2s=1$

$q=0.2$ **A1** **N4**

[6 marks]

Question 3

(a) evidence of valid approach **(M1)**

e.g. $92+52$, line on graph at $x=31$

$p=144$ **A1** **N2**

[2 marks]

(b) (i) evidence of valid approach **(M1)**

e.g. line on graph, 0.8×160, using complement

$=29.5$ **A1** **N2**

(ii) $Q_1=23; Q_3=29$ **(A1)(A1)**

IQR $=6$ (accept any notation that suggests an interval) **A1** **N3**

[5 marks]

Total [7 marks]

Question 4

(a) $t = 0.3$ *A1* *N1*

[1 mark]

(b) (i) correct values *A1*
e.g. $0.3 + 0.6 - 0.7$; $0.9 - 0.7$

$r = 0.2$ *AG* *N0*

(ii) $q = 0.1$, $s = 0.4$ *A1A1* *N2*

[3 marks]

(c) (i) 0.4 *A1* *N1*

(ii) $\mathrm{P}(A|B') = \frac{1}{4}$ *A2* *N2*

[3 marks]

Total [7 marks]

Question 5

(a) evidence of median position *(M1)*
e.g. 50, line on sketch

median is 56 *A1* *N2*

[2 marks]

(b) lower quartile $= 40$, upper quartile $= 70$ *(A1)(A1)*

interquartile range $= 30$ *A1* *N3*

[3 marks]

Total [5 marks]

Question 6

(a) attempt to substitute $P(X>1)=0.5$ ***(M1)***

e.g. $r+0.2=0.5$

$r=0.3$ ***A1*** ***N2***

[2 marks]

(b) correct substitution into $E(X)$ (seen anywhere) ***(A1)***

e.g. $0\times p+1\times q+2\times r+3\times 0.2$

correct equation ***A1***

e.g. $q+2\times 0.3+3\times 0.2=1.4,\ q+1.2=1.4$

$q=0.2$ ***A1*** ***N1***

evidence of choosing $\sum p_i=1$ ***M1***

e.g. $p+0.2+0.3+0.2=1,\ p+q=0.5$

correct working ***(A1)***

$p+0.7=1, 1-0.2-0.3-0.2\ ,\ p+0.2=0.5$

$p=0.3$ ***A1*** ***N2***

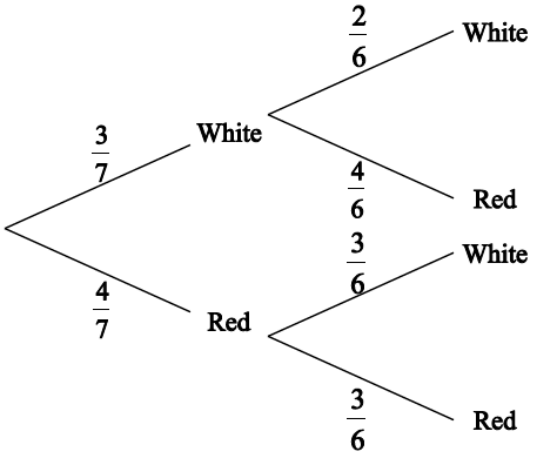
Note: Exception to the ***FT*** rule. Award ***FT*** marks on an incorrect value of q, even if q is an inappropriate value. Do not award the final ***A*** mark for an inappropriate value of p.

[6 marks]

Total [8 marks]

Question 7

(a) (i)



$\frac{4}{6}, \frac{3}{6}$ and $\frac{3}{6}$ $\left(\frac{2}{3}, \frac{1}{2}$ and $\frac{1}{2}\right)$ ***A1A1A1*** ***N3***

(ii) multiplying along the correct branches (may be seen on diagram) ***(A1)***

e.g. $\frac{3}{7} \times \frac{2}{6}$

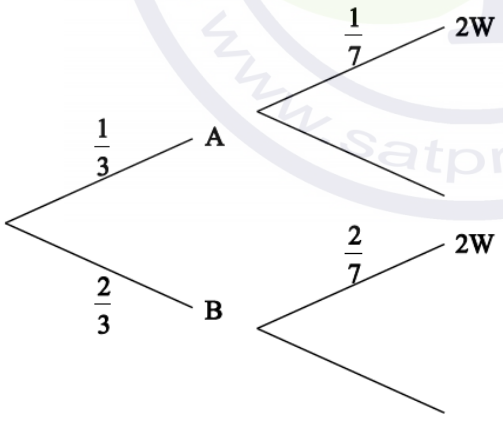
$\frac{6}{42}$ $\left(=\frac{1}{7}\right)$ ***A1*** ***N2***

[5 marks]

(b) $\mathrm{P}(\text{bag A}) = \frac{2}{6}$ $\left(=\frac{1}{3}\right)$, $\mathrm{P}(\text{bag B}) = \frac{4}{6}$ $\left(=\frac{2}{3}\right)$ (seen anywhere) ***(A1)(A1)***

appropriate approach ***(M1)***

e.g. $\mathrm{P}(WW \cap A) + \mathrm{P}(WW \cap B)$



correct calculation ***A1***

e.g. $\frac{1}{3} \times \frac{1}{7} + \frac{2}{3} \times \frac{2}{7}, \frac{2}{42} + \frac{8}{42}$

$\mathrm{P}(2W) = \frac{60}{252}$ $\left(=\frac{5}{21}\right)$ ***A1*** ***N3***

[5 marks]

(c) recognizing conditional probability **(M1)**

e.g. $\frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$, $\mathrm{P}(A \mid WW) = \frac{\mathrm{P}(WW \cap A)}{\mathrm{P}(WW)}$

correct numerator **(A1)**

e.g. $\mathrm{P}(A \cap WW) = \frac{6}{42} \times \frac{2}{6}$, $\frac{1}{21}$

correct denominator **(A1)**

e.g. $\frac{60}{252}, \frac{5}{21}$

probability $\frac{84}{420} \left(= \frac{1}{5} \right)$ **A1** **N3**

[4 marks]

Question 8

(a) evidence of summing to 1 **(M1)**

e.g. $\sum p = 1$, $0.3 + k + 2k + 0.1 = 1$

correct working **(A1)**

e.g. $0.4 + 3k$, $3k = 0.6$

$k = 0.2$ **A1** **N2**

[3 marks]

(b) correct substitution into $\mathrm{E}(X)$ formula **(A1)**

e.g. $0(0.3) + 2(k) + 5(2k) + 9(0.1), 12k + 0.9$

correct working **(A1)**

e.g. $0(0.3) + 2(0.2) + 5(0.4) + 9(0.1)$, $0.4 + 2.0 + 0.9$

$\mathrm{E}(X) = 3.3$ **A1** **N2**

[3 marks]

Total [6 marks]

Question 9

(a) (i) median weekly wage = 400 (dollars) *A1* *N1*

(ii) lower quartile = 330, upper quartile = 470 *(A1)(A1)*

IQR = 140 (dollars) (accept any notation suggesting interval 330 to 470) *A1* *N3*

Note: Exception to the ***FT*** rule. Award ***A1(FT)*** for an incorrect IQR **only** if both quartiles are explicitly noted.

