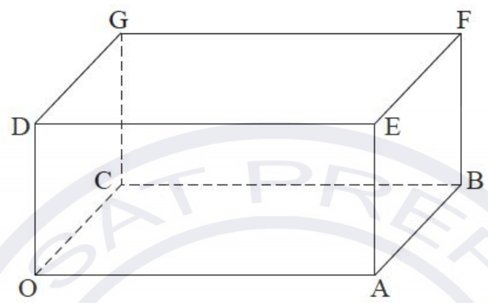
Subject – Math (Standard Level)
Topic - Vector
Year - Nov 2011 – Nov 2019
Paper- 2

Question 1

[Maximum mark: 16]

The following diagram shows the cuboid (rectangular solid) OABCDEFG, where O is the origin, and $\vec{OA} = 4\boldsymbol{i}$, $\vec{OC} = 3\boldsymbol{j}$, $\vec{OD} = 2\boldsymbol{k}$.



(a) (i) Find $\vec{OB}$.

(ii) Find $\vec{OF}$.

(iii) Show that $\vec{AG} = -4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$. *[5 marks]*

(b) Write down a vector equation for

(i) the line OF;

(ii) the line AG. *[4 marks]*

(c) Find the obtuse angle between the lines OF and AG. *[7 marks]*

Question 2

[Maximum mark: 7]

Line L_1 has equation $\boldsymbol{r}_1 = \begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix}$ and line L_2 has equation $\boldsymbol{r}_2 = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$.

Lines L_1 and L_2 intersect at point A. Find the coordinates of A.

Question 3

[Maximum mark: 15]

Consider the points $\mathrm{A}(5, 2, 1)$, $\mathrm{B}(6, 5, 3)$, and $\mathrm{C}(7, 6, a+1)$, where $a \in \mathbb{R}$.

(a) Find

(i) $\overrightarrow{\mathrm{AB}}$;

(ii) $\overrightarrow{\mathrm{AC}}$. *[3 marks]*

Let q be the angle between $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$.

(b) Find the value of a for which $q = \dfrac{\pi}{2}$. *[4 marks]*

(c) (i) Show that $\cos q = \dfrac{2a+14}{\sqrt{14a^2+280}}$.

(ii) Hence, find the value of a for which $q = 1.2$. *[8 marks]*

Question 4

[Maximum mark: 17]

Consider the lines L_1 and L_2 with equations $L_1 : \boldsymbol{r} = \begin{pmatrix} 11 \\ 8 \\ 2 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$ and $L_2 : \boldsymbol{r} = \begin{pmatrix} 1 \\ 1 \\ -7 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 11 \end{pmatrix}$.

The lines intersect at point P.

(a) Find the coordinates of P. *[6]*

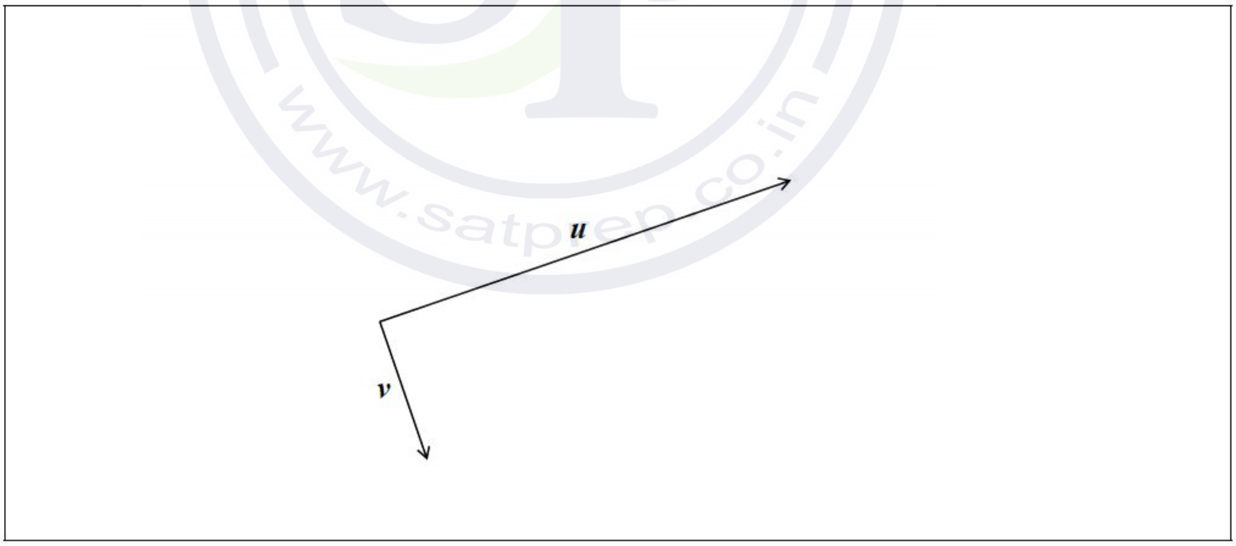
(b) Show that the lines are perpendicular. *[5]*