[4 marks]

(b) (i) 330 (dollars) *A1* *N1*

(ii) 400 (dollars) *A1* *N1*

(iii) 700 (dollars) *A1* *N1*

[3 marks]

(c) valid approach *(M1)*

e.g. $\text{hours} = \frac{\text{wages}}{\text{rate}}$

correct substitution *(A1)*

e.g. $\frac{400}{20}$

median hours per week = 20 *A1* *N2*

[3 marks]

(d) attempt to find wages for 25 hours per week *(M1)*

e.g. wages = hours × rate

correct substitution *(A1)*

e.g. 25×20

finding wages = 500 *(A1)*

65 people (earn ≤ 500) *(A1)*

15 people (work more than 25 hours) *A1* *N3*

[5 marks]

Total [15 marks]

Question 10

(a) (i) attempt to find $\mathrm{P}(\text{red}) \times \mathrm{P}(\text{red})$ ***(M1)***

eg $\frac{3}{8} \times \frac{2}{7}, \frac{3}{8} \times \frac{3}{8}, \frac{3}{8} \times \frac{2}{8}$

$\mathrm{P}(\text{none green}) = \frac{6}{56} \left(= \frac{3}{28}\right)$ ***A1*** ***N2***

(ii) attempt to find $\mathrm{P}(\text{red}) \times \mathrm{P}(\text{green})$ ***(M1)***

eg $\frac{5}{8} \times \frac{3}{7}, \frac{3}{8} \times \frac{5}{8}, \frac{15}{56}$

recognizing two ways to get one red, one green ***(M1)***

eg $2\mathrm{P}(R) \times \mathrm{P}(G), \frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7}, \frac{3}{8} \times \frac{5}{8} \times 2$

$\mathrm{P}(\text{exactly one green}) = \frac{30}{56} \left(= \frac{15}{28}\right)$ ***A1*** ***N2***

[5 marks]

(b) $\mathrm{P}(\text{both green}) = \frac{20}{56}$ (seen anywhere) ***(A1)***

correct substitution into formula for $\mathrm{E}(X)$ ***A1***

eg $0 \times \frac{6}{56} + 1 \times \frac{30}{56} + 2 \times \frac{20}{56}, \frac{30}{64} + \frac{50}{64}$

expected number of green marbles is $\frac{70}{56} \left(= \frac{5}{4}\right)$ ***A1*** ***N2***

[3 marks]

(c) (i) $\mathrm{P}(\text{jar B}) = \frac{4}{6} \left(= \frac{2}{3}\right)$ ***A1*** ***N1***

(ii) $\mathrm{P}(\text{red} \mid \text{jar B}) = \frac{6}{8} \left(= \frac{3}{4}\right)$ ***A1*** ***N1***

[2 marks]

(d) recognizing conditional probability ***(M1)***

eg $\mathrm{P}(A|R)$, $\frac{\mathrm{P}(\text{jar A and red})}{\mathrm{P}(\text{red})}$, tree diagram

attempt to multiply along either branch (may be seen on diagram) ***(M1)***

eg $\mathrm{P}(\text{jar A and red}) = \frac{1}{3} \times \frac{3}{8} \left(= \frac{1}{8}\right)$

attempt to multiply along **other** branch ***(M1)***

eg $\mathrm{P}(\text{jar B and red}) = \frac{2}{3} \times \frac{6}{8} \left(= \frac{1}{2}\right)$

adding the probabilities of two mutually exclusive paths ***(A1)***

eg $\mathrm{P}(\text{red}) = \frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{6}{8} \left(= \frac{5}{8}\right)$

correct substitution

eg $\mathrm{P}(\text{jar A}|\text{red}) = \frac{\frac{1}{3} \times \frac{3}{8}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{6}{8}}, \frac{\frac{1}{8}}{\frac{5}{8}}$ ***A1***

$\mathrm{P}(\text{jar A}|\text{red}) = \frac{1}{5}$ ***A1*** ***N3***

[6 marks]

Total [16 marks]

Question 11

(a) attempt to find p ***(M1)***
eg $120-70,\ 50+20+x=120$

$p=50$ ***A1*** ***N2***

attempt to find q ***(M1)***
eg $180-20,\ 200-20-20$

$q=160$ ***A1*** ***N2***
[4 marks]

(b) (i) $\frac{70}{200}\ \left(=\frac{7}{20}\right)$ ***A1*** ***N1***

(ii) valid approach ***(M1)***
eg $20+20,\ 200-160$

$\frac{40}{200}\ \left(=\frac{1}{5}\right)$ ***A1*** ***N2***
[3 marks]

(c) (i) attempt to find number of girls ***(M1)***
eg $0.4,\ \frac{40}{100}\times 200$

80 are not selected ***A1*** ***N2***

(ii) 120 are selected ***(A1)***
$x=20$ ***A1*** ***N2***
[4 marks]

(d) (i) 30 given second chance ***A1*** ***N1***

(ii) 20 took less than 20 minutes ***(A1)***

attempt to find **their** selected total (may be seen in % calculation) ***(M1)***
eg $120+20\ (=140),\ 120+$ their answer from (d)(i)

70 (%) ***A1*** ***N3***
[4 marks]

Total [15 marks]

Question 12

(a) attempt to find number who took less than 45 minutes **(M1)**
eg line on graph (vertical at approx 45, or horizontal at approx 70)

70 students (accept 69) **A1** **N2**
[2 marks]

(b) 55 students completed task in less than 35 minutes **(A1)**

subtracting **their** values **(M1)**
eg $70 - 55$

15 students **A1** **N2**
[3 marks]

(c) correct approach **(A1)**
eg line from y-axis on 50

$k = 33$ **A1** **N2**
[2 marks]

[Total 7 marks]

Question 13

recognize need for intersection of Y and F **(R1)**
eg $\mathrm{P}(Y \cap F)$, 0.3×0.4

valid approach to find $\mathrm{P}(Y \cap F)$ **(M1)**
eg $\mathrm{P}(Y) + \mathrm{P}(F) - \mathrm{P}(Y \cup F)$, Venn diagram

correct working (may be seen in Venn diagram) **(A1)**
eg $0.4 + 0.3 - 0.6$

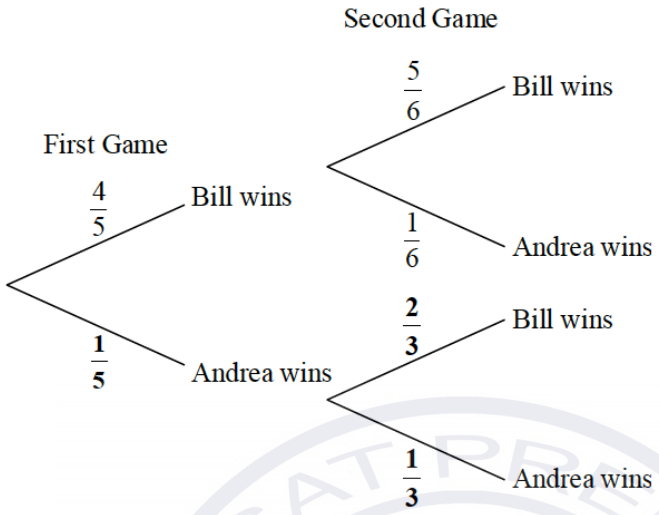
$\mathrm{P}(Y \cap F) = 0.1$ **A1**

recognize need for complement of $Y \cap F$ **(M1)**
eg $1 - \mathrm{P}(Y \cap F)$, $1 - 0.1$

$\mathrm{P}\left((Y \cap F)'\right) = 0.9$ **A1** **N3**
[6 marks]

Question 14

(a)



A1A1A1 *N3*

Note: Award *A1* for each correct **bold** probability.

[3 marks]

(b) multiplying along the branches (may be seen on diagram) *(M1)*

eg $\frac{4}{5}\times\frac{1}{6}$

$\frac{4}{30}\left(\frac{2}{15}\right)$ *A1* *N2*

[2 marks]

(c) **METHOD 1**

multiplying along the branches (may be seen on diagram) *(M1)*

eg $\frac{4}{5}\times\frac{5}{6}, \frac{4}{5}\times\frac{1}{6}, \frac{1}{5}\times\frac{2}{3}$

adding their probabilities of three mutually exclusive paths *(M1)*

eg $\frac{4}{5}\times\frac{5}{6}+\frac{4}{5}\times\frac{1}{6}+\frac{1}{5}\times\frac{2}{3}, \frac{4}{5}+\frac{1}{5}\times\frac{2}{3}$

correct simplification *(A1)*

eg $\frac{20}{30}+\frac{4}{30}+\frac{2}{15}, \frac{2}{3}+\frac{2}{15}+\frac{2}{15}$

$\frac{28}{30}\left(=\frac{14}{15}\right)$ *A1* *N3*

METHOD 2

recognizing "Bill wins at least one" is complement of "Andrea wins 2" ***(R1)***

eg finding P (Andrea wins 2)

P (Andrea wins both) $= \frac{1}{5} \times \frac{1}{3}$ ***(A1)***

evidence of complement ***(M1)***

eg $1-p$, $1-\frac{1}{15}$

$\frac{14}{15}$ ***A1*** ***N3***

[4 marks]

(d) P (B wins both) $= \frac{4}{5} \times \frac{5}{6} \left(= \frac{2}{3} \right)$ ***A1***

evidence of recognizing conditional probability ***(R1)***

eg $\mathrm{P}(A \mid B)$, P (Bill wins both |Bill wins at least one), tree diagram

correct substitution ***(A2)***

eg $\frac{\frac{4}{5} \times \frac{5}{6}}{\frac{14}{15}}$

$\frac{20}{28} \left(= \frac{5}{7} \right)$ ***A1*** ***N3***

[5 marks]

Total [14 marks]

Question 15

(a) correct working *(A1)*

eg $1-\frac{1}{6}$

$p=\frac{5}{6}$ *A1* *N2*

[2 marks]

(b) multiplying along correct branches *(A1)*

eg $\frac{1}{2}\times\frac{1}{6}$

$\mathrm{P}(C\cap L)=\frac{1}{12}$ *A1* *N2*

[2 marks]

(c) multiplying along the other branch *(M1)*

eg $\frac{1}{2}\times\frac{1}{3}$

adding probabilities of their 2 mutually exclusive paths *(M1)*

eg $\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{3}$

correct working *(A1)*

eg $\frac{1}{12}+\frac{1}{6}$

$\mathrm{P}(L)=\frac{3}{12}\left(=\frac{1}{4}\right)$ *A1* *N3*

[4 marks]

(d) recognizing conditional probability (seen anywhere) ***(M1)***

eg $\mathrm{P}(C \mid L)$

correct substitution of **their** values into formula ***(A1)***

eg $\dfrac{\frac{1}{12}}{\frac{3}{12}}$

$\mathrm{P}(C \mid L) = \dfrac{1}{3}$ ***A1*** ***N2***

[3 marks]

(e) valid approach ***(M1)***

eg $X \sim \mathrm{B}\left(3, \frac{1}{4}\right)$, $\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^2$, $\binom{3}{1}$, three ways it could happen

correct substitution ***(A1)***

eg $\binom{3}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^2$, $\frac{1}{4}\times\frac{3}{4}\times\frac{3}{4} + \frac{3}{4}\times\frac{1}{4}\times\frac{3}{4} + \frac{3}{4}\times\frac{3}{4}\times\frac{1}{4}$

correct working ***(A1)***

eg $3\left(\frac{1}{4}\right)\left(\frac{9}{16}\right)$, $\frac{9}{64}+\frac{9}{64}+\frac{9}{64}$

$\dfrac{27}{64}$ ***A1*** ***N2***

[4 marks]

Total [15 marks]