(c) The point Q(7, 5, 3) lies on L_1. The point R is the reflection of Q in the line L_2. Find the coordinates of R. *[6]*

Question 5

[Maximum mark: 6]

The following diagram shows two perpendicular vectors $\boldsymbol{u}$ and $\boldsymbol{v}$.



(a) Let $\boldsymbol{w} = \boldsymbol{u} - \boldsymbol{v}$. Represent $\boldsymbol{w}$ on the diagram above. *[2]*

(b) Given that $\boldsymbol{u} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 5 \\ n \\ 3 \end{pmatrix}$, where $n \in \mathbb{Z}$, find n. *[4]*

Question 6

[Maximum mark: 7]

Let $\boldsymbol{u} = 6\boldsymbol{i} + 3\boldsymbol{j} + 6\boldsymbol{k}$ and $\boldsymbol{v} = 2\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$.

(a) Find

(i) $\boldsymbol{u} \bullet \boldsymbol{v}$;

(ii) $|\boldsymbol{u}|$;

(iii) $|\boldsymbol{v}|$. [5]

(b) Find the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$. [2]

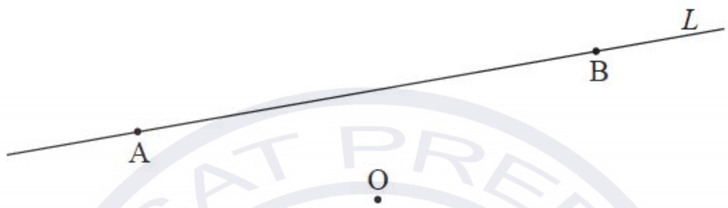
Question 7

[Maximum mark: 16]

The points A and B lie on a line L, and have position vectors $\begin{pmatrix}-3\\-2\\2\end{pmatrix}$ and $\begin{pmatrix}6\\4\\-1\end{pmatrix}$ respectively.

Let O be the origin. This is shown on the following diagram.

diagram not to scale



(a) Find $\overrightarrow{AB}$. [2]

The point C also lies on L, such that $\overrightarrow{AC} = 2\overrightarrow{CB}$.

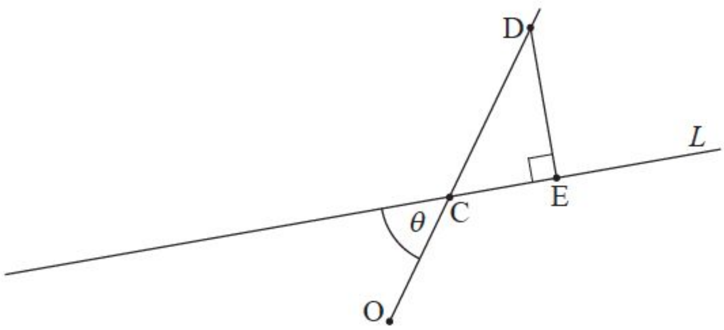
(b) Show that $\overrightarrow{OC} = \begin{pmatrix}3\\2\\0\end{pmatrix}$. [3]

Let θ be the angle between $\overrightarrow{AB}$ and $\overrightarrow{OC}$.