Question 16

(a) evidence of summing probabilities to 1 **(M1)**

eg $\frac{5}{20}+\frac{4}{20}+\frac{1}{20}+p=1,\ \sum=1$

correct working **(A1)**

eg $p=1-\frac{10}{20}$

$p=\frac{10}{20}\left(=\frac{1}{2}\right)$ **A1** **N2**

[3 marks]

(b) correct substitution into $\mathrm{E}(X)$ **(A1)**

eg $\frac{4}{20}(q)+\frac{1}{20}(10)+\frac{10}{20}(-3)$

valid reasoning for fair game (seen anywhere, including equation) **(M1)**

eg $\mathrm{E}(X)=0$, points lost = points gained

correct working **(A1)**

eg $4q+10-30=0,\ \frac{4}{20}q+\frac{10}{20}=\frac{30}{20}$

$q=5$ **A1** **N2**

[4 marks]

Total [7 marks]

Question 17

(a) summing probabilities to 1 **(M1)**

eg $\sum=1,\ 3+4+2+x=10$

correct working **(A1)**

$\frac{3}{10}+\frac{4}{10}+\frac{2}{10}+p=1,\ p=1-\frac{9}{10}$

$p=\frac{1}{10}$ **A1** **N3**

[3 marks]

(b) correct substitution into formula for $\mathrm{E}(X)$ **(A1)**

eg $0\left(\frac{3}{10}\right)+\ldots+3(p)$

correct working **(A1)**

eg $\frac{4}{10}+\frac{4}{10}+\frac{3}{10}$

$\mathrm{E}(X)=\frac{11}{10}(1.1)$ **A1** **N2**

[3 marks]

Total [6 marks]

Question 18

(a) recognizing Ann rolls green **(M1)**
eg P(G)

$\frac{3}{8}$ **A1** **N2**

[2 marks]

(b) (i) $p=\frac{4}{8}, q=\frac{5}{8}$ or $q=\frac{4}{8}, p=\frac{5}{8}$ **A1A1** **N2**

(ii) recognizes Ann and Bob lose 9 times **(M1)**

eg $\overline{A_L B_L}\ \overline{A_L B_L} \ldots \overline{A_L B_L}$ 9 times, $\underbrace{\left(\frac{5}{8}\times\frac{4}{8}\right)\times\ldots\times\left(\frac{5}{8}\times\frac{4}{8}\right)}_{9 \text{ times}}$

$k=9$ (seen anywhere) **A1** **N2**

correct working **(A1)**

eg $\left(\frac{5}{8}\times\frac{4}{8}\right)^9\times\frac{3}{8}, \left(\frac{5}{8}\times\frac{4}{8}\right)\times\ldots\times\left(\frac{5}{8}\times\frac{4}{8}\right)\times\frac{3}{8}$

$r=\frac{20}{64}\ \left(=\frac{5}{16}\right)$ **A1** **N2**

[6 marks]

(c) recognize the probability is an infinite sum **(M1)**
eg Ann wins on her 1st roll or 2nd roll or 3rd roll..., S_∞

recognizing GP **(M1)**

$u_1=\frac{3}{8}$ (seen anywhere) **A1**

$r=\frac{20}{64}$ (seen anywhere) **A1**

correct substitution into infinite sum of GP **A1**

eg $\frac{\frac{3}{8}}{1-\frac{5}{16}}, \frac{3}{8}\left(\frac{1}{1-\left(\frac{5}{8}\times\frac{4}{8}\right)}\right), \frac{1}{1-\frac{5}{16}}$

correct working **(A1)**

eg $\frac{\frac{3}{8}}{\frac{11}{16}}, \frac{3}{8}\times\frac{16}{11}$

P (Ann wins) $=\frac{48}{88}\ \left(=\frac{6}{11}\right)$ **A1** **N1**

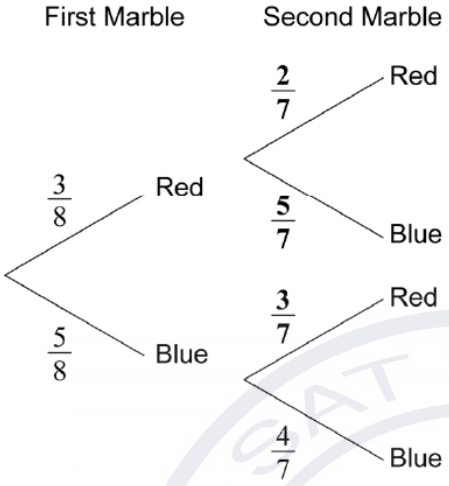
[7 marks]
Total [15 marks]

Question 19

(a) $\frac{3}{8}$ ***A1 N1***

[1 mark]

(b)



A1A1A1 N3

Note: Award ***A1*** for each correct **bold** value.

[3 marks]

(c) multiplying along the blue branches ***(M1)***

eg $\frac{5}{8}\times\frac{4}{7}$

$\frac{20}{56}\left(=\frac{5}{14}\right)$ ***A1 N2***

[2 marks]

Total [6 marks]

Question 20

(a) evidence of approach (may be seen on graph) **(M1)**
eg 80, $(3, 80)$

Note: Award ***M0*** for an incorrect approach such as $\frac{0+6}{2}$, which leads to the correct answer, even if $(3, 80)$ is indicated on graph.

median $= 3$ **A1** **N2**
[2 marks]

(b) (i) $p = 30$ **A1** **N1**

(ii) attempt to set up an expression to find q **(M1)**
eg cumulative frequency for 4.5 indicated on graph

correct expression to find q **(A1)**
eg $160 - 20 - 50 - 30$, $140 - 50 - p$, $140 - 80$

$q = 60$ **A1** **N2**
[4 marks]

Total [6 marks]

Question 21

recognizing fair game (seen anywhere) **(M1)**
eg $\mathrm{E}(X) = 10$, $\mathrm{E}(X) = 0$, money spent = money gained

correct substitution **(A2)**
eg $0(0.6) + k(0.4)$, $0.4(k - 10) + 0.6(-10)$

correct equation **(A2)**
eg $0(0.6) + k(0.4) = 10$, $0.4(k - 10) + 0.6(-10) = 0$, $k(0.4) = 10$

correct work towards solving equation **(A1)**
eg $k = \frac{10}{0.4}, \frac{100}{4}$

$k = 25$ **A1** **N3**
[7 marks]

Question 22

(a) 60 **A1** **N1**
[1 mark]

(b) (i) valid approach **(M1)**
eg $\max - \min = \text{range},\ c = 40 + 47$

$c = 87$ **A1** **N2**

(ii) valid approach **(M1)**
eg $Q3 - Q1 = IQR,\ 74 - 22$

$d = 52$ **A1** **N2**
[4 marks]

Total [5 marks]

Question 23

(a) (i) $q = 0.1$ **A1** **N1**

(ii) appropriate approach **(M1)**
eg $\mathrm{P}(A) - q,\ 0.4 - 0.1$

$p = 0.3$ **A1** **N2**
[3 marks]

(b) valid approach **(M1)**
eg $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B),\ \mathrm{P}(A \cap B) + \mathrm{P}(B \cap A')$

correct values **(A1)**
eg $0.8 = 0.4 + \mathrm{P}(B) - 0.1,\ 0.1 + 0.4$

$\mathrm{P}(B) = 0.5$ **A1** **N2**
[3 marks]

Total [6 marks]

Question 24

(a) valid approach **(M1)**

eg between 10th and 11th, $\frac{8+8}{2}$

median = 38 **A1** **N2**

[2 marks]

(b) (i) $a = 20$ **A1** **N1**

(ii) valid approach **(M1)**

eg $Q_3 - Q_1,\ Q_1 + 14,\ b - 30 = 14$

$b = 44$ **A1** **N2**

[3 marks]

(c) valid approach **(M1)**

eg $40 \times 20,\ \frac{x + 745}{20},\ 40 - \frac{745}{20}$

correct working **(A1)**

eg $800 - 745,\ 20 \times 2.75$

55 (more cans) **A1** **N2**

[3 marks]

(d) (i) most cans in Sam's class = 50 **(A1)**

5 ($) **A1** **N2**

(ii) correct value of 64 or 16 **A1**

valid approach **(M1)**

eg $\frac{64}{80},\ 80\%,\ 80 - 64,\ \frac{16}{80}$

20% **A1** **N2**

[5 marks]

(e) (i) 41.4 (exact) **A1** **N1**

(ii) 18.5 **A1** **N1**

[2 marks]

Total [15 marks]

Question 25

(a) correct approach **(A1)**

eg $\frac{60}{10}$

mean $= 6$ **A1** **N2**

[2 marks]

(b) (i) new mean $= 24$ **A1** **N1**

(ii) valid approach **(M1)**

eg variance $\times (4)^2$, 3×16, new standard deviation $= 4\sqrt{3}$

new variance $= 48$ **A1** **N2**

[3 marks]

Total [5 marks]

Question 26

(a) (i) $p = 3$ **A1** **N1**

(ii) valid approach **(M1)**
eg $(12+10+3)-21,\ 22-18$

$q = 4$ **A1** **N2**

(iii) $r = 8,\ s = 6$ **A1A1** **N2**

[5 marks]

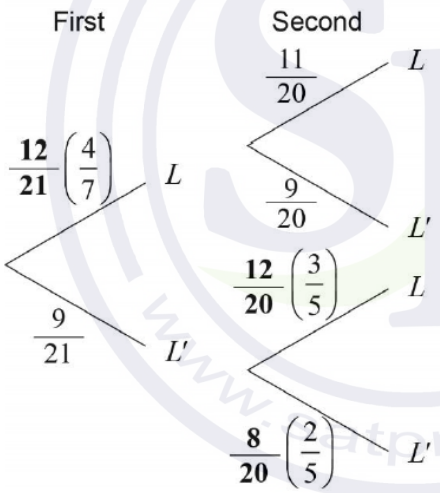
(b) (i) $\frac{12}{21}\left(=\frac{4}{7}\right)$ **A2** **N2**

(ii) valid approach **(M1)**
eg $8+6,\ r+s$

$\frac{14}{21}\left(=\frac{2}{3}\right)$ **A1** **N2**

[4 marks]

(c) (i)



A1A1A1 **N3**

Note: Award ***A1*** for each correct **bold** answer.

(ii) $\frac{11}{20}$ **A1** **N1**

[4 marks]

Total [13 marks]

Question 27

(a) valid interpretation (may be seen on a Venn diagram) ***(M1)***
eg $\mathrm{P}(A\cap B)+\mathrm{P}(A'\cap B)$, $0.2+0.6$

$\mathrm{P}(B)=0.8$ ***A1*** ***N2***
[2 marks]

(b) valid attempt to find $\mathrm{P}(A)$ ***(M1)***
eg $\mathrm{P}(A\cap B)=\mathrm{P}(A)\times\mathrm{P}(B)$, $0.8\times A=0.2$

correct working for $\mathrm{P}(A)$ ***(A1)***
eg 0.25, $\frac{0.2}{0.8}$

correct working for $\mathrm{P}(A\cup B)$ ***(A1)***
eg $0.25+0.8-0.2, 0.6+0.2+0.05$

$\mathrm{P}(A\cup B)=0.85$ ***A1*** ***N3***
[4 marks]

[Total 6 marks]

Question 28

(a) (i) t ***A1*** ***N1***

(ii) 105 ***A1*** ***N1***
[2 marks]

(b) -0.992 ***A2*** ***N2***
[2 marks]

(c) valid approach ***(M1)***

eg $\frac{\mathrm{d}d}{\mathrm{d}t}=-2.24;\ 2\times2.24,\ 2\times-2.24,\ d(2)=-2\times2.24+105,$
finding $d(t_2)-d(t_1)$ where $t_2=t_1+2$

4.48 (degrees) ***A1*** ***N2***

Notes:	Award no marks for answers that **directly** use the table to find the decrease in temperature for 2 minutes *eg* $\frac{105-98.4}{2}=3.3$.

[2 marks]

Total [6 marks]

Question 29

(a) evidence of summing to 1 ***(M1)***

eg $\sum p = 1$

correct equation ***A1***

eg $\cos\theta + 2\cos 2\theta = 1$

correct equation in $\cos\theta$ ***A1***

eg $\cos\theta + 2\left(2\cos^2\theta - 1\right) = 1$, $4\cos^2\theta + \cos\theta - 3 = 0$

evidence of valid approach to solve quadratic ***(M1)***

eg factorizing equation set equal to 0, $\frac{-1 \pm \sqrt{1 - 4 \times 4 \times (-3)}}{8}$

correct working, clearly leading to required answer ***A1***

eg $(4\cos\theta - 3)(\cos\theta + 1)$, $\frac{-1 \pm 7}{8}$

correct reason for rejecting $\cos\theta \neq -1$ ***R1***

eg $\cos\theta$ is a probability (value must lie between 0 and 1), $\cos\theta > 0$

Note: Award ***R0*** for $\cos\theta \neq -1$ without a reason.