(c) Find θ. [5]

Let D be a point such that $\overrightarrow{OD} = k\overrightarrow{OC}$, where $k > 1$. Let E be a point on L such that $\hat{CED}$ is a right angle. This is shown on the following diagram.

diagram not to scale



(d) (i) Show that $\left|\overrightarrow{DE}\right| = (k-1)\left|\overrightarrow{OC}\right|\sin\theta$.

(ii) The distance from D to line L is less than 3 units. Find the possible values of k. [6]

Question 8

[Maximum mark: 15]

Consider the points $A(1, 5, -7)$ and $B(-9, 9, -6)$.

(a) Find $\vec{AB}$. [2]

Let C be a point such that $\vec{AC} = \begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$.

(b) Find the coordinates of C. [2]

The line L passes through B and is parallel to (AC).

(c) Write down a vector equation for L. [2]

(d) Given that $\left|\vec{AB}\right| = k\left|\vec{AC}\right|$, find k. [3]

(e) The point D lies on L such that $\left|\vec{AB}\right| = \left|\vec{BD}\right|$. Find the possible coordinates of D. [6]

Question 9

[Maximum mark: 6]

Let $\boldsymbol{v} = \begin{pmatrix} -10 \\ 2 \\ 1 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$. Find the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$, giving your answer correct to one decimal place.

Question 10

[Maximum mark: 6]

Let $\vec{AB} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$.

(a) Find $\left|\vec{AB}\right|$. [2]

(b) Let $\vec{AC} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$. Find $B\hat{A}C$. [4]

Question 11

[Maximum mark: 13]

Two points P and Q have coordinates $(3, 2, 5)$ and $(7, 4, 9)$ respectively.

(a) (i) Find $\vec{PQ}$.

(ii) Find $\left|\vec{PQ}\right|$. [4]

Let $\vec{PR} = 6\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$.

(b) Find the angle between PQ and PR. [4]

(c) Find the area of triangle PQR. [2]

(d) Hence or otherwise find the shortest distance from R to the line through P and Q. [3]

Question 12

[Maximum mark: 16]

Consider the points $\mathrm{A}(-3, 4, 2)$ and $\mathrm{B}(8, -1, 5)$.

(a) (i) Find $\vec{\mathrm{AB}}$.

(ii) Find $\left|\vec{\mathrm{AB}}\right|$. [4]

A line L has vector equation $r = \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$. The point $\mathrm{C}(5, y, 1)$ lies on line L.

(b) (i) Find the value of y.

(ii) Show that $\vec{\mathrm{AC}} = \begin{pmatrix} 8 \\ -10 \\ -1 \end{pmatrix}$. [5]

(c) Find the angle between $\vec{\mathrm{AB}}$ and $\vec{\mathrm{AC}}$. [5]

(d) Find the area of triangle ABC. [2]

Question 13

[Maximum mark: 6]

The vector equation of line L is given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} + t\begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix}$.

Point P is the point on L that is closest to the origin. Find the coordinates of P.

Question 14

[Maximum mark: 5]

Consider the lines L_1 and L_2 with respective equations

$$L_1 : y = -\frac{2}{3}x + 9 \text{ and } L_2 : y = \frac{2}{5}x - \frac{19}{5}.$$

(a) Find the point of intersection of L_1 and L_2. [2]

A third line, L_3, has gradient $-\frac{3}{4}$.

(b) Write down a direction vector for L_3. [1]

L_3 passes through the intersection of L_1 and L_2.

(c) Write down a vector equation for L_3. [2]