$\cos\theta = \frac{3}{4}$ ***AG*** ***N0***

[6 marks]

(b) valid approach ***(M1)***

eg sketch of right triangle with sides 3 and 4, $\sin^2 x + \cos^2 x = 1$

correct working ***(A1)***

eg missing side $= \sqrt{7}$, $\frac{\frac{\sqrt{7}}{4}}{\frac{3}{4}}$

$\tan\theta = \frac{\sqrt{7}}{3}$ ***A1*** ***N2***

[3 marks]

(c) attempt to substitute either limits or the function into formula involving f^2 **(M1)**

eg $\pi\int_{\theta}^{\frac{\pi}{4}} f^2$, $\int\left(\frac{1}{\cos x}\right)^2$

correct substitution of both limits and function **(A1)**

eg $\pi\int_{\theta}^{\frac{\pi}{4}}\left(\frac{1}{\cos x}\right)^2 \mathrm{dx}$

correct integration **(A1)**

eg $\tan x$

substituting **their** limits into **their** integrated function and subtracting **(M1)**

eg $\tan\frac{\pi}{4} - \tan\theta$

Note: Award ***M0*** if they substitute into original or differentiated function.

$\tan\frac{\pi}{4} = 1$ **(A1)**

eg $1 - \tan\theta$

$V = \pi - \frac{\pi\sqrt{7}}{3}$ **A1** **N3**

[6 marks]

[Total: 15 marks]

Question 30

(a) $\mathrm{P}(X > 107) = 0.24 \left(= \frac{6}{25}, 24\%\right)$ **A1** **N1**

[1 mark]

(b) valid approach **(M1)**

eg $\mathrm{P}(X > 100) = 0.5$, $\mathrm{P}(X > 100) - \mathrm{P}(X > 107)$

correct working **(A1)**

eg $0.5 - 0.24$, $0.76 - 0.5$

$\mathrm{P}(100 < X < 107) = 0.26 \left(= \frac{13}{50}, 26\%\right)$ **A1** **N2**

[3 marks]

(c) valid approach **(M1)**

eg 2×0.26, $1 - 2(0.24)$, $\mathrm{P}(93 < X < 100) = \mathrm{P}(100 < X < 107)$

$\mathrm{P}(93 < X < 107) = 0.52 \left(= \frac{13}{25}, 52\%\right)$ **A1** **N2**

[2 marks]

[Total 6 marks]

Question 31

(a) (i) evidence of median position *(M1)*
eg 80th employee

40 hours *A1* *N2*

(ii) 130 employees *A1* *N1*
[3 marks]

(b) (i) £320 *A1* *N1*

(ii) splitting into 40 and 3 *(M1)*
eg 3 hours more, 3×10

correct working *(A1)*
eg $320+3\times10$

£350 *A1* *N3*
[4 marks]

(c) valid approach *(M1)*
eg 200 is less than 320 so 8 pounds/hour, $200\div8$, 25, $\frac{200}{320}=\frac{x}{40}$

18 employees *A2* *N3*
[3 marks]

(d) valid approach *(M1)*
eg $160-10$

60 hours worked *(A1)*

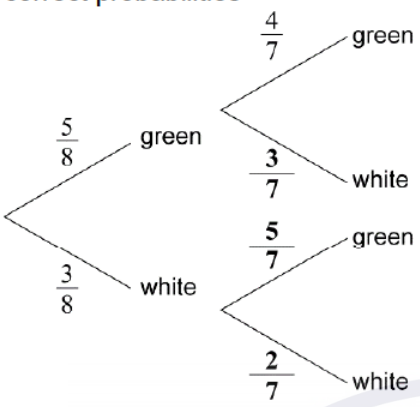
correct working *(A1)*
eg $40(8)+20(10)$, $320+200$

$k=520$ *A1* *N3*
[4 marks]

[Total 14 marks]

Question 32

(a) correct probabilities



A1A1A1 ***N3***

Note: Award ***A1*** for each correct **bold** answer.

[3 marks]

(b) multiplying along branches ***(M1)***

eg $\frac{5}{8}\times\frac{3}{7}$, $\frac{3}{8}\times\frac{5}{7}$, $\frac{15}{56}$

adding probabilities of correct mutually exclusive paths ***(A1)***

eg $\frac{5}{8}\times\frac{3}{7}+\frac{3}{8}\times\frac{5}{7}$, $\frac{15}{56}+\frac{15}{56}$

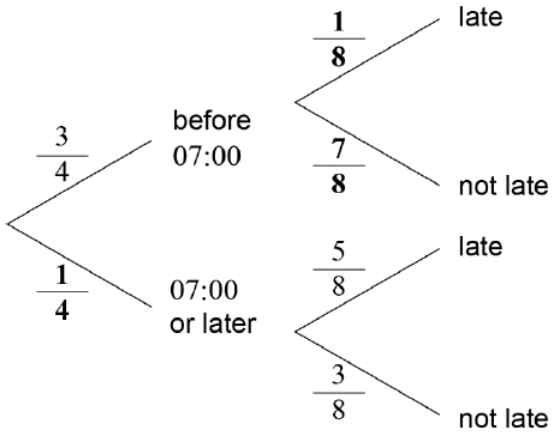
$\frac{30}{56}\left(=\frac{15}{28}\right)$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

Question 33

(a)



A1A1A1 ***N3***

Note: Award ***A1*** for each bold fraction.

[3 marks]

(b) multiplying along correct branches ***(A1)***

eg $\frac{3}{4}\times\frac{1}{8}$

$\mathrm{P}(\text{leaves before } 07{:}00 \cap \text{late}) = \frac{3}{32}$ ***A1*** ***N2***

[2 marks]

(c) multiplying along other "late" branch ***(M1)***

eg $\frac{1}{4}\times\frac{5}{8}$

adding probabilities of two mutually exclusive late paths ***(A1)***

eg $\left(\frac{3}{4}\times\frac{1}{8}\right)+\left(\frac{1}{4}\times\frac{5}{8}\right), \frac{3}{32}+\frac{5}{32}$

$\mathrm{P}(L) = \frac{8}{32}\left(=\frac{1}{4}\right)$ ***A1*** ***N2***

[3 marks]

(d) recognizing conditional probability (seen anywhere) ***(M1)***

eg $\mathrm{P}(A \mid B)$, $\mathrm{P}(\text{before } 7 \mid \text{late})$

correct substitution of **their** values into formula ***(A1)***

eg $\dfrac{\frac{3}{32}}{\frac{1}{4}}$

$\mathrm{P}(\text{left before } 07{:}00 \mid \text{late}) = \dfrac{3}{8}$ ***A1*** ***N2***

[3 marks]

(e) valid approach ***(M1)***

eg $1 - \mathrm{P}(\text{not late twice})$, $\mathrm{P}(\text{late once}) + \mathrm{P}(\text{late twice})$

correct working ***(A1)***

eg $1 - \left(\frac{3}{4} \times \frac{3}{4}\right)$, $2 \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4}$

$\dfrac{7}{16}$ ***A1*** ***N2***

[3 marks]

[Total: 14 marks]

Question 34

(a) correct approach **(A1)**

eg $\frac{800}{n} = 20$

40 **A1** **N2**

[2 marks]

(b) (i) 200 **A1** **N1**

(ii) **METHOD 1**

recognizing variance $= \sigma^2$ **(M1)**

eg $3^2 = 9$

correct working to find new variance **(A1)**

eg $\sigma^2 \times 10^2$, 9×100

900 **A1** **N3**

METHOD 2

new standard deviation is 30 **(A1)**

recognizing variance $= \sigma^2$ **(M1)**

eg $3^2 = 9$, 30^2

900 **A1** **N3**

[4 marks]

[Total: 6 marks]

Question 35

(a) recognizing Q_1 or Q_3 (seen anywhere) **(M1)**

eg 4, 11, indicated on diagram

IQR = 7 **A1** **N2**

[2 marks]

(b) recognizing the need to find 1.5 IQR **(M1)**

eg $1.5 \times \text{IQR}$, 1.5×7

valid approach to find k **(M1)**

eg $10.5 + 11$, $1.5 \times \text{IQR} + Q_3$

21.5 **(A1)**

$k = 22$ **A1** **N3**

Note: If no working shown, award ***N2*** for an answer of 21.5.

[4 marks]

Total [6 marks]

Question 36

(a) (i) $\frac{2}{n}$ **A1** **N1**

(ii) correct probability for one of the draws **A1**

eg $\mathrm{P}(\text{not blue first}) = \frac{n-2}{n}$, blue second $= \frac{2}{n-1}$

valid approach **(M1)**

eg recognizing loss on first in order to win on second,
$\mathrm{P}(B' \text{ then } B)$, $\mathrm{P}(B') \times \mathrm{P}(B \mid B')$, tree diagram

correct expression in terms of n **A1** **N3**

eg $\frac{n-2}{n} \times \frac{2}{n-1}$, $\frac{2n-4}{n^2-n}$, $\frac{2(n-2)}{n(n-1)}$

[4 marks]

(b) (i) correct working **(A1)**

eg $\frac{3}{5} \times \frac{2}{4} \times \frac{2}{3}$

$\frac{12}{60} \left(= \frac{1}{5}\right)$ **A1** **N2**

(ii) correct working **(A1)**

eg $\frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2}$

$\frac{6}{60} \left(= \frac{1}{10}\right)$ **A1** **N2**

[4 marks]

(c) correct probabilities (seen anywhere) ***(A1)(A1)***

eg $\mathrm{P}(1)=\frac{2}{5}$, $\mathrm{P}(2)=\frac{6}{20}$ (may be seen on tree diagram)

valid approach to find $\mathrm{E}(M)$ or expected winnings using **their** probabilities ***(M1)***

eg $\mathrm{P}(1)\times(0)+\mathrm{P}(2)\times(20)+\mathrm{P}(3)\times(8k)+\mathrm{P}(4)\times(12k)$,
$\mathrm{P}(1)\times(-20)+\mathrm{P}(2)\times(0)+\mathrm{P}(3)\times(8k-20)+\mathrm{P}(4)\times(12k-20)$

correct working to find $\mathrm{E}(M)$ or expected winnings ***(A1)***

eg $\frac{2}{5}(0)+\frac{3}{10}(20)+\frac{1}{5}(8k)+\frac{1}{10}(12k)$,
$\frac{2}{5}(-20)+\frac{3}{10}(0)+\frac{1}{5}(8k-20)+\frac{1}{10}(12k-20)$

correct equation for fair game ***A1***

eg $\frac{3}{10}(20)+\frac{1}{5}(8k)+\frac{1}{10}(12k)=20$, $\frac{2}{5}(-20)+\frac{1}{5}(8k-20)+\frac{1}{10}(12k-20)=0$

correct working to combine terms in k ***(A1)***

eg $-8+\frac{14}{5}k-4-2=0$, $6+\frac{14}{5}k=20$, $\frac{14}{5}k=14$

$k=5$ ***A1*** ***N0***

Note: Do not award the final ***A1*** if the candidate's ***FT*** probabilities do not sum to 1.

[7 marks]

Total [15 marks]

Question 37

(a) valid approach **(M1)**
eg $16+8,\ a-8$

24 (hours) **A1** **N2**
[2 marks]

(b) valid approach **(M1)**
eg $20-15,\ Q_3-Q_1,\ 15-20$

IQR $=5$ **A1** **N2**
[2 marks]

(c) correct working **(A1)**
eg $\frac{180}{10}, \frac{180}{n}, \frac{\sum x}{10}$

mean $=18$ (hours) **A1** **N2**
[2 marks]

(d) (i) attempt to find total hours for group B **(M1)**
eg $\overline{x} \times n$

group B total hours $=420$ (seen anywhere) **A1** **N2**

(ii) attempt to find sum for combined group (may be seen in working) **(M1)**
eg $180+420,\ 600$

correct working **(A1)**
eg $\frac{180+420}{30}, \frac{600}{30}$

mean $=20$ (hours) **A1** **N2**

[5 marks]

(e) (i) valid approach to find the new mean **(M1)**

eg $\frac{1}{2}\mu, \frac{1}{2}\times 21$

mean $= \frac{21}{2} (= 10.5)$ (hours) **A1** **N2**

(ii) variance $= \sigma^2$ (seen anywhere) **(A1)**

eg $\sigma^2 = 9$, $3^2 = 9$, $\left(\frac{3}{2}\right)^2$, 3^2

valid attempt to find new standard deviation or variance **(M1)**

eg $\frac{1}{4}\times 3^2$, $\frac{1}{2}\times 3$, $\frac{3}{2}$

variance $= \frac{9}{4} (= 2.25)$ (hours) **A1** **N2**

[5 marks]

Total [16 marks]

Question 38

(a) evidence of using $\sum p = 1$ **(M1)**

correct working **(A1)**

eg $\frac{3}{13}+\frac{1}{13}+\frac{4}{13}+k=1,\ 1-\frac{8}{13}$

$k = \frac{5}{13}$ **A1** **N2**

[3 marks]

(b) valid approach to find $\mathrm{E}(X)$ **(M1)**

eg $1\times\frac{1}{13}+2\times\frac{4}{13}+3\times k$, $0\times\frac{3}{13}+1\times\frac{1}{13}+2\times\frac{4}{13}+3\times\frac{5}{13}$

correct working **(A1)**

eg $\frac{1}{13}+\frac{8}{13}+\frac{15}{13}$

$\mathrm{E}(X) = \frac{24}{13}$ **A1** **N2**

[3 marks]

Total [6 marks]

Question 39

(a) recognizing area under curve = 1 ***(M1)***

eg $a+x+b=1,\ 100-a-b\ ,\ 1-a+b$

$\mathrm{P}(-1.6<z<2.4)=1-a-b\ \ (=1-(a+b))$ ***A1*** ***N2***

[2 marks]

(b) $\mathrm{P}(z>-1.6)=1-a$ (seen anywhere) ***(A1)***

recognizing conditional probability ***(M1)***

eg $\mathrm{P}(A\,|\,B),\ \mathrm{P}(B\,|\,A)$

correct working ***(A1)***

eg $\dfrac{\mathrm{P}(z<2.4\cap z>-1.6)}{\mathrm{P}(z>-1.6)},\ \dfrac{\mathrm{P}(-1.6<z<2.4)}{\mathrm{P}(z>-1.6)}$

$\mathrm{P}(z<2.4\,|\,z>-1.6)=\dfrac{1-a-b}{1-a}$ ***A1*** ***N4***

Note: Do not award the final ***A1*** if correct answer is seen followed by incorrect simplification.

[4 marks]

(c) $z=-1.6$ (may be seen in part (d)) ***A1*** ***N1***

Note: Depending on the candidate's interpretation of the question, they may give $\dfrac{1-m}{s}$ as the answer to part (c). Such answers should be awarded the first ***(M1)*** in part (d), even when part (d) is left blank. If the candidate goes on to show $z=-1.6$ as part of their working in part (d), the ***A1*** in part (c) may be awarded.

[1 mark]

(d) attempt to standardize x (do not accept $\frac{x-\mu}{\sigma}$) **(M1)**

eg $\frac{1-m}{s}$ (may be seen in part (c)), $\frac{m-2}{s}$, $\frac{x-m}{\sigma}$

correct equation with each z-value **(A1)(A1)**

eg $-1.6=\frac{1-m}{s}$, $2.4=\frac{2-m}{s}$, $m+2.4s=2$

valid approach (to set up equation in one variable) **M1**

eg $2.4=\frac{2-(1.6s+1)}{s}$, $\frac{1-m}{-1.6}=\frac{2-m}{2.4}$

correct working **(A1)**

eg $1.6s+1=2-2.4s$, $4s=1$, $m=\frac{7}{5}$

$s=\frac{1}{4}$ **A1** **N2**

[6 marks]

Total [13 marks]

Question 40

(a) valid approach **(M1)**

eg $0.3-0.1$, $p+0.1=0.3$

$p=0.2$ **A1** **N2**

[2 marks]

(b) valid approach **(M1)**

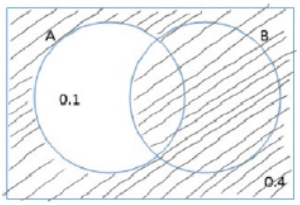
eg $1-(0.3+0.4)$, $1-0.4-0.1-p$

$q=0.3$ **A1** **N2**

[2 marks]

(c) valid approach **(M1)**

eg $0.7+0.5-0.3$, $p+q+0.4$, $1-0.1$, $\mathrm{P}(A'\cup B)=\mathrm{P}(A')+\mathrm{P}(B)-\mathrm{P}(A'\cap B)$,



$\mathrm{P}(A'\cup B)=0.9$ **A1** **N2**

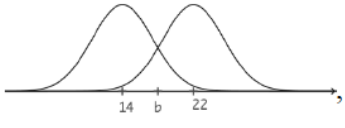
[2 marks]

Total [6 marks]

Question 41

(a) **METHOD 1**

recognizing that b is midway between the means of 14 and 22. ***(M1)***

eg 

, $b = \frac{14+22}{2}$

$b = 18$ ***A1*** ***N2***

METHOD 2

valid attempt to compare distributions ***(M1)***

eg $\frac{b-14}{a} = -\frac{b-22}{a}$, $b-14 = 22-b$

$b = 18$ ***A1*** ***N2***

[2 marks]

(b) valid attempt to compare distributions (seen anywhere) ***(M1)***

eg Y is a horizontal translation of X of 8 units to the right,
$\mathrm{P}(16 < Y < 28) = \mathrm{P}(8 < X < 20)$, $\mathrm{P}(Y > 22+6) = \mathrm{P}(X > 14+6)$

valid approach using symmetry ***(M1)***

eg $1-2\mathrm{P}(X > 20)$, $1-2\mathrm{P}(Y < 16)$, $2\times\mathrm{P}(14 < x < 20)$, $\mathrm{P}(X < 8) = \mathrm{P}(X > 20)$

correct working ***(A1)***

eg $1-2(0.112)$, $2\times(0.5-0.112)$, 2×0.388, $0.888-0.112$

$\mathrm{P}(16 < Y < 28) = 0.776$ ***A1*** ***N3***

[4 marks]

Total [6 marks]

Question 42

(a) $q = 5$ **A1** **N1**

[1 mark]

(b) valid approach **(M1)**

eg $(18+10+5)-30$, $28-25$, $18+10-n=25$

$n = 3$ **A1** **N2**

[2 marks]

(c) valid approach for finding m or p (may be seen in part (b)) **(M1)**

eg $18-3$, $3+p=10$

$m=15$, $p=7$ **A1A1** **N3**

[3 marks]

[Total 6 marks